Finite Unification and the Higgs mass prediction

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Abstract

Finite Unified Theories (FUTs) are N=1 supersymmetric Grand Unified Theories, which can be made all-loop finite, both in the gauge and Yukawa couplings and in the soft supersymmetry breaking sector. This remarkable property, based on the reduction of couplings at the quantum level, provides a drastic reduction in the number of free parameters, which in turn leads to an accurate prediction of the top quark mass, and predictions for the Higgs boson mass and the supersymmetric spectrum. Here we examine the predictions of three FUTs taking into account a number of theoretical and experimental constraints.

1. Introduction

The theoretical efforts to establish a deeper understanding of Nature have led to very interesting frameworks such as String theories and Non-commutative Geometry both of which aim to describe physics at the Planck scale. Looking for the origin of the idea that coordinates might not commute we might have to go back to the days of Heisenberg. In the recent years the birth of such speculations can be found in refs. [1, 2]. In the spirit of Non-commutative Geometry also particle models with non-commutative gauge theory were explored [3] (see also [4]), [5, 6]. On the other hand the present intensive research has been triggered by the natural realization of non-commutativity of space in the string theory context of D-branes in the presence of a constant background antisymmetric field [7]. After the work of Seiberg and Witten [8], where a map (SW map) between non-commutative and commutative gauge theories has been described, there has been a lot of activity also in the construction of non-commutative phenomenological

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Lagrangians, for example various non-commutative standard model like Lagrangians have been proposed [9, 10][1]. In particular in ref. [10], following the SW map methods developed in refs. [11], a non-commutative standard model with $SU(3) \times SU(2) \times U(1)$ gauge group has been presented. These non-commutative models represent interesting generalizations of the SM and hint at possible new physics. However they do not address the usual problem of the SM, the presence of a plethora of free parameters mostly related to the ad hoc introduction of the Higgs and Yukawa sectors in the theory. At this stage it is worth recalling that various schemes, with the Coset Space Dimensional Reduction (CSDR) [14, 15, 16, 17] being pioneer, were suggesting that a unification of the gauge and Higgs sectors can be achieved in higher dimensions. Moreover the addition of fermions in the higher-dimensional gauge theory leads naturally after CSDR to Yukawa couplings in four dimensions. In the successes of the CSDR scheme certainly should be added the possibility to obtain chiral theories in four dimensions [18, 19, 20, 21] as well as softly broken supersymmetric or non-supersymmetric theories starting from a supersymmetric gauge theory defined in higher dimensions [22].

Finite Unified Theories are $N = 1$ supersymmetric Grand Unified Theories (GUTs) which can be made finite even to all-loop orders, including the soft supersymmetry breaking sector. The method to construct GUTs with reduced independent parameters [25, 26] consists of searching for renormalization group invariant (RGI) relations holding below the Planck scale, which in turn are preserved down to the GUT scale. Of particular interest is the possibility to find RGI relations among couplings that guarantee finiteness to all-orders in perturbation theory [27, 28]. In order to achieve the latter it is enough to study the uniqueness of the solutions to the one-loop finiteness conditions [27, 28]. The constructed *finite unified* $N = 1$ supersymmetric SU(5) GUTs, using the above tools, predicted correctly from the dimensionless sector (Gauge-Yukawa unification), among others, the top quark mass [29]. The search for RGI relations and finiteness has been extended to the soft supersymmetry breaking sector (SSB) of these theories [30, 31], which involves parameters of dimension one and two. Eventually, the full theories can be made all-loop finite and their predictive power is extended to the Higgs sector and the supersymmetric spectrum (s-spectrum).

