Photon as a black hole

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Abstract

The photon has zero rest mass, spin value of h and the constant velocity of c. An upper limit imposed on the energy of the photon is derived using these three unique characteristics and extending the concept of Schwarzschild radius to the photon. The Schwarzschild radius for the photon turns out to be the Planck length and the energy of the photon at this radius is the Planck energy. At this maximum energy, the photon behaves as a black hole. Further, the possible phase change that occurs to the photon at the Schwarzschild radius, from energy to matter as a result of vacuum fluctuations is indicated. Finally, with in the present theory, Planck length and Planck time are shown to be the meaningful minimum values of space and time.

The photon, as an elementary particle, is unique. It is the only elementary particle of energy where as there are hundreds of elementary particles of matter. Also photon is the only known elementary particle with out an antiparticle. There are three characteristics for the photon.

1. In the free space, a photon always propagates with the constant velocity, velocity of light c, irrespective of the energy it possesses.
2. The rest mass of the photon is zero.
3. The photon has a spin value of h.

In this paper we make only order of magnitude calculations and therefore no distinction is made between h and $\hbar$. Throughout this paper h is used even where $\hbar$ appears in the original context, such as in the spin value of the photon and in uncertainty relations.

It will be shown below that the above three properties put an upper limit on the energy of the photon. The upper limit of the photon energy sets lower limits for time and space.

The size of the universe is an unsettled problem [1, 2]. But everything else is finite, starting from giant structures like galaxies and stars to microscopic

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objects like protons and electrons. These objects often have lower and upper limits for their physical parameters like mass, volume, density etc. A well known example is the theoretical limit for the mass of the white dwarf. Chandrasekhar showed that the upper limit for the mass of any white dwarf will be 1.4 solar mass [3]. Similarly, an upper limit for the mass of the star has been suggested. This upper limit is estimated in the range of 150 times the solar mass [4]. Therefore, it seems natural to expect an upper limit for the energy of a photon.

The special theory of relativity postulates constancy [5] for the velocity of light in free space and hence on the total energy possible for a photon.

Relativistic equation for the energy of a particle is given by

$$E^2 = p^2c^2 + (m_oc^2)^2, \tag{1}$$

where E is energy of the particle, P is momentum of the particle, c is velocity of light and m_o is rest mass of the particle.

For a photon, the rest mass is zero and therefore the energy of the photon is given by the equation

$$E^2 = P^2c^2. \tag{2}$$

It is well known that photon has one degree of freedom of rotation and two degrees of freedom of translation [6, 7]. Since the photon has no rest mass, the total energy of the photon must be stored as rotational kinetic energy and translational kinetic energy.

The first postulate of relativity [5] (or the Galilean invariance) states that no object is aware of its motion when moving with uniform velocity and light is no exception to this. Therefore, for the photon itself moving with the velocity of light, its kinetic energy due to translatory motion will be zero. But this does not say that the photon does not exist for itself. The photon has a kinetic energy of rotation and therefore the photon can feel its existence. Therefore, from the point of view of the photon itself, since the photon has no rest mass, the total energy of the photon is stored as rotational kinetic energy. Thus we can associate rotation even to a particle with zero rest mass, provided the particle has spatial extension. So far there is no particle with spin zero and mass zero. Incidentally, this also shows the necessity of a nonzero spin value for any particle with zero rest mass provided it is moving with constant velocity. Even the neutrino appears to have a very small mass though its estimate is very uncertain. The implications as to the energy-momentum transformation under special relativity and interpretation of Compton effect, photoelectric effect and pair production need further investigation.

The energy of the photon is given by the expression

$$E = h\nu = \frac{hc}{\lambda}, \tag{3}$$

where h is the Planck's constant, ν is the frequency of the photon and λ is the wavelength of the photon.

