Initial Singularity, Λ-Problem and Crossing the Phantom Divide in Scale Invariant TMT Model*

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Abstract

In the framework of the scale invariant model of the Two Measures Field Theory (TMT), we study the dilaton-gravity sector in the context of spatially flat FRW cosmology. The scale invariance is spontaneously broken due to the intrinsic features of the TMT dynamics. If no fine tuning is made, the effective ϕ-Lagrangian $p(\phi, X)$ depends quadratically upon the kinetic term X. Hence TMT represents an explicit example of *the effective k-essence resulting from first principles without any exotic term* in the underlying action intended for obtaining this result. Depending of the choice of regions in the parameter space (but without fine tuning), TMT exhibits interesting outputs for cosmological dynamics, for example: a) *Absence of initial singularity of the curvature while its time derivative is singular.* This is a sort of "sudden" singularities studied by Barrow on purely kinematic grounds. b) Power law inflation in the subsequent stage of evolution which ends with a *graceful exit* into the state with zero cosmological constant (CC). c) Possibility of *resolution of the old CC problem.* From the point of view of TMT, it becomes clear why the old CC problem cannot be solved (without fine tuning) in conventional field theories; d) There is a wide range of the parameters such that in the late time universe: the equation-of-state $w = p/\rho < -1$; *w asymptotically* (as $t \to \infty$) *approaches -1 from below*; ρ approaches a constant, the smallness of which does not require fine tuning of dimensionfull parameters.

1. Basis of Two Measures Field Theory

TMT is a generally coordinate invariant theory where *all the difference from the standard field theory in curved space-time consists only of the following three additional assumptions*:

1. The main supposition is that for describing the effective action for 'gravity + matter' at energies below the Planck scale, the usual form of the action $S = \int L\sqrt{-g}d^4x$ is not complete. Our positive hypothesis is that the effective action has to be of the form[1]

$$S = \int L_1 \Phi d^4x + \int L_2 \sqrt{-g} d^4x \tag{1}$$

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including two Lagrangian densities L_1 and L_2 and two measures of integration $\sqrt{-g}$ and Φ. One is the usual measure of integration $\sqrt{-g}$ in the 4-dimensional space-time manifold equipped with the metric $g_{\mu\nu}$. Another is the new measure of integration Φ in the same 4-dimensional space-time manifold. The measure Φ being a scalar density and a total derivative may be defined by means of four scalar fields φ_a $(a = 1, 2, 3, 4)$:

$$\Phi = \varepsilon^{\mu\nu\alpha\beta}\varepsilon_{abcd}\partial_\mu\varphi_a\partial_\nu\varphi_b\partial_\alpha\varphi_c\partial_\beta\varphi_d. \tag{2}$$

2. It is assumed that the Lagrangian densities L_1 and L_2 are functions of all matter fields, the dilaton field, the metric, the connection but not of the "measure fields" φ_a. In such a case, i.e. when the measure fields enter in the theory only via the measure Φ, the action (1) possesses an infinite dimensional symmetry[1]). One can hope that this symmetry should prevent emergence of a measure fields dependence in L_1 and L_2 after quantum effects are taken into account.

3. Important feature of TMT that is responsible for many interesting and desirable results of the field theory models studied so far[1] consists of the assumption that all fields, including also metric, connection and the *measure fields* φ_a are independent dynamical variables. All the relations between them are results of equations of motion. In particular, the independence of the metric and the connection means that we proceed in the first order formalism and the relation between connection and metric is not necessarily according to Riemannian geometry.

We want to stress that except for the listed three assumptions we do not make any changes as compared with principles of the standard field theory in curved space-time. In other words, all the freedom in constructing different models in the framework of TMT consists of the choice of the concrete matter content and the Lagrangians L_1 and L_2 that is quite similar to the standard field theory.

Variation of the measure fields φ_a results in the equation of the form $B^\mu_a\partial_\mu L_1 = 0$ where the matrix B^μ_a is non-degenerate if $\Phi \neq 0$. Then we get

$$L_1 = sM^4 = const, \tag{3}$$

where $s = \pm 1$ and M is a constant of integration with the dimension of mass. In what follows we make the choice $s = 1$.

Applying the Palatini formalism in TMT one can show that in addition to the Christoffel's connection coefficients, the resulting relation between connection and metric includes also the gradient of the ratio of the two measures

$$\zeta \equiv \frac{\Phi}{\sqrt{-g}} \tag{4}$$

which is a scalar field. Consequently geometry of the space-time with the metric $g_{\mu\nu}$ is non-Riemannian. The gravity and matter field equations obtained by means of the first order formalism contain both ζ and its gradient.

