Extension of Conformal (Super)Symmetry using Heisenberg and Parabose operators*

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Abstract

In this paper we investigate a particular possibility to extend $C(1,3)$ conformal symmetry using Heisenberg operators, and a related possibility to extend conformal supersymmetry using parabose operators. The symmetry proposed is of a simple mathematical form, as is the form of necessary symmetry breaking that reduces it to the conformal (super)symmetry. It turns out that this extension of conformal superalgebra can be obtained from standard non-extended conformal superalgebra by allowing anticommutators $\{Q_\eta, Q_\xi\}$ and $\{\overline{Q}_{\dot\eta}, \overline{Q}_{\dot\xi}\}$ to be nonzero operators and then by closing the algebra. In regard of the famous Coleman and Mandula theorem (and related Haag-Lopuszanski-Sohnius theorem), the higher symmetries that we consider do not satisfy the requirement for finite number of particles with masses below any given constant. However, we argue that in the context of theories with broken symmetries, this constraint may be unnecessarily strong.

1. Introduction

Prospect of finding larger symmetries that would embed observable Poincare symmetry and possibly some of the internal symmetries (in a non trivial way) has attracted physicists for a long time. Early attempts ended in formulation of famous Coleman and Mandula no-go theorem [1], but only to be soon evaded by the idea of supersymmetry. The Coleman and Mandula theorem was then replaced by Haag-Lopuszanski-Sohnius theorem [2] that put the now standard super-Poincare and super-conformal symmetries at the place of maximal supersymmetries of realistic models (up to multiplication by an internal symmetry group). However, the attempts to go around these no-go theorems never truly ceased, mostly trying to weaken the mathematical requirements of the theorems [4, 5].

We will here consider an extension of conformal (super)symmetry which does not meet one of the physical premises of the theorem [1] - namely the "particle finiteness" premise: "for any finite M, there are only finite number

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of particle types with mass less than M". Regarding this requirement, S. Coleman in paper [3] comments: "We would probably be willing to accept a theory with an infinite number of particles, as long as they were spread out in mass in such a way that experiments conducted at limited energy could only detect a finite number of them". Our point is that a proper symmetry breaking can, in principle, induce such mass splitting. Besides, this does not have to imply increasing of the complexity of a theory, since symmetry breaking is already an inescapable component of most of the contemporary physical models.

The mathematical motivation for choice of the particular symmetries that we are going to investigate comes from the fact that $C(1,3)$ conformal symmetry is in an interesting way contained in algebra that is formed by all (hermitian) quadratic polynomials of four pairs of Heisenberg operators [this algebra, constructed in the following section, is isomorphic to $sp(2n)$ where $n = 4$]. Namely, reduction from $sp(8)$ to $c(1,3)$ can be seen as a consequence of one $SU(2)$ to $U(1)$ symmetry breaking. Though this construction based on Heisenberg operators has many interesting properties itself, we will actually use it as an intermediate step, serving to introduce bosonic part of a larger symmetry.

In the next step (section 3.) we will show that by replacing the starting Heisenberg algebra with relations of parabose algebra, we arrive to an extension of super-conformal algebra. Relation of this algebra with standard conformal superalgebra is rather interesting: it is the algebra that is obtained from conformal superalgebra when we remove the algebraic "constraints" $\{Q_\eta, Q_\xi\} = 0$ (allowing these and adjoint anticommutators to be new symmetry generators) and appropriately close the algebra. Intriguing is that enlarging the conformal superalgebra in this way simplifies the algebra instead of complicating it, as the structural relations of the larger symmetry are determined by two defining relations of parabose algebra.