The wavelength guarantees the extension of the photon. The expression $E = \frac{hc}{\lambda}$ shows that the greater the energy of the photon the shorter its wavelength. This implies that with the increase in energy, the photon will be more and more localized. Can this localization go an ad infinitum. The idea of infinite amount of energy packed in an infinitely small volume is quite unphysical and it cannot be true. How is the property of space altered by the ever increasing energy density? Do the constant parameters h and c put an upper limit for the possible energy of a photon? What are the consequences of this upper limit on the space time geometry.

Kip Thorne has pointed out that a very high concentration of energy in a very small region can lead to the formation of a black hole. This is known as the Hoop conjecture [8]. Also several authors have pointed out that the minimum length consistent with classical relativity and quantum mechanics is the Planck length. Recently Xavier Calmet et al. [9] have established that the minimum length that can be measured is the Planck length.

The Harward Tower experiment [10] has clearly demonstrated that the energy of a photon is increased under gravitational attraction. Can the energy of the photon be increased continuously without limit by gravitational attraction? Since the velocity of the photon is not increased as a result of gravitational attraction, the increased energy of the photon, when attracted by a gravitational field, is entirely stored as rotational kinetic energy. Spin is a quantum mechanical analogue of classical angular momentum. Since the photon has a spin of 1 h, we can attribute an angular momentum to the photon in the semiclassical analysis.

This assumption requires some further explanation. Even at the time of postulating the existence of spin by Uhlenbeck and Goudsmit [12], H. A. Lorentz by a simple calculation showed that if we consider the spin of the electron as a rotation, the linear velocity of the electron at the circumference of the electron will exceed the velocity of light and this will contradict the special theory of relativity. Ever since spin is usually termed as the intrinsic degree of freedom and there is a general reluctance among the physics community to consider spin as rotation. But it is to be pointed out that Lorentz's calculation was based on the assumption that the electron is a rigid body. There is not any justification, either theoretical or experimental, to consider the photon as a rigid body. Again, the relativity with finite signal speed rules out the existence of perfect rigid bodies. Therefore here we assume, as pointed out in [6, 7], the rotational nature of photon.

When L, I and w are the angular momentum, moment of inertia and angular velocity of a particle, we have the expression

$$L = Iw. \tag{4}$$

When m, r and v are the mass, radius and velocity of a particle respectively, we have

$$L = mrv\,. \tag{5}$$

Special theory of relativity puts c as the upper limit for the velocity of any signal. Therefore no particle can rotate with a tangential linear velocity v

greater than the velocity of light. When we put this limiting value of c to v in equation (5), we have

$$L = mrc\,. \tag{6}$$

When the particle we consider is the photon, L is replaced by the spin s of the photon and m is the equivalent mass of the photon. Therefore,

$$s = mrc.$$

For a photon

$$\begin{aligned} \therefore\ h &= mrc\,, \\ m &= \frac{h}{rc}\,. \end{aligned} \tag{7}$$

Now, we assume that as in the case of material particles, the concept of Schwarzschild radius is valid for the photon also.

The Schwarzschild radius of a black hole is given by the expression

$$r = \frac{2Gm}{c^2}\,, \tag{8}$$

$$\frac{r}{m} = \frac{2G}{c^2}\,. \tag{9}$$

The right hand side of equation (9) is constant. Therefore, for any black hole, the increase in radius must be accompanied by an increase in mass. But when we consider the photon as a black hole, Schwarzschild radius is the minimum radius that a photon can have. In the case of material particles, we can have black holes of varying masses with the corresponding radii. This is possible because the right hand side value of equation (9) can be maintained by proportional increase in the value of r and m. This is impossible for the photon because with the increase in the equivalent mass of the photon, the radius of the photon decreases. Therefore, unlike in the case of material particles, there is a unique value of radius and equivalent mass, for a photon to behave as a black hole.

Substituting the value of the radius r from equation (8) into equation (7), we get

$$m = \sqrt{\frac{hc}{2G}}\,. \tag{10}$$

This is the expression for Planck mass. Planck mass is usually defined as the minimum mass corresponding to the Schwarzschild radius of a classical object. Planck mass, in the case of the photon, is the maximum equivalent mass permitted by a combination of classical mechanics and relativity for the photon.