It turns out that at least at the classical level, the measure fields φ_a affect the theory only through the scalar field ζ. The consistency condition of equations of motion has the form of a constraint which determines $\zeta(x)$ as a function of matter fields. By an appropriate change of the dynamical variables which includes a transformation of the metric, one can formulate the theory in a Riemannian space-time. The corresponding frame we call "the Einstein frame". The big advantage of TMT is that in the very wide class of models, *the gravity and all matter fields equations of motion take canonical GR form in the Einstein frame.* All the novelty of TMT in the Einstein frame as compared with the standard GR is revealed only in an unusual structure of the scalar fields effective potential (produced in the Einstein frame), masses of fermions and their interactions with scalar fields as well as in the unusual structure of fermion contributions to the energy-momentum tensor: all these quantities appear to be ζ dependent[1]-[3]. This is why the scalar field $\zeta(x)$ determined by the constraint as a function of matter fields, has a key role in the dynamics of TMT models.

2. Scale invariant model

Our TMT model includes gravity and a dilaton field ϕ. In the early universe ϕ plays the role of inflaton and in the late universe it is is a part of the dark energy. We postulate the global scale invariance:

$$g_{\mu\nu} \to e^{\theta} g_{\mu\nu}, \quad \Gamma^{\mu}_{\alpha\beta} \to \Gamma^{\mu}_{\alpha\beta}, \quad \varphi_a \to \lambda_{ab}\varphi_b, \quad \phi \to \phi - \frac{M_p}{\alpha}\theta, \tag{5}$$

where $\det(\lambda_{ab}) = e^{2\theta}$ and the Lagrangian L_1, L_2 in the action (1) are:

$$L_1 = e^{\alpha\phi/M_p}\left[-\frac{1}{\kappa}R(\Gamma,g) + \frac{1}{2}g^{\mu\nu}\phi_{,\mu}\phi_{,\nu} - V_1 e^{\alpha\phi/M_p}\right],$$
$$L_2 = e^{\alpha\phi/M_p}\left[-\frac{b_g}{\kappa}R(\Gamma,g) + \frac{b_\phi}{2}g^{\mu\nu}\phi_{,\mu}\phi_{,\nu} - V_2 e^{\alpha\phi/M_p}\right]. \tag{6}$$

One should point out that there are two types of the gravitational terms and of the "kinetic-like terms" which respect the scale invariance : the terms of the one type coupled to the measure Φ and those of the other type coupled to the measure $\sqrt{-g}$. Using the freedom in normalization of the measure fields φ_a we set the coupling constant of the scalar curvature to the measure Φ to be $-\frac{1}{\kappa}$. Normalizing all the fields such that their couplings to the measure Φ have no additional factors, we are not able in general to provide the same in the terms describing the appropriate couplings to the measure $\sqrt{-g}$. This fact explains the need to introduce the dimensionless real parameters b_g and b_ϕ and we will only assume that *they are positive and have close orders of magnitudes.* Note that in the case of the choice $b_g = b_\phi$ we would proceed with *the fine tuned model.* The real positive parameter α is assumed to be of the order of unity; in all numerical solutions presented in this talk we set $\alpha = 0.2$. For the Newton constant we use $\kappa = 16\pi G$, $M_p = (8\pi G)^{-1/2}$. Note also the possibility of introducing two different pre-potentials which are exponential functions of the dilaton ϕ coupled to the measures Φ and $\sqrt{-g}$ with factors V_1 and V_2 respectively. Such ϕ-dependence provides the scale symmetry (5). However V_1 and V_2 might be Higgs-dependent and then they play the role of the Higgs pre-potentials[2].

The common feature of all equations of motion in the original frame is that the measure Φ degrees of freedom appear only through dependence upon the scalar field ζ, Eq.(4). In particular, all the equations of motion and the solution for the connection coefficients include terms proportional to $\partial_\mu\zeta$, that makes space-time non Riemannian and all equations of motion - noncanonical. It turns out that when working with the new metric

$$\tilde{g}_{\mu\nu} = e^{\alpha\phi/M_p}(\zeta + b_g)g_{\mu\nu}, \tag{7}$$

which we call the Einstein frame, the connection becomes Riemannian. Since $\tilde{g}_{\mu\nu}$ is invariant under the scale transformations (5), spontaneous breaking of the scale symmetry (by means of Eq.(3)) is reduced in the Einstein frame to the *spontaneous breakdown of the shift symmetry* $\phi \to \phi + const$. Notice that the Goldstone theorem generically is not applicable in this model[4].