As consecutive action of the same supersymmetry generator no longer annihilates a state, it is obvious that number of particles in a supermultiplet becomes infinite. However, we argue that such an obvious disagreement with experimental data is a problem of the qualitatively same type as occurs in the standard Poincare supersymmetry. Namely, whereas in models with Poincare supersymmetry we need a symmetry breaking to induce mass differences between finite number of super-partners, here the symmetry breaking should provide ascending masses for infinite series of super-partners. We will demonstrate existence of simple form of symmetry breaking that reduces the symmetry down to Poincare group, altogether with providing the mass splitting.

In this paper we do not intend to present details of any realistic model, but rather to point out some theoretical possibilities which are, to our opinion, unrightfully ignored. In the moment when experimental confirmation of existence of supersymmetry is being expected, we should be aware of all possible variations of supersymmetry idea.

Throughout the text, Latin indices $i, j, k, \ldots$ will take values 1, 2 and 3, Greek indices from the beginning of alphabet $\alpha, \beta, \ldots$ will take values from

1 to 4 and will in general denote Dirac-like spinor indices, η and ξ will be two-dimensional Weyl spinor indices, while Greek indices from the middle of alphabet $\mu, \nu, \ldots$ will denote Lorentz four-vector indices.

2. Heisenberg Operators and Extension of Conformal Symmetry

Let operators κ^α and α satisfy Heisenberg algebra in four dimensions[1]:

$$[\kappa^\alpha, \,\beta] = i\delta^\alpha_\beta, \qquad [\kappa^\alpha, \kappa^\beta] = [\,\alpha, \,\beta] = 0. \tag{1}$$

There are three types of quadratic combinations of these operators: quadratic in κ^α, quadratic in α and mixed. Hermitian operators of each of these kinds can be written in matrix notation, respectively as:

$$\begin{aligned} \widehat{(A)}_{\kappa\kappa} &\equiv A_{\alpha\beta}\tfrac{1}{2}\{\kappa^\alpha, \kappa^\beta\}, \\ \widehat{(A)}_{\pi\pi} &\equiv A^{\alpha\beta}\tfrac{1}{2}\{\,\alpha, \,\beta\}, \\ \widehat{(A)}_{\pi\kappa} &\equiv A^\alpha{}_\beta\tfrac{1}{2}\{\,\alpha, \kappa^\beta\}, \end{aligned} \tag{2}$$

where A is an arbitrary four by four real matrix, with restriction that matrices appearing in definitions $\widehat{(A)}_{\kappa\kappa}$ and $\widehat{(A)}_{\pi\pi}$ are implied to be symmetric.[2] Such quadratic operators form an algebra with commutation relations easily derivable from the Heisenberg algebra relations. This *algebra of quadratic operators* $\mathcal{A}_2$ we wish to consider as an extension of conformal algebra.

To reveal the conformal subalgebra in this structure, first we choose a set of six real matrices σ_i and $\tau_{\underline{i}}$, $i, \underline{i} = 1, 2, 3$ (four dimensional analogs of Pauli matrices) satisfying

$$[\sigma_i, \sigma_j] = 2\varepsilon_{ijk}\sigma_k, \qquad [\tau_{\underline{i}}, \tau_{\underline{j}}] = 2\varepsilon_{\underline{i}\underline{j}\underline{k}}\tau_{\underline{k}}, \qquad [\sigma_i, \tau_{\underline{j}}] = 0, \tag{3}$$

as a basis of antisymmetric four by four real matrices[3] (we distinct tau indices from sigma indices by underlining the former). However, unlike Pauli matrices, these matrices are anti-hermitian, satisfying $\sigma_i^2 = \tau_{\underline{i}}^2 = -1$. As a basis for symmetric matrices we choose nine matrices $\alpha_{\underline{i}j} \equiv \tau_{\underline{i}}\sigma_j$ and unit matrix denoted as α_0.

[1] In spite of this, we stress that these operators do not represent coordinates and momenta. Furthermore, they will turn out to transform like Dirac spinors.

[2] A hat sign over a matrix will be used to emphasize the difference between the operator obtained from a matrix in the sense of definition (2) and the matrix itself.