We have

$$\begin{aligned} c &= 2.99 \times 10^{8}\ \mathrm{m\ s^{-1}} \\ h &= 6.62 \times 10^{-34}\ \mathrm{Js} \\ G &= 6.67 \times 10^{-11}\ \mathrm{N\ m^{2}\ kg^{-2}}. \end{aligned}$$

Substituting these values in equation (10) we get

$$m = 1.54 \times 10^{-7} \text{ kg.}$$

The maximum energy possible for the photon will be

$$mc^2 = \sqrt{\frac{hc}{2G}}\, c^2 = 8.61 \times 10^{22} \text{ MeV.}$$

This is the Planck energy. The highest energy so far observed [11] for a photon is of order of 10^{13} eV.

A simple dimensional analysis in terms of the Planck's natural units will be illuminating here. The dimension of energy is ML^2T^{-2}. When we substitute for M, L and T in terms of c, G and h we get the expression

$$E = ML^2T^{-2} = \sqrt{\frac{hC}{G}} \cdot \frac{hG}{c^3} \cdot \frac{c^5}{hG} = \sqrt{\frac{hc}{G}}\, c^2. \tag{11}$$

What happens to the photon when it reaches its upper limit of energy and lower limit of radius? Now for the photon, with maximum energy density, Hoop conjecture [8] becomes a reality and the photon is a black hole. Energy and matter are two phases of the same physical entity. Hawking [13, 14] has shown that in the case of primordial radius, spontaneous pair production takes place. At the upper limit of energy permitted for the photon, having reached the Schwarzschild radius, it is legitimate to presume that the photon will undergo pair production or some other processes and transformed to material particles. It is also an open question whether the photon at the maximum energy limit can be disintegrated into many photons with smaller energies in the turbulent sea of vacuum fluctuations. Of course, these are pure speculations either to be proved or disproved by future theoretical and experimental studies.

By eliminating m from equation (7) and (8) we get the expression for the Schwarzschild radius r. That is,

$$r = \sqrt{\frac{2Gh}{c^3}}\,. \tag{12}$$

This is the expression for Planck length. Thus we see that Planck length is the minimum length allowed for a photon and at Planck length photon attains maximum energy and is transformed into a black hole. Substituting the values of h, c and G we see that Planck length is of the order of 10^{-35} metres.

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References

[1] Frank H. Shu, *The Physical Universe. An Introduction to Astronomy*, University Science Books, Hill Valley, California, 1982.

[2] Cotsakis S. and Gibbons G.W. (Eds), *Mathematical and Quantum Aspects of Cosmology*, Springer, Berlin, 2000, p. 180.

[3] Chandrasekhar S., *Philos. Mag.* **11** (1931) 592.

[4] Donald F., Figer, *Nature* **434** (2005) 192–194.

[5] Einstein A., *Ann. Phys.* **L 140** (1905) 217.

[6] Hann D., Kim Y. S., and Son D., *Phys. Rev. D* **25** (1982) 461.

[7] Kim Y. S. and Wigner E. P., *J. Maths. Phys.* **31** (1990) 65.

[8] Thorne K. S., *Magic Without Magic*, Ed. J. Klauder, W.H. Freeman, San Fransisco 1972, pp. 231–258.

[9] Xavier Calmet, Michael Graesser and Stephen D.H. Hsu, *Phys. Rev. Lett.* **93** (2004) 21101.

[10] Pound R. V. and Rebka, Jr., G. A. *Phys. Rev. Lett.* **4** (1960) 337.

[11] Horan D. and Weekes T. C., *New Astro Rev.* **48** (2004) 527.

[12] Uhlenbeck G. E. and Goudsmit S., *Die Naturwissen Schaften* **13** (1925) 953.

[13] Hawking S. W., *Nature* **248** (1974) 30.

[14] Hawking S. W., *Commun. Maths. Phys.* **43** (1975) 199.