2. Reduction of Couplings and Finiteness in $N = 1$ SUSY Gauge Theories

Here let us review the main points and ideas concerning the *reduction of couplings* and *finiteness* in $N = 1$ supersymmetric theories. A RGI relation among couplings g_i, $\Phi(g_1, \cdots, g_N) = 0$, has to satisfy the partial differential equation $\mu\, d\Phi/d\mu = \sum_{i=1}^{N} \beta_i\, \partial\Phi/\partial g_i = 0$, where β_i is the β-function of g_i. There exist $(N-1)$ independent Φ's, and finding the complete set of these solutions is equivalent to solve the so-called reduction equations

[1] These SM actions are mainly considered as effective actions because they are not renormalizable. The effective action interpretation is consistent with the SM in [10] being anomaly free [12]. Non-commutative phenomenology has been discussed in [13].

(REEs) [26], $\beta_g\,(dg_i/dg) = \beta_i$, $i = 1, \cdots, N$, where g and β_g are the primary coupling and its β-function. Using all the $(N-1)\,\Phi$'s to impose RGI relations, one can in principle express all the couplings in terms of a single coupling g. The complete reduction, which formally preserves perturbative renormalizability, can be achieved by demanding a power series solution, whose uniqueness can be investigated at the one-loop level.

Finiteness can be understood by considering a chiral, anomaly free, $N = 1$ globally supersymmetric gauge theory based on a group G with gauge coupling constant g. The superpotential of the theory is given by

$$W = \frac{1}{2}\, m^{ij}\,\Phi_i\,\Phi_j + \frac{1}{6}\, C^{ijk}\,\Phi_i\,\Phi_j\,\Phi_k \ , \tag{1}$$

where m^{ij} (the mass terms) and C^{ijk} (the Yukawa couplings) are gauge invariant tensors and the matter field Φ_i transforms according to the irreducible representation R_i of the gauge group G.

The one-loop β-function of the gauge coupling g is given by

$$\beta_g^{(1)} = \frac{dg}{dt} = \frac{g^3}{16\pi^2}\left[\sum_i l(R_i) - 3\,C_2(G)\right] \ , \tag{2}$$

where $l(R_i)$ is the Dynkin index of R_i and $C_2(G)$ is the quadratic Casimir of the adjoint representation of the gauge group G. The β-functions of C^{ijk}, by virtue of the non-renormalization theorem, are related to the anomalous dimension matrix γ_i^j of the matter fields Φ_i as:

$$\beta_C^{ijk} = \frac{d}{dt}\,C^{ijk} = C^{ijp}\sum_{n=1}\frac{1}{(16\pi^2)^n}\,\gamma_p^{k(n)} + (k\leftrightarrow i) + (k\leftrightarrow j) \ . \tag{3}$$

At one-loop level γ_i^j is given by

$$\gamma_i^{j(1)} = \frac{1}{2}\,C_{ipq}\,C^{jpq} - 2\,g^2\,C_2(R_i)\delta_i^j \ , \tag{4}$$

where $C_2(R_i)$ is the quadratic Casimir of the representation R_i, and $C^{ijk} = C^*_{ijk}$.

All the one-loop β-functions of the theory vanish if the β-function of the gauge coupling $\beta_g^{(1)}$, and the anomalous dimensions $\gamma_i^{j(1)}$, vanish, i.e.

$$\sum_i \ell(R_i) = 3C_2(G)\,, \qquad \frac{1}{2}\,C_{ipq}C^{jpq} = 2\delta_i^j g^2 C_2(R_i) \ , \tag{5}$$

where $l(R_i)$ is the Dynkin index of R_i, and $C_2(G)$ is the quadratic Casimir invariant of the adjoint representation of G.

A very interesting result is that the conditions (5) are necessary and sufficient for finiteness at the two-loop level [32, 33].

The one- and two-loop finiteness conditions (5) restrict considerably the possible choices of the irreducible representations R_i for a given group G as well as the Yukawa couplings in the superpotential (1). Note in particular that the finiteness conditions cannot be applied to the supersymmetric standard model (SSM), since the presence of a $U(1)$ gauge group is incompatible with the condition (5), due to $C_2[U(1)] = 0$. This leads to the expectation that finiteness should be attained at the grand unified level only, the SSM being just the corresponding low-energy, effective theory.