[3] One possible realization of such matrices is, for example: $\sigma_1 = -i\sigma_y \times \sigma_x$, $\sigma_2 = -iI_2 \times \sigma_y$, $\sigma_3 = -i\sigma_y \times \sigma_z$, $\tau_1 = i\sigma_x \times \sigma_y$, $\tau_2 = -i\sigma_z \times \sigma_y$, $\tau_3 = -i\sigma_y \times I_2$, where σ_x, σ_y and σ_z are standard two dimensional Pauli matrices and I_2 is a two dimensional unit matrix.

Now, set of 36 operators

$$\left\{\widehat{(\tau_{\underline{i}})}_{\pi\kappa}, \widehat{(\sigma_j)}_{\pi\kappa}, \widehat{(\alpha_0)}_{\pi\kappa}, \widehat{(\alpha_{\underline{i}j})}_{\pi\kappa}, \widehat{(\alpha_0)}_{\pi\pi}, \widehat{(\alpha_{\underline{i}j})}_{\pi\pi}, \widehat{(\alpha_0)}_{\kappa\kappa}, \widehat{(\alpha_{\underline{i}j})}_{\kappa\kappa}\right\} \tag{4}$$

can be chosen as basis of algebra of quadratic operators.

To obtain conformal subalgebra let us discard all operators from this set with underlined index having values $\underline{1}$ and $\underline{2}$. What we are left with is a subalgebra isomorphic with conformal algebra C(1,3) plus one additional generator that commutes with the rest of the subalgebra. Thus we introduce new notation for the remaining generators:

$$\begin{aligned} &J_k \equiv \widehat{\left(\frac{\sigma_i}{2}\right)}_{\pi\kappa}, \quad N_i \equiv \widehat{\left(\frac{\alpha_{\underline{3}i}}{2}\right)}_{\pi\kappa}, \quad D \equiv \widehat{\left(\frac{\alpha_0}{2}\right)}_{\pi\kappa}, \\ &P_i \equiv \widehat{\left(\frac{\alpha_{\underline{3}i}}{2}\right)}_{\pi\pi}, \quad P_0 \equiv \widehat{\left(\frac{\alpha_0}{2}\right)}_{\pi\pi}, \\ &K_i \equiv \widehat{\left(\frac{\alpha_{\underline{3}i}}{2}\right)}_{\kappa\kappa}, \quad K_0 \equiv -\widehat{\left(\frac{\alpha_0}{2}\right)}_{\kappa\kappa}, \end{aligned} \tag{5}$$

where J_i, N_i, D, P_μ and K_μ play roles of rotation generators, boost generators, dilatation generator, momenta and pure conformal generators, respectively. The additional remaining operator is

$$Y_{\underline{3}} \equiv \widehat{\left(\frac{\tau_{\underline{3}}}{2}\right)}_{\pi\kappa}, \tag{6}$$

which commutes with all of the conformal generators. We will call it the third component of the *dual spin*.

If we consider the way in which the recognized conformal subalgebra fits into the larger algebra $\mathcal{A}_2$, we see that spatial momenta, being equal to $(\widehat{\frac{\alpha_{\underline{3}j}}{2}})_{\pi\pi}$ naturally fit into a set of nine operators $(\widehat{\frac{\alpha_{\underline{i}j}}{2}})_{\pi\pi}$, spatial components of pure conformal generators fit into a set of nine $(\widehat{\frac{\alpha_{\underline{i}j}}{2}})_{\kappa\kappa}$, boosts into set of nine $(\widehat{\frac{\alpha_{\underline{i}j}}{2}})_{\pi\kappa}$ and the third component of the dual spin fits into set of three operators $(\widehat{\frac{\tau_{\underline{i}}}{2}})_{\pi\kappa}$. Having this on mind, we can extend notation convention (5) to cover whole algebra $\mathcal{A}_2$:

$$N_{\underline{i}j} \equiv (\widehat{\frac{\alpha_{\underline{i}j}}{2}})_{\pi\kappa}, \quad P_{\underline{i}j} \equiv (\widehat{\frac{\alpha_{\underline{i}j}}{2}})_{\pi\pi}, \quad K_{\underline{i}j} \equiv (\widehat{\frac{\alpha_{\underline{i}j}}{2}})_{\kappa\kappa}, \quad Y_{\underline{i}} \equiv (\widehat{\frac{\tau_{\underline{i}}}{2}})_{\pi\kappa}, \tag{7}$$

while for spatial components of conformal generators it holds $N_i = N_{\underline{3}i}$, $P_i = P_{\underline{3}i}$ and $K_i = K_{\underline{3}i}$.

Alternatively, we could have obtained conformal subalgebra by keeping operators with underlined index equal to 1 or 2, instead 3. As the matter in fact, if we pick any linear combination of operators $Y_{\underline{i}}$, or of operators J_i, the subalgebra of $\mathcal{A}_2$ that commutes with the chosen operator will be $c(1,3)$ isomorphic. On the other hand, operators $Y_{\underline{i}}$ and J_i constitute two, mutually commuting $su(2)$ isomorphic subalgebras (a consequence of $so(4) = su(2) \oplus su(2)$ identity). The two corresponding $SU(2)$ isomorphic groups act, respectively, on underlined and on non-underlined indices of algebra operators. The situation slightly looks like as if we had two independent rotation groups, while "momenta", "boosts" and "pure conformal operators" were here determined by two independent three-vector directions, each related to its own "rotation" group. And the symmetry reduction from $\mathcal{A}_2$ group to its conformally isomorphic subgroup can be therefore understood as a consequence of symmetry breaking of one of these two $SU(2)$ subgroups. Without loss of generality, we have assumed breaking of the group generated by $Y_{\underline{i}}$, with $Y_{\underline{3}}$ generating the remaining $U(1)$ symmetry.

As a more concrete example of such symmetry breaking, we can assume existence of effective potential being an increasing function of absolute value of $Y_{\underline{3}}$ [e.g. proportional to the $(Y_{\underline{3}})^2$]. If the potential is sufficiently strong, all low energy physics would be constrained to subspace of $Y_{\underline{3}}$ eigenvalue equal to zero, and the remaining symmetry would be conformal symmetry. Moreover, since such a potential would have to break dilatational symmetry, overall symmetry would be reduced to the observable Poincare group. (Unfortunately, this particular symmetry breaking example seems to be oversimplified for more delicate physical reasons that we will not analyze in this paper.)

Note that such symmetry breaking also automatically fixes metric of space-time (i.e. of the remained symmetry) to be Minkowskian [the remaining J_i and $N_{\underline{3}i}$ hermitian operators form exactly $so(1,3)$ algebra]. It is also interesting that energy operator P_0 singles out among other momentum operators (i.e. among the rest of operators quadratic in) even before the symmetry reduction. Indeed, this operator, being the sum of squares of α, stands out as a positive operator, and there is no algebra automorphism that takes any other "momentum component" $P_{\underline{i}j}$ into the P_0 or vice-versus. This gives us some right to interpret the full group generated by $\mathcal{A}_2$ as a symmetry that differs from the observable space-time symmetry in the first place by existence of two "spatial-like" rotations, whereas it possesses something that looks like unique role of one axis (to be interpreted as the time axis). And the symmetry breaking only gets us rid of one of the "rotation-like" groups.