Equations of motion in the Einstein frame take the standard GR form:

$$G_{\mu\nu}(\tilde{g}_{\alpha\beta}) = \frac{\kappa}{2}T^{eff}_{\mu\nu}, \tag{8}$$

where $G_{\mu\nu}(\tilde{g}_{\alpha\beta})$ is the Einstein tensor in the Riemannian space-time with the metric $\tilde{g}_{\mu\nu}$; the energy-momentum tensor $T^{eff}_{\mu\nu}$ reads

$$T^{eff}_{\mu\nu} = \frac{\zeta + b_\phi}{\zeta + b_g}\left(\phi_{,\mu}\phi_{,\nu} - \tilde{g}_{\mu\nu}X\right) + \tilde{g}_{\mu\nu}V_{eff}(\phi;\zeta,M) - \tilde{g}_{\mu\nu}\frac{\delta\cdot b_g}{\zeta + b_g}X\,, \tag{9}$$

where $X \equiv \frac{1}{2}\,\tilde{g}^{\alpha\beta}\phi_{,\alpha}\phi_{,\beta}$, $\delta = (b_g - b_\phi)/b_g$ and

$$V_{eff}(\phi;\zeta,M) = \frac{b_g\left[M^4e^{-2\alpha\phi/M_p} + V_1\right] - V_2}{(\zeta + b_g)^2}. \tag{10}$$

The scalar field ζ is determined by means of the constraint which appears as the consistency condition of the equations of motion:

$$(b_g - \zeta)\left[M^4e^{-2\alpha\phi/M_p} + V_1\right] - 2V_2 - \delta\cdot b_g(\zeta + b_g)X = 0\,. \tag{11}$$

Applying the constraint (11) to the ϕ-equation one can reduce it to the form

$$\frac{1}{\sqrt{-\tilde{g}}}\,\partial_\mu\left[\frac{\zeta + b_\phi}{\zeta + b_g}\sqrt{-\tilde{g}}\tilde{g}^{\mu\nu}\partial_\nu\phi\right] - \frac{2\alpha\zeta}{(\zeta + b_g)^2M_p}\,M^4e^{-2\alpha\phi/M_p} = 0. \tag{12}$$

The effective energy-momentum tensor (9) can be represented in a form of that of a perfect fluid with the following energy and pressure densities resulting from Eqs.(9) and (10) after inserting the solution $\zeta = \zeta(\phi, X; M)$ of Eq.(11):

$$\rho(\phi, X; M) = X + \frac{(M^4 e^{-2\alpha\phi/M_p} + V_1 - \delta \cdot b_g X)^2 - 4\delta^2 b_g^2 X^2}{4[b_g(M^4 e^{-2\alpha\phi/M_p} + V_1) - V_2]}, \tag{13}$$

$$p(\phi, X; M) = X - \frac{\left(M^4 e^{-2\alpha\phi/M_p} + V_1 + \delta b_g X\right)^2}{4[b_g(M^4 e^{-2\alpha\phi/M_p} + V_1) - V_2]}. \tag{14}$$

Now, we will proceed in a spatially flat FRW universe with metric $\tilde{g}_{\mu\nu}= diag(1, -a^2, -a^2, -a^2)$ filled with homogeneous scalar field $\phi(t)$. Then the ϕ equation reads

$$Q_1 \ddot{\phi} + 3HQ_2\dot{\phi} - \frac{\alpha}{M_p} Q_3 M^4 e^{-2\alpha\phi/M_p} = 0 \tag{15}$$

where H is the Hubble parameter, $\dot{\phi} \equiv d\phi/dt$ and we have used the following notations

$$Q_1 = (b_g + b_\phi)(M^4 e^{-2\alpha\phi/M_p} + V_1) - 2V_2 - 3\delta^2 b_g^2 X, \tag{16}$$

$$Q_2 = (b_g + b_\phi)(M^4 e^{-2\alpha\phi/M_p} + V_1) - 2V_2 - \delta^2 b_g^2 X, \tag{17}$$

$$-\frac{\alpha}{M_p} Q_3 M^4 e^{-2\alpha\phi/M_p} = 2[b_g(M^4 e^{-2\alpha\phi/M_p} + V_1) - V_2]\rho_{,\phi}\,. \tag{18}$$