The finiteness conditions impose relations between gauge and Yukawa couplings. Therefore, we have to guarantee that such relations leading to a reduction of the couplings hold at any renormalization point. The necessary, but also sufficient, condition for this to happen is to require that such relations are solutions to the reduction equations (REs) to all orders. The all-loop order finiteness theorem of ref.[27] is based on: (a) the structure of the supercurrent in $N = 1$ SYM and on (b) the non-renormalization properties of $N = 1$ chiral anomalies [27]. Alternatively, similar results can be obtained [28, 34] using an analysis of the all-loop NSVZ gauge beta-function [35].

3. Soft supersymmetry breaking and finiteness

The above described method of reducing the dimensionless couplings has been extended [31, 30] to the soft supersymmetry breaking (SSB) dimensionful parameters of $N = 1$ supersymmetric theories. Recently very interesting progress has been made [36, 37, 38, 39, 40, 41, 42, 43, 44] concerning the renormalization properties of the SSB parameters, based conceptually and technically on the work of ref. [38]. In this work the powerful supergraph method [41] for studying supersymmetric theories has been applied to the softly broken ones by using the "spurion" external space-time independent superfields [42]. In the latter method a softly broken supersymmetric gauge theory is considered as a supersymmetric one in which the various parameters such as couplings and masses have been promoted to external superfields that acquire "vacuum expectation values". Based on this method the relations among the soft term renormalization and that of an unbroken supersymmetric theory have been derived. In particular the β-functions of the parameters of the softly broken theory are expressed in terms of partial differential operators involving the dimensionless parameters of the unbroken theory. The key point in the strategy of refs. [36]-[44] in solving the set of coupled differential equations so as to be able to express all parameters in a RGI way, was to transform the partial differential operators involved to total derivative operators [36]. It is indeed possible to do this on the RGI surface which is defined by the solution of the reduction equations. In addition it was found that RGI SSB scalar masses in Gauge-Yukawa unified models satisfy a universal sum rule at one-loop [40]. This result was generalized to two-loops for finite theories [44], and then to all-loops for general Gauge-Yukawa and Finite Unified Theories [37].

In order to obtain a feeling of some of the above results, consider the superpotential given by (1) along with the Lagrangian for SSB terms

$$-\mathcal{L}_{\rm SB} = \frac{1}{6}\, h^{ijk}\, \phi_i\phi_j\phi_k + \frac{1}{2}\, b^{ij}\, \phi_i\phi_j + \frac{1}{2}\, (m^2)^j_i\, \phi^{*\,i}\phi_j + \frac{1}{2}\, M\, \lambda\lambda + \text{H.c.}, \tag{6}$$

where the ϕ_i are the scalar parts of the chiral superfields Φ_i , λ are the gauginos and M their unified mass. Since only finite theories are considered here, it is assumed that the gauge group is a simple group and the one-loop β-function of the gauge coupling g vanishes. It is also assumed that the reduction equations admit power series solutions of the form

$$C^{ijk} = g \sum_{n=0} \rho^{ijk}_{(n)} g^{2n} \ . \tag{7}$$

According to the finiteness theorem [27], the theory is then finite to all-orders in perturbation theory, if, among others, the one-loop anomalous dimensions $\gamma_i^{j(1)}$ vanish. The one- and two-loop finiteness for h^{ijk} can be achieved by [33]

$$h^{ijk} = -MC^{ijk} + \cdots = -M\rho^{ijk}_{(0)}\, g + O(g^5) \ . \tag{8}$$

An additional constraint in the SSB sector up to two-loops [44], concerns the soft scalar masses as follows

$$\frac{(\ m_i^2 + m_j^2 + m_k^2\ )}{MM^\dagger} = 1 + \frac{g^2}{16\pi^2}\, \Delta^{(2)} + O(g^4) \tag{9}$$

for i, j, k with $\rho^{ijk}_{(0)} \neq 0$, where $\Delta^{(2)}$ is the two-loop correction

$$\Delta^{(2)} = -2 \sum_l [(m_l^2/MM^\dagger) - (1/3)]\ T(R_l), \tag{10}$$

which vanishes for the universal choice [33], i.e. when all the soft scalar masses are the same at the unification point.