Structural relations of algebra $\mathcal{A}_2$ are:

$$
\begin{aligned}
&[J_i, J_j] = i\,\varepsilon_{ijk} J_k, \quad [Y_{\underline{i}}, Y_{\underline{j}}] = i\,\varepsilon_{\underline{ijk}} Y_{\underline{k}}, \quad [J_i, Y_{\underline{j}}] = 0,\\
&[J_i, N_{\underline{j}k}] = i\varepsilon_{ikl} N_{\underline{j}l}, \qquad [Y_{\underline{i}}, N_{\underline{j}k}] = i\varepsilon_{ijl} N_{\underline{l}k},\\
&[N_{\underline{i}j}, N_{\underline{k}l}] = -i\left(\delta_{jl}\varepsilon_{\underline{ikm}} Y_{\underline{m}} + \delta_{\underline{ik}}\varepsilon_{jlm} J_m\right),\\
&[J_i, D] = [Y_{\underline{i}}, D] = [N_{\underline{i}j}, D] = 0,
\end{aligned}
$$

$$
\begin{aligned}
&[J_i, P_{\underline{j}k}] = i\varepsilon_{ikl} P_{\underline{j}l}, \qquad [Y_{\underline{i}}, P_{\underline{j}k}] = i\varepsilon_{\underline{i}\underline{j}\underline{l}} P_{\underline{l}k}, \qquad\qquad (8)\\
&[N_{\underline{i}j}, P_{\underline{k}l}] = i\delta_{\underline{i}\underline{k}}\delta_{jl} P_0 + i\varepsilon_{\underline{i}\underline{k}\underline{m}}\varepsilon_{jln} P_{\underline{m}n},\\
&[N_{\underline{i}j}, P_0] = iP_{\underline{i}j}, \qquad [D, P_{\underline{i}j}] = iP_{\underline{i}j},\\
&[D, P_0] = iP_0, \qquad [J_i, P_0] = [Y_{\underline{i}}, P_0] = 0,\\
&[J_i, K_{\underline{j}k}] = i\varepsilon_{ikl} K_{\underline{j}l}, \qquad [Y_{\underline{i}}, K_{\underline{j}k}] = i\varepsilon_{\underline{i}\underline{j}\underline{l}} K_{\underline{l}k}, \qquad \ldots\\
&[P_{\underline{i}j}, K_{\underline{k}l}] = 2i\left(-\delta_{\underline{i}\underline{k}}\delta_{jl} D - \varepsilon_{\underline{i}\underline{k}\underline{m}}\varepsilon_{jln} N_{\underline{m}n} + \delta_{\underline{i}\underline{k}}\varepsilon_{jlm} J_m + \delta_{jl}\varepsilon_{\underline{i}\underline{k}\underline{m}} Y_{\underline{m}}\right),\\
&[P_{\underline{i}j}, K_0] = -2iN_{\underline{i}j}, \qquad [P_0, K_{\underline{i}j}] = -2iN_{\underline{i}j},\\
&[P_0, K_0] = -2iD.
\end{aligned}
$$

We will not investigate this construction, based on Heisenberg operators, in more detail. We just note that realization of conformal symmetry by quadratic Heisenberg operators has a number of interesting features, e.g. operator $Y_{\underline{3}}$ turns out to be helicity operator (appearing here on the same footage with the rest of conformal generators) generating both chirality symmetry and electromagnetic duality symmetry [6], while free equations of motion show up as direct consequences of identities connecting algebra generators [7].[4]

In the next section we will replace starting Heisenberg algebra with relations of parabose algebra, obtaining extension of conformal supersymmetry. The case of Heisenberg algebra is actually one representation (the simplest nontrivial) of that superalgebra.

3. Parabose Operators and Extension of Conformal Supersymmetry

Relations (5, 7) defining one basis of algebra $\mathcal{A}_2$ can be written the other way around in the following anticommutator form:

$$
\begin{aligned}
&\{\ \alpha,\ \beta\} = (\alpha_0)_{\alpha\beta}\, P_0 + (\alpha_{\underline{i}j})_{\alpha\beta}\, P_{\underline{i}j},\\
&\{\kappa^\alpha, \kappa^\beta\} = -(\alpha_0)^{\alpha\beta}\, K_0 + (\alpha_{\underline{i}j})^{\alpha\beta}\, K_{\underline{i}j}, \qquad\qquad (9)\\
&\{\kappa^\alpha,\ \beta\} = (\alpha_0)^\alpha{}_\beta\, D + (\alpha_{\underline{i}j})^\alpha{}_\beta\, N_{\underline{i}j} + (\sigma_i)^\alpha{}_\beta\, J_i + (\tau_{\underline{i}})^\alpha{}_\beta\, Y_{\underline{i}}.
\end{aligned}
$$