It is interesting that the non-linear X-dependence appears here in the framework of the underlying model without exotic terms in the Lagrangians L_1 and L_2. This effect follows just from the fact that there are no reasons to choose the parameters b_g and b_ϕ in the action (6) to be equal in general; on the contrary, the choice $b_g = b_\phi$ would be a fine tuning. Besides one should stress that the ϕ dependence in ρ, p and in equations of motion emerges only in the form $M^4 e^{-2\alpha\phi/M_p}$ where M is the integration constant (see Eq.(3)), i.e. due to the spontaneous breakdown of the scale symmetry (5) (or the shift symmetry $\phi \to \phi + const.$ in the Einstein frame). Thus the above equations represent an *explicit example of k-essence*[5] *resulting from first principles including the scale invariance.* The system of equations (8), (13)-(15) can be obtained from the k-essence type effective action

$$S_{eff} = \int \sqrt{-\tilde{g}} d^4x \left[-\frac{1}{\kappa} R(\tilde{g}) + p\,(\phi, X; M) \right], \tag{19}$$

where the scalar field effective Lagrangian $p(\phi, X; M)$, Eq.(14), can be represented in the form

$$p\,(\phi, X; M) = K(\phi)X + L(\phi)X^2 - \frac{[V_1 + M^4 e^{-2\alpha\phi/M_p}]^2}{4[b_g\left(V_1 + M^4 e^{-2\alpha\phi/M_p}\right) - V_2]}, \tag{20}$$

where $K(\phi)$ and $L(\phi)$ depend on ϕ only via $M^4 e^{-2\alpha\phi/M_p}$. Note that the Lagrangian (20) differs from that of Ref.[6] by the sign of $L(\phi)$: in our case $L(\phi) < 0$ provided the effective potential (the last term in (20)) is non-negative. *This result cannot be removed by a choice of the parameters* of the underlying action (6) while in Ref.[6] the positivity of $L(\phi)$ was an essential *assumption*. This difference plays a crucial role in a number of specific features of the scalar field dynamics, e.g. *the absence of the initial singularity of the curvature is directly related to the negative sign of* $L(\phi)$.

3. Resolution of the Old Cosmological Constant Problem

The cosmological dynamics is significantly simplified if $\delta = 0$. Note that studying the cosmological constant (CC) problem we are in fact interested in a regime where $X \to 0$. Therefore in what it concerns the CC problem, the results of the fine tuned model ($\delta = 0$ is equivalent to $b_g = b_\phi$) coincide with those of the generic case $\delta \neq 0$. So let us consider a spatially flat FRW cosmological model governed by Eqs.(13)-(15) (where we set $\delta = 0$) and $\dot{a}^2/a^2 = (3M_p)^{-2}\rho$. In the fine tuned case under consideration, the constraint (11) yields

$$\zeta = \zeta(\phi, X; M)|_{\delta=0} \equiv b_g - \frac{2V_2}{V_1 + M^4 e^{-2\alpha\phi/M_p}} \,. \qquad (21)$$

The energy density and pressure take then the canonical form,

$$\rho = \dot{\phi}^2/2 + V_{eff}^{(0)}(\phi) \,, \qquad p = \dot{\phi}^2/2 - V_{eff}^{(0)}(\phi),$$

where the effective potential of the scalar field ϕ results from Eqs.(10) and (21)

$$V_{eff}^{(0)}(\phi) \equiv V_{eff}(\phi; \zeta, M)|_{\delta=0} = \frac{[V_1 + M^4 e^{-2\alpha\phi/M_p}]^2}{4[b_g\left(V_1 + M^4 e^{-2\alpha\phi/M_p}\right) - V_2]} \qquad (22)$$

and the ϕ-equation (15) is reduced to $\ddot{\phi} + 3H\dot{\phi} + dV_{eff}^{(0)}/d\phi = 0$. Notice that $V_{eff}^{(0)}(\phi)$ is non-negative for any ϕ provided $b_g V_1 \geq V_2$ that we will assume in what follows. We assume also that $b_g > 0$.

The most remarkable feature of the effective potential (22) is that it is proportional to the square of $V_1 + M^4 e^{-2\alpha\phi/M_p}$ which is a straightforward consequence of our basic assumption that L_1 and L_2 are independent of the measure fields. Due to this, as $V_1 < 0$ and $V_2 < 0$, *the effective potential has a minimum where it equals zero automatically*, without any further tuning of the parameters V_1 and V_2. This occurs in the process of evolution of the field ϕ at the value of $\phi = \phi_0$ where $V_1 + M^4 e^{-2\alpha\phi_0/M_p} = 0$. This means that the universe evolves into the state with zero cosmological constant without any additional tuning of the parameters and initial conditions.