If we know higher-loop β-functions explicitly, we can follow the same procedure and find higher-loop RGI relations among SSB terms. However, the β-functions of the soft scalar masses are explicitly known only up to two loops. In order to obtain higher-loop results, we need something else instead of knowledge of explicit β-functions, e.g. some relations among β-functions.

The recent progress made using the spurion technique [41, 42] leads to the following all-loop relations among SSB β-functions, [36, 37, 38, 39, 40, 41, 42, 43, 44]

$$
\begin{aligned}
\beta_M &= 2\mathcal{O}\left(\frac{\beta_g}{g}\right) \;, && (11)\\
\beta_h^{ijk} &= \gamma^i{}_l h^{ljk} + \gamma^j{}_l h^{ilk} + \gamma^k{}_l h^{ijl} \\
&\quad -2\gamma_1^i{}_l C^{ljk} - 2\gamma_1^j{}_l C^{ilk} - 2\gamma_1^k{}_l C^{ijl} \;, && (12)\\
(\beta_{m^2})^i{}_j &= \left[\Delta + X\frac{\partial}{\partial g}\right]\gamma^i{}_j \;, && (13)\\
\mathcal{O} &= \left(Mg^2\frac{\partial}{\partial g^2} - h^{lmn}\frac{\partial}{\partial C^{lmn}}\right) \;, && (14)\\
\Delta &= 2\mathcal{O}\mathcal{O}^* + 2|M|^2 g^2\frac{\partial}{\partial g^2} && (15)\\
&\quad +\tilde{C}_{lmn}\frac{\partial}{\partial C_{lmn}} + \tilde{C}^{lmn}\frac{\partial}{\partial C^{lmn}} \;, && (16)
\end{aligned}
$$

where $(\gamma_1)^i{}_j = \mathcal{O}\gamma^i{}_j$, $C_{lmn} = (C^{lmn})^*$, and

$$\tilde{C}^{ijk} = (m^2)^i{}_l C^{ljk} + (m^2)^j{}_l C^{ilk} + (m^2)^k{}_l C^{ijl} \;. \tag{17}$$

It was also found [43] that the relation

$$h^{ijk} = -M(C^{ijk})' \equiv -M\frac{dC^{ijk}(g)}{d\ln g} \;, \tag{18}$$

among couplings is all-loop RGI. Furthermore, using the all-loop gauge β-function of Novikov *et al.* [35] given by

$$\beta_g^{\rm NSVZ} = \frac{g^3}{16\pi^2}\left[\frac{\sum_l T(R_l)(1-\gamma_l/2) - 3C(G)}{1 - g^2C(G)/8\pi^2}\right] \;, \tag{19}$$

it was found the all-loop RGI sum rule [37],

$$
\begin{aligned}
m_i^2 + m_j^2 + m_k^2 &= |M|^2\{\, \frac{1}{1 - g^2C(G)/(8\pi^2)}\frac{d\ln C^{ijk}}{d\ln g} && (20)\\
&\quad + \frac{1}{2}\frac{d^2\ln C^{ijk}}{d(\ln g)^2}\,\} + \sum_l \frac{m_l^2 T(R_l)}{C(G) - 8\pi^2/g^2}\frac{d\ln C^{ijk}}{d\ln g} \;.
\end{aligned}
$$

In addition the exact-β-function for m^2 in the NSVZ scheme has been obtained [37] for the first time and is given by

$$
\begin{aligned}
\beta_{m_i^2}^{\rm NSVZ} &= \Big[\, |M|^2\{\, \frac{1}{1 - g^2C(G)/(8\pi^2)}\frac{d}{d\ln g} + \frac{1}{2}\frac{d^2}{d(\ln g)^2}\,\} \\
&\quad + \sum_l \frac{m_l^2 T(R_l)}{C(G) - 8\pi^2/g^2}\frac{d}{d\ln g}\,\Big]\;\gamma_i^{\rm NSVZ} \;. && (21)
\end{aligned}
$$