In addition to the fact that operators and κ satisfy set of anticommutation relations, their commutators with algebra $\mathcal{A}_2$ generators:

$$
\begin{aligned}
&[J_i,\ \alpha] = -i(\tfrac{\sigma_i}{2})_\alpha{}^\beta\ \beta, \qquad [Y_{\underline{i}},\ \alpha] = -i(\tfrac{\tau_{\underline{i}}}{2})_\alpha{}^\beta\ \beta,\\
&[D,\ \alpha] = \tfrac{i}{2}\,\alpha, \qquad [N_{\underline{i}j},\ \alpha] = i(\tfrac{\alpha_{\underline{i}j}}{2})_\alpha{}^\beta\ \beta, \qquad\qquad (10)\\
&[K_0,\ \alpha] = -i(\alpha_0)_{\alpha\beta}\,\kappa^\beta, \qquad [K_{\underline{i}j},\ \alpha] = i(\alpha_{\underline{i}j})_{\alpha\beta}\,\kappa^\beta,\\
&[P_0,\ \alpha] = [P_{\underline{i}j},\ \alpha] = 0 \quad \text{(and similar } \kappa \text{ relations)}
\end{aligned}
$$

[4]We call attention that, had we utilized Heisenberg operators to directly construct, in a standard fashion, an $so(2,4)$ conformal algebra (instead of $sp(8)$) none of these would be present.

show that α and κ^α transform as Dirac spinors under the Lorentz subgroup. To see this more clearly, we can introduce the following (Majorana) representation of Dirac matrices:

$$\gamma_0 = i\tau_{\underline{2}}, \quad \gamma_i = \gamma_0\,\alpha_{\underline{3}i} = i\tau_{\underline{1}}\sigma_i, \quad \gamma_5 = -i\gamma_0\gamma_1\gamma_2\gamma_3 = i\tau_{\underline{3}}, \tag{11}$$

so that Lorentz part of (10) can be written as $[M_{\mu\nu},\,\alpha] = -i(\frac{1}{4}[\gamma_\mu,\gamma_\nu])_\alpha{}^\beta\,\beta$ (where $M_{ij} = \varepsilon_{ijk}J_k$, $M_{i0} = N_{\underline{3}i}$).

Therefore, α and κ^α are spinors whose anticommutation relations close on algebra $\mathcal{A}_2$. If there were no commutation relations of starting Heisenberg algebra this would have been a structure of a graded algebra. We now show that it is possible to modify the whole idea as to obtain a true graded algebra.

If the operators κ^α and α satisfy Heisenberg algebra, their linear combinations $a_\alpha \equiv \frac{1}{\sqrt{2}}(\kappa^\alpha + i\,\alpha)$ and $a_\alpha^\dagger \equiv \frac{1}{\sqrt{2}}(\kappa^\alpha - i\,\alpha)$, satisfy Bose algebra. However, in this section we will change our starting point and assume that linear combinations a_α and $a_\alpha^\dagger$ satisfy parabose instead of Bose algebra (i.e. that κ and satisfy some "para-Heisenberg" instead of Heisenberg algebra):

$$[\{a_\alpha, a_\beta\}, a_\gamma] = 0, \quad [\{a_\alpha, a_\beta^\dagger\}, a_\gamma] = -2\delta_\beta^\gamma a_\alpha. \tag{12}$$

(Relations obtained from these by generalized Jacobi identities and by hermitian conjugation are also implied.)