If such type of the structure for the scalar field potential in a conventional (non TMT) model would be chosen "by hand" it would be a sort of fine tuning. But in our TMT model it is not the starting point, *it is rather a result* obtained in the Einstein frame of TMT models with spontaneously broken global scale symmetry including the shift symmetry $\phi \to \phi + const$. Note that the assumption of scale invariance is not necessary for the effect of appearance of the perfect square in the effective potential in the Einstein frame and therefore for the described mechanism of disappearance of the cosmological constant, see Ref.[1]. To provide the global scale invariance (5), the prepotentials V_1 and V_2 enter in the action (6) with factor $e^{2\alpha\phi/M_p}$. However, if quantum effects (considered in the original frame) break the scale invariance of the action via modification of existing prepotentials or by means of generation of other prepotentials with arbitrary ϕ dependence (and in particular a "normal" cosmological constant term $\int \tilde{\Lambda}\sqrt{-g}d^4x$), this cannot change the result of TMT that the effective potential generated in the Einstein frame is proportional to a perfect square.

At the first glance this effect contradicts the Weinberg's no-go theorem[7] which states that there cannot exist a field theory model where the cosmological constant is zero without fine tuning. However one can show that conditions of the Weinberg's theorem do not hold in our model. Note that $\zeta = \zeta_0(\phi)$, Eq.(21), becomes singular

$$|\zeta| \approx \frac{2|V_2|}{|V_1 + M^4 e^{-2\alpha\phi/M_p}|} \to \infty \qquad as \qquad \phi \to \phi_0. \tag{23}$$

In this limit the effective potential (10) (or Eq.(22)) behaves as $V_{eff} \approx \frac{|V_2|}{\zeta^2}$. Thus, disappearance of the cosmological constant occurs in the regime where $|\zeta| \equiv |\Phi|/\sqrt{-g} \to \infty$. In this limit, the dynamical role of the terms of the Lagrangian L_2 (coupled with the measure $\sqrt{-g}$) in the action (6) becomes negligible in comparison with the terms of the Lagrangian L_1 (see also the general form of the action (1)). It is evident that **the limit of the TMT action (1) as $|\zeta| \to \infty$ is opposite to the conventional field theory (with only measure $\sqrt{-g}$) limit of the TMT action**. From the point of view of TMT, this is **the answer to the question why the old CC problem cannot be solved (without fine tuning) in theories with only the measure of integration $\sqrt{-g}$ in the action**.

Recall that one of the basic assumptions of the Weinberg's no-go theorem is that all fields in the vacuum must be constant. This is also assumed for the metric tensor, components of which in the vacuum must be **nonzero** constants. However, this is not the case in the underlying TMT action (6) defined in the original (non Einstein) frame if we ask what is the metric tensor $g_{\mu\nu}$ in the $\Lambda = 0$ vacuum . To see this let us note that in the Einstein frame all the terms in the cosmological equations are regular. This means that the metric tensor in the Einstein frame $\tilde{g}_{\mu\nu}$ is always well defined, including the $\Lambda = 0$ vacuum state $\phi = \phi_0$ where ζ is infinite. Taking this into account and using the transformation to the Einstein frame (7) we see that **all components of the metric in the original frame $g_{\mu\nu}$ go**

to zero overall in space-time as ϕ approaches the $\Lambda = 0$ vacuum state:

$$g_{\mu\nu} \sim \frac{1}{\zeta} \sim V_1 + M^4 e^{-2\alpha\phi/M_p} \to 0 \quad (\mu, \nu = 0, 1, 2, 3) \quad as \quad \phi \to \phi_0. \tag{24}$$

This result shows that the Weinberg's analysis based on the study of the trace of the energy-momentum tensor misses any sense in the case $g_{\mu\nu} = 0$.

It follows immediately from (24) that $\sqrt{-g}$ tends to zero like $\sqrt{-g} \sim \zeta^{-2}$ as $\phi \to \phi_0$. Then the definition (5) implies that the integration measure Φ also tends to zero but rather like $\Phi \sim \zeta^{-1}$ as $\phi \to \phi_0$. Thus both the measure Φ and the measure $\sqrt{-g}$ become degenerate in the $\Lambda = 0$ vacuum state $\phi = \phi_0$. However $\sqrt{-g}$ tends to zero more rapidly than Φ.