4. Finite Unified Theories

In this section we examine two concrete $SU(5)$ finite models, where the reduction of couplings in the dimensionless and dimensionful sector has been achieved. For other interesting Finite Unified Theories based on cross group structure see ref.[45]. A predictive Gauge-Yukawa unified $SU(5)$ model which is finite to all orders, in addition to the requirements mentioned already, should also have the following properties:

1. One-loop anomalous dimensions are diagonal, i.e., $\gamma_i^{(1)\,j} \propto \delta_i^j$.
2. Three fermion generations, in the irreducible representations $\overline{\mathbf{5}}_i, \mathbf{10}_i$ $(i = 1, 2, 3)$, which obviously should not couple to the adjoint $\mathbf{24}$.
3. The two Higgs doublets of the MSSM should mostly be made out of a pair of Higgs quintet and anti-quintet, which couple to the third generation.

In the following we discuss two versions of the all-order finite model. The model of ref. [29], which will be labeled **A**, and a slight variation of this model (labeled **B**), which can also be obtained from the class of the models suggested by Kazakov *et al.* [36] with a modification to suppress non-diagonal anomalous dimensions [2].

The superpotential which describes the two models takes the form [29, 44]

$$\begin{aligned} W \;=\; & \sum_{i=1}^{3} \Big[\, \frac{1}{2} g_i^u \, \mathbf{10}_i \mathbf{10}_i H_i + g_i^d \, \mathbf{10}_i \overline{\mathbf{5}}_i \, \overline{H}_i \,\Big] + g_{23}^u \, \mathbf{10}_2 \mathbf{10}_3 H_4 + g_{23}^d \, \mathbf{10}_2 \overline{\mathbf{5}}_3 \, \overline{H}_4 \\ & + g_{32}^d \, \mathbf{10}_3 \overline{\mathbf{5}}_2 \, \overline{H}_4 + \sum_{a=1}^{4} g_a^f \, H_a \, \mathbf{24} \, \overline{H}_a + \frac{g^\lambda}{3} \, (\mathbf{24})^3 \;, \end{aligned} \tag{22}$$

where H_a and $\overline{H}_a$ $(a = 1, \ldots, 4)$ stand for the Higgs quintets and anti-quintets.

[2] An extension to three families, and the generation of quark mixing angles and masses in Finite Unified Theories has been addressed in [46], where several realistic examples are given. These extensions are not considered here.

The non-degenerate and isolated solutions to $\gamma_i^{(1)} = 0$ for the models $\{\mathbf{A}\ ,\ \mathbf{B}\}$ are:

$$
\begin{aligned}
(g_1^u)^2 &= \{\frac{8}{5},\frac{8}{5}\}g^2\ ,\ (g_1^d)^2 = \{\frac{6}{5},\frac{6}{5}\}g^2\ ,\\
(g_2^u)^2 &= (g_3^u)^2 = \{\frac{8}{5},\frac{4}{5}\}g^2\ ,\\
(g_2^d)^2 &= (g_3^d)^2 = \{\frac{6}{5},\frac{3}{5}\}g^2\ ,\\
(g_{23}^u)^2 &= \{0,\frac{4}{5}\}g^2\ ,\ (g_{23}^d)^2 = (g_{32}^d)^2 = \{0,\frac{3}{5}\}g^2\ ,\\
(g^\lambda)^2 &= \frac{15}{7}g^2\ ,\ (g_2^f)^2 = (g_3^f)^2 = \{0,\frac{1}{2}\}g^2\ ,\\
(g_1^f)^2 &= 0\ ,\ (g_4^f)^2 = \{1,0\}g^2\ .
\end{aligned}
\tag{23}
$$

According to the theorem of ref. [27] these models are finite to all orders. After the reduction of couplings the symmetry of W is enhanced [29, 44].