It is not difficult to verify that this change in starting algebra does not influence relations (8) and (10), if we keep the definitions (9) [i.e. definitions (5, 7)]. As the commutators between α and κ^α are no longer fixed, the structure now presents a standard graded (mod 2) algebra.

Just like the algebra (8) is extension of conformal algebra, the graded algebra given by relations (8), (9) and (10) [isomorphic to $osp(1,8)$] can be seen as an extension of $N = 1$ conformal superalgebra. Indeed, by setting $P_{\underline{1}i} = P_{\underline{2}i} = K_{\underline{1}i} = K_{\underline{2}i} = N_{\underline{1}i} = N_{\underline{2}i} = Y_{\underline{1}} = Y_{\underline{2}} = 0$ we obtain standard conformal superalgebra. (Together with this change the coefficient multiplying operator $Y_{\underline{3}}$ in the third relation of (9) must also be altered in order to satisfy generalized Jacobi identities. Comparison is then made using gamma matrices (11), identification (5) and by replacing: $\alpha = Q_\alpha/\sqrt{2}$, $\kappa^\alpha = \overline{s}^\alpha/\sqrt{2}$, $Y_{\underline{3}} = \frac{1}{2}R$).

The connection in the opposite direction (from conformal superalgebra to this extension) can be established if we notice that anticommutator of two left-handed operators $\{\eta,\,\xi\}$, or of two right-handed operators $\{\dot\eta,\,\dot\xi\}$ yields linear combination of operators $P_{\underline{1}i}$ and $P_{\underline{2}i}$ (and similarly for κ operators). Therefore, our graded algebra can be seen as a special conformal superalgebra where all anticommutators of supersymmetry generators are allowed to be nonzero operators. (It is enough to allow existence of nonzero $P_{\underline{1}i}$ and $P_{\underline{2}i}$ – the introduction of the rest of the operators (7) is necessary to close the algebra.)

As we already announced in the introduction, by relaxing the constraint $\{\eta, \xi\} = 0$ (i.e. $\{Q_\eta, Q_\xi\} = 0$) supermultiplets become infinite. Nevertheless, the simple symmetry breaking assumption, discussed in the previous section, breaks not only extra bosonic generators (7), but also the supersymmetry generators α and κ^α. Since action of operators η and $\dot{\eta}$ change value of Y_3 for $\frac{1}{2}$, each following member of a supermultiplet would gain higher and higher mass, whereas the low-energy space-time symmetry would be given by Poincare group.

4. Conclusion

In this paper we analyzed an extension of the conformal supersymmetry. It is interesting that, although the proposed symmetry is higher and mathematical structure thus richer, the algebra relations are simplified. Namely, commutators of bosonic with fermionic operators (10) are nothing more than simple relations of parabose algebra written in a complicated basis. And the fermionic anticommutators (9) are relations that describe this new basis, so these relations can be seen as a specific naming convention for linear combinations of and κ anticommutators. The idea is that this complicated basis becomes physically relevant due to the symmetry breaking, analyzed in section 2. The relatively simple symmetry breaking is therefore responsible not only for reduction of the starting symmetry and for introduction of mass splitting, but also for superficial complexity that hides simplicity of the starting parabose algebra. Bosonic algebra $\mathcal{A}_2$ relations (8) are direct consequence of (9) and (10).

From the perspective of this higher symmetry, those relations of standard conformal superalgebra that set some of the anticommutators to zero appear as a kind of artificial constraints – constraints that are, in this picture, consequences of a symmetry breaking. This fact that some linear combinations of anticommutators are zero makes it impossible to see anticommutators of fermionic generators simply as a naming convention, as it was possible for (9). On the other hand, extension from the conformal superalgebra to the symmetry discussed here can be done by allowing all anticommutators of supersymmetry generators to be nonzero operators. By doing so we end up with an algebra determined by only two parabose relations (12).

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