As we have discussed in detail[1], with the original set of variables used in the underlying TMT action (1), (6) it is very hard or may be even impossible to display the physical meaning of TMT models. One of the reasons is that in the framework of the postulated need to use the Palatini formalism, the original metric $g_{\mu\nu}$ and connection $\Gamma^{\alpha}_{\mu\nu}$ appearing in the underlying TMT action describe a non-Riemannian space-time. The transformation to the Einstein frame (7) enables to see the physical meaning of TMT because the space-time becomes Riemannian in the Einstein frame. Now we see that the transformation to the Einstein frame (7) plays also the role of a **regularization of the space-time metric**: the singular behavior of the transformation (7) as $\phi \approx \phi_0$ compensates the disappearance of the original metric $g_{\mu\nu}$ in the vacuum $\phi = \phi_0$. As a result of this the metric in the Einstein frame $\tilde{g}_{\mu\nu}$ turns out to be well defined in all physical states including the $\Lambda = 0$ vacuum state.

4. Absence of Initial Singularity of the Curvature and Inflationary Cosmology with Graceful Exit to $\Lambda = 0$ Vacuum

Let us start from the analysis of Eq.(15). The interesting feature of this equation is that each of the factors $Q_i(\phi, X)$ $(i = 1, 2, 3)$ can get to zero and this effect depends on the range of the parameter space chosen.
For $Q_1 \neq 0$, Eq. (15) results in the well known equation[9]

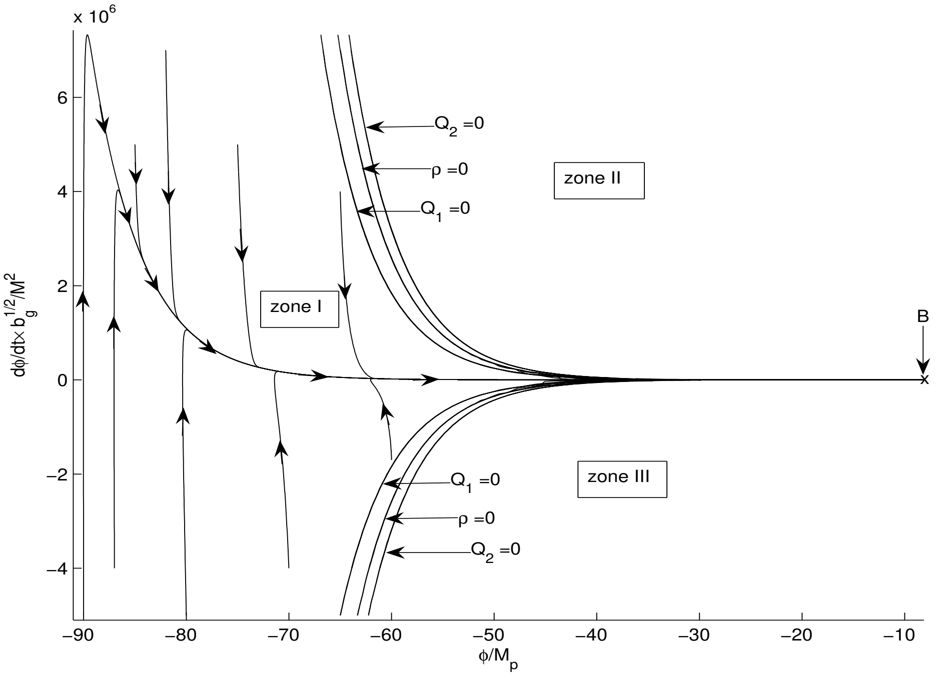
$$\ddot{\phi} + \frac{\sqrt{3\rho}}{M_p} c_s^2 \dot{\phi} + \frac{\rho_{,\phi}}{\rho_{,X}} = 0, \tag{25}$$

where c_s is the effective sound speed of perturbations

$$c_s^2 = \frac{p_{,X}}{\rho_{,X}} = \frac{Q_2}{Q_1} \qquad and \qquad \frac{\rho_{,\phi}}{\rho_{,X}} = -\frac{\alpha}{M_p} \cdot \frac{Q_3}{Q_1} \cdot M^4 e^{-2\alpha\phi/M_p}, \tag{26}$$

ρ and p are defined by Eqs.(13), (14) and Q_i $(i = 1, 2, 3)$ - by Eqs.(16)-(18).

It follows from the definitions of Q_1 and Q_2 that $c_s^2 > 1$ when $Q_1 > 0$ and $X > 0$ (that implies $Q_2 > 0$). However $c_s^2 < 1$ when $Q_1 < 0$ and $Q_2 < 0$.