The main difference of the models $\mathbf{A}$ and $\mathbf{B}$ is that three pairs of Higgs quintets and anti-quintets couple to the $\mathbf{24}$ for $\mathbf{B}$ so that it is not necessary to mix them with H_4 and $\overline{H}_4$ in order to achieve the triplet-doublet splitting after the symmetry breaking of $SU(5)$.

In the dimensionful sector, the sum rule gives us the following boundary conditions at the GUT scale [44]:

$$
m_{H_u}^2 + 2m_{\mathbf{10}}^2 = m_{H_d}^2 + m_{\overline{\mathbf{5}}}^2 + m_{\mathbf{10}}^2 = M^2 \quad \text{for} \quad \mathbf{A}\ ; \tag{24}
$$

$$
\begin{aligned}
m_{H_u}^2 + 2m_{\mathbf{10}}^2 &= M^2\ ,\ m_{H_d}^2 - 2m_{\mathbf{10}}^2 = -\frac{M^2}{3}\ ,\\
m_{\overline{\mathbf{5}}}^2 + 3m_{\mathbf{10}}^2 &= \frac{4M^2}{3} \quad \text{for} \quad \mathbf{B},
\end{aligned}
\tag{25}
$$

where we use as free parameters $m_{\overline{\mathbf{5}}} \equiv m_{\overline{\mathbf{5}}_3}$ and $m_{\mathbf{10}} \equiv m_{\mathbf{10}_3}$ for the model $\mathbf{A}$, and $m_{\mathbf{10}} \equiv m_{\mathbf{10}_3}$ for $\mathbf{B}$, in addition to M.

5. Predictions of Low Energy Parameters

Since the gauge symmetry is spontaneously broken below M_{GUT}, the finiteness conditions do not restrict the renormalization properties at low energies, and all it remains are boundary conditions on the gauge and Yukawa couplings (23), the $h = -MC$ relation, and the soft scalar-mass sum rule (9) at M_{GUT}, as applied in the two models. Thus we examine the evolution of these parameters according to their RGEs up to two-loops for dimensionless parameters and at one-loop for dimensionful ones with the relevant boundary conditions. Below M_{GUT} their evolution is assumed to be governed by the MSSM. We further assume a unique supersymmetry breaking scale M_s (which we define as the average of the stop masses) and therefore below that scale the effective theory is just the SM.

The predictions for the top quark mass M_t are ~ 183 and ~ 173 GeV in models **A** and **B** respectively, as can be seen if fig.**??**. Comparing these predictions with the most recent experimental value $M_t^{exp} = (172.7 \pm 2.9)$ GeV [47], and recalling that the theoretical values for M_t may suffer from a correction of $\sim 4\%$ [48], we see that clearly model **B** is preferred. In addition the value of $\tan\beta$ is found to be $\tan\beta \sim 54$ and ~ 48 for models **A** and **B** respectively.

In the SSB sector, besides the constraints imposed by finiteness there are further restrictions imposed by phenomenology. In the case where all the soft scalar masses are universal at the unfication scale, there is no region of M below $O(few\ TeV)$ in which $m_{\tilde{\tau}} > m_{\chi^0}$ is satisfied (where $m_{\tilde{\tau}}$ is the lightest $\tilde{\tau}$ mass, and m_{χ^0} the lightest neutralino mass, which is the lightest supersymmetric particle). But once the universality condition is relaxed this problem can be solved naturally (thanks to the sum rule). More specifically, using the sum rule (9) and imposing the conditions a) successful radiative electroweak symmetry breaking, b) $m^2_{\tilde{\tau}} > 0$ and c) $m_{\tilde{\tau}} > m_{\chi^0}$, a comfortable parameter space for both models (although model **B** requires large $M \sim 1$ TeV) is found.

As additional constraints, we consider the following observables: the anomalous magnetic moment of the muon, $(g-2)_\mu$, rare b decays $\mathrm{BR}(b \to s\gamma)$ and $\mathrm{BR}(B_s \to \mu^+\mu^-)$, as well as the density of cold dark matter in the Universe, assuming it consists mainly of neutralinos.

For the branching ratio $BR(b \to s\gamma)$ [49], we take the present experimental value estimated by the Heavy Flavour Averaging Group (HFAG) is [50]

$$\mathrm{BR}(b \to s\gamma) = (3.54^{+0.30}_{-0.28}) \times 10^{-4}, \tag{26}$$

where the error includes an uncertainty due to the decay spectrum, as well as the statistical error. In the case of the anomalous magnetic moment of the muon $a_\mu \equiv (g-2)_\mu$, we compare our different models with

$$a_\mu^{\mathrm{exp}} - a_\mu^{\mathrm{theo}} = (25.2 \pm 9.2) \times 10^{-10}. \tag{27}$$

For the branching ratio $\mathrm{BR}(B_s \to \mu^+\mu^-)$, the SM prediction is $(3.4 \pm 0.5) \times 10^{-9}$ [51], and the present experimental upper limit from the Fermilab Tevatron collider is 3.4×10^{-7} at the 95% C.L. [52], providing the possibility for the MSSM to dominate the SM contribution.

The lightest supersymmetric particle (LSP) is an excellent candidate for cold dark matter (CDM) [53], with a density that falls naturally within the range

$$0.094 < \Omega_{\mathrm{CDM}} h^2 < 0.129 \tag{28}$$

favoured by a joint analysis of WMAP and other astrophysical and cosmological data [54].

The results for $\mu > 0$ and $\mu < 0$ are different for the models. For **FUTA** with $\mu < 0$ the spectrum starts with an LSP around 750 GeV, whereas for $\mu > 0$ the spectrum starts around 500 GeV. For **FUTB** and $\mu < 0$ the spectrum starts around 300 GeV, and for $\mu > 0$ around 400 GeV.

The main difference, though, is in the value of the running bottom mass $m_{bot}(m_{bot})$, where we have included the corrections coming from bottom squark-gluino loops and top squark-chargino loops [55].

We give the predictions for the running bottom quark mass evaluated at M_Z, $m_{bot}(M_Z) = 2.825 \pm 0.1$ [57], to avoid the large QCD uncertainties at the pole mass. The value of m_{bot} depends strongly on the sign of μ, due to the above mentioned susy radiative corrections. Both for models **A** and **B** the values for $\mu > 0$ are above the central experimental value, with $m_{bot}(M_Z) \sim 4.0 - 5.0$ GeV . For $\mu < 0$ on the other hand, model **B** has overlap with the experimental allowed values, $m_{bot}(M_Z) \sim 2.5 - 2.8$ whereas for model **A**, $m_{bot}(M_Z) \sim 2.0 - 2.6$, there is only a small region of allowed parameter space at two sigma level.
In fig.**??** we give the prediction for the Higgs mass M_h in terms of the unified gaugino mass M for model **FUTB**, which is the one who passes best the set of high energy physics constraints.
The Higgs mass prediction of the two models is, although the details of each of the models differ, in the following range

$$m_h = \quad \sim 115 - 132 \quad GeV, \tag{29}$$

where the uncertainty comes from variations of the gaugino mass M and the soft scalar masses, and from finite (i.e. not logarithmically divergent) corrections in changing renormalization scheme. The one-loop radiative corrections have been included [58] for m_h, but not for the rest of the spectrum. In making the analysis, the value of M was varied from $200-2000$ GeV. We have also included a small variation, due to threshold corrections at the GUT scale, of up to 5% of the FUT boundary conditions. This small variation does not give a noticeable effect in the results at low energies. The requirement $m_h > 114.4$ GeV [59] (neglecting the theoretical uncertainties) excludes the possibility of $M = 200$ GeV for FUTA, as seen also from the graph. In Table 1 we present a representative example of the spectrum for model **FUTB**.
A more detailed numerical analysis, where the results of our program and of the known programs FeynHiggs [60] and Suspect [61] are combined, is currently in progress [62].