In the model with $V_1 < 0$ and $V_2 < 0$, the structure of the phase plane is presented in the figure above for the following set of the parameters: $\alpha = 0.2$, $\delta = 0.1$, $V_1 = -30$, $V_2 = -50$. With such a choice of the parameters the following condition is satisfied

$$(b_g + b_\phi)V_1 - 2V_2 > 0 \tag{27}$$

that determines the structure of the phase plane. By the two branches of the line $Q_1 = 0$, the phase plane is divided into three large dynamically disconnected zones. In the most part of two of them (II and III), the energy density is negative ($\rho < 0$). In the couple of regions between the lines $Q_1 = 0$ and $\rho = 0$, $\rho > 0$ but $Q_1 < 0$.

The zone I of the phase plane (where $Q_1 > 0$, $\rho > 0$ and $Q_2 > 0$) is of a great cosmological interest: All the phase curves start with very steep approach to an attractor and they arrive the attractor asymptotically. One can show that the attractor does not intersect the line $Q_1 = 0$. In the neighborhood of the line $Q_1 = 0$ all the phase curves exhibit a *repulsive* behavior from this line. In other words, the shape of two branches of $Q_1 = 0$ do not allow a classical dynamical continuation of the phase curves to the past in time without crossing the classical barrier formed by the line $Q_1 = 0$. This is true for all finite values of the initial conditions ϕ_{in}, $\dot{\phi}_{in}$ in zone I. The cosmological evolution corresponding to the phase curves with $\phi_{in} \ll -M_p$ is a power law inflation which ends with the graceful exit to a zero CC vacuum state without fine tuning (point B in the figure marks the corresponding oscillatory regime).

In two regions between the lines $Q_1 = 0$ and $Q_2 = 0$ that we refer for short as ($Q_1 \to Q_2$)-regions,, the squared sound speed of perturbations is negative, $c_s^2 < 0$. This means that on the right hand side of the classical barrier $Q_1 = 0$, the model is absolutely unstable. Moreover, this pure imaginary sound speed becomes infinite in the limit $Q_1 \to 0-$. Thus the branches of the line $Q_1 = 0$ divide zone I (of the classical dynamics) from the ($Q_1 \to Q_2$)-regions where the physical significance of the model is unclear. Note that the line $\rho = 0$ divides the ($Q_1 \to Q_2$)-regions into two subregions with opposite signs of the classical energy density.

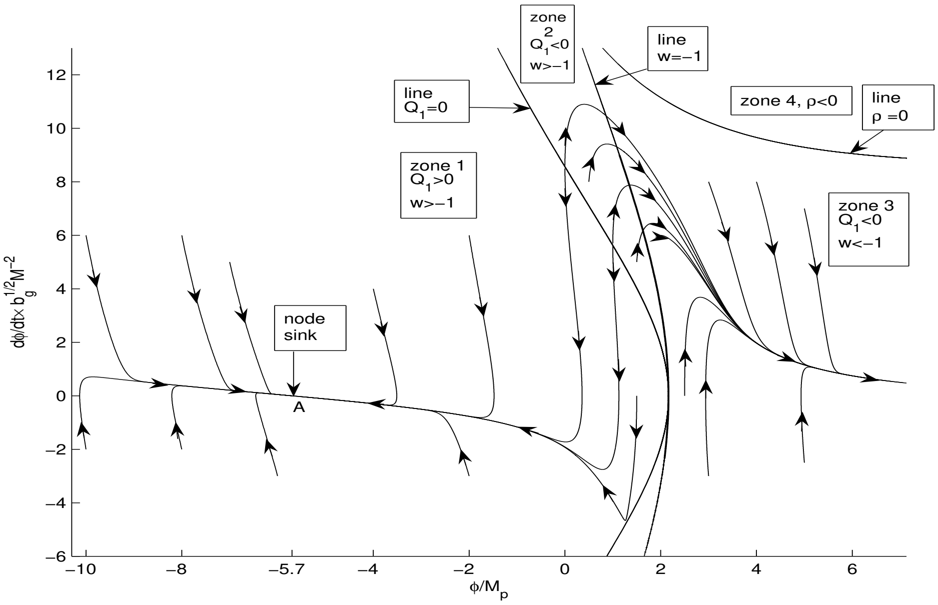
Thus the structure of the phase plane yields a conclusion that the starting point of the classical history in the phase plane can be only in zone I and **the line $Q_1 = 0$ is the limiting set of points where the classical history might begin**. For any finite initial values of ϕ_{in} and $\dot{\phi}_{in}$ at the initial cosmic time t_{in}, the duration $t_{in} - t_s$ of the continuation of the evolution into the past up to the moment t_s when the phase trajectory arrives the line $Q_1 = 0$, is finite. Note that $\dot{\phi}_s = \dot{\phi}(t_s)$, the energy density $\rho_s = \rho(t_s)$ and the pressure $p_s = p(t_s)$ are finite in all points of the line $Q_1 = 0$ with finite $\phi_s = \phi(t_s)$. One can also show that the strong energy condition is satisfied in regions of zone I close to the line $Q_1 = 0$ including the line itself. The detailed analysis yields the conclusion that the singularity as $t \to t_s$ is a sort of sudden singularities studied by Barrow[8] on purely kinematic grounds: $\dot{\phi} \propto \sqrt{t - t_s}$, the scalar curvature is finite but its time derivative $\dot{R} \propto -a^{-1}d^3a/dt^3 \propto -(t - t_s)^{-1/2} \to -\infty$.