6. FUTs based on product gauge groups

Let us now examine the possibility of constructing realistic FUTs based on product gauge groups [45]. Consider the gauge group $\mathrm{SU}(N)_1 \times \mathrm{SU}(N)_2 \times \cdots \times \mathrm{SU}(N)_k$ with n_f copies of the supersymmetric multiplet $(N, N^*, 1, \ldots, 1) + (1, N, N^*, \ldots, 1) + \cdots + (N^*, 1, 1, \ldots, N)$. The one-loop β-function coefficient in the renormalization-group equation of each $\mathrm{SU}(N)$ gauge coupling

M_{top}	173	$m_{bot}(M_Z)$	2.7
$\tan\beta =$	44	α_s	.116
m_{χ_1}	730	$m_{\tilde{\tau}_2}$	1077
m_{χ_2}	916	$m_{\tilde{\nu}_3}$	1040
m_{χ_3}	1345	μ	-1951
m_{χ_4}	1956	B	429
$m_{\chi_1^\pm}$	1345	m_A	1048
$m_{\chi_2^\pm}$	1956	$m_{H^\pm}$	1051
$m_{\tilde{t}_1}$	2216	m_H	1049
$m_{\tilde{t}_2}$	2604	m_h	123.7
$m_{\tilde{b}_1}$	2277	M_1	730
$m_{\tilde{b}_2}$	2584	M_2	1348
$m_{\tilde{\tau}_1}$	809	M_3	3385

Table 1: A representative example of the bottom (running) and top (pole) masses, plus the supersymmetric spectrum for Model FUTB, with $m_{10} =$ 1006 GeV, $M = 1587$ GeV and susy breaking scale $M_{susy} = 2457$ GeV, $\mu < 0$. All masses in the Table are in GeV.

is simply given by

$$b = \left(-\frac{11}{3} + \frac{2}{3}\right) N + n_f \left(\frac{2}{3} + \frac{1}{3}\right) \left(\frac{1}{2}\right) 2N = -3N + n_f N\,. \qquad (30)$$

This means that $n_f = 3$ is a solution of the equation $b = 0$, independently of the values of N and k. Since $b = 0$ is a necessary condition for a finite field theory, the existence of three families of quarks and leptons is natural in such models. This is true of course only if the matter content is exactly as given above.

The model of this type with best phenomenology is the $SU(3)^3$ model discussed in ref.[45], where the details of the model are given. Let us just point out that the finiteness conditions impose in this model the following boundary conditions for the adimensional couplings

$$f^2 = f_{111}^2 = f_{222}^2 = f_{333}^2 = \frac{16}{9} g^2\,. \qquad (31)$$

where the f's are the Yukawa couplings associated to each generation. For the soft susy breaking sector we have the following conditions

$$m_{H_u}^2 + m_{\tilde{t}^c}^2 + m_{\tilde{q}}^2 = M^2 \qquad (32)$$

$$m_{H_d}^2 + m_{\tilde{b}^c}^2 + m_{\tilde{q}}^2 = M^2\,. \qquad (33)$$

A numerical analysis putting as low energy constraints successful radiative electroweak symmetry breaking, $m^2_{\tilde{\tau},\tilde{b},\tilde{t}} > 0$, and correct branching ratio

BR($b \to s\gamma$) shows that it is possible to find regions of parameter space which comply with all the above requirements. In this case with $\mu < 0$, we obtain $m_t \sim 183$ GeV, $\tan\beta \sim 47 - 54$, and the Higgs mass is $\sim 130 - 132$ GeV. The sparticle spectrum starts with an LSP at $m_{\chi^0} \sim 450\,\mathrm{GeV}$, for a boundary condition of $M \sim 1800\,\mathrm{GeV}$. In the case that $\mu > 0$ we do not find solutions which satisfy all the above requirements.

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