5. Crossing the Phantom Divide

Equations $Q_i(\phi, \dot{\phi}) = 0 \quad (i = 1, 2, 3)$ determine lines in the phase plane $(\phi, \dot{\phi})$. In terms of a mechanical interpretation of Eq.(15), the change of the sign of Q_1 can be treated as the change of the mass of "the particle". Therefore one can think of situation where "the particle" climbs up in the potential with acceleration. It turns out that when the scalar field is behaving in this way, the flat FRW universe may undergo a super-acceleration.
With simple algebra one can see that the following "sign rule" holds for the equation-of-state: $sign(w + 1) = sign(Q_2)$. Therefore if in the one side from the line $Q_2(\phi, \dot{\phi}) = 0 \quad w > -1$ then in the other side $w < -1$. To incorporate the region of the phase plane where $w < -1$ into the cosmological dynamics one should provide that the line $Q_2(\phi, \dot{\phi}) = 0$ lies in a dynamically permitted zone. As we have seen in Sec.4, the condition (27) disposes the line $Q_2(\phi, \dot{\phi}) = 0$ in the physically unacceptable region where $\rho < 0$. It turns out that if the opposite condition holds

$$(b_g + b_\phi)V_1 - 2V_2 < 0 \tag{28}$$

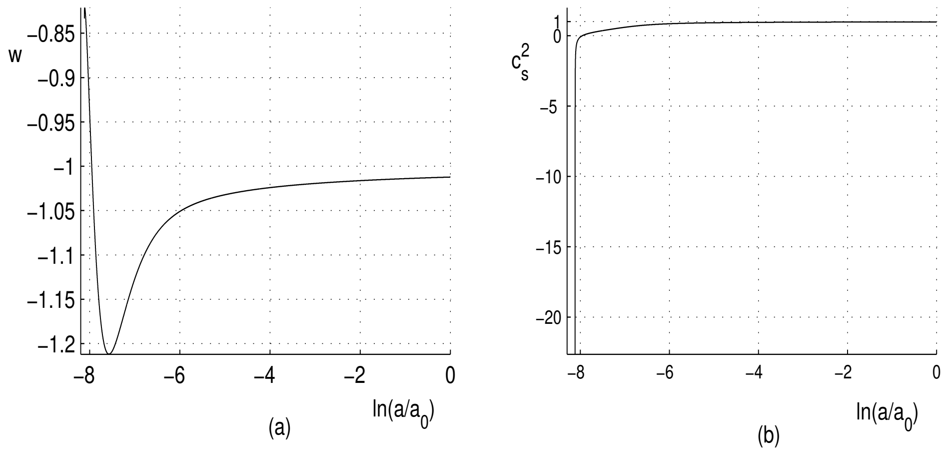
then the line $Q_2(\phi, \dot{\phi}) = 0$ may be in a dynamically permitted zone.



There are a lot of sets of parameters providing the $w < -1$ phase in the late universe. For example we are demonstrating here this effect with the following set of the parameters of the underlying action(6): $\alpha = 0.2$, $V_1 = 10M^4$ and $V_2 = 9.9b_g M^4$, $\delta = 0.1$. The results of the numerical solution are presented in the figure above. The phase plane is divided into two regions by the line $\rho = 0$. The region $\rho > 0$ is divided into two dynamically disconnected regions by the line $Q_1(\phi, \dot{\phi}) = 0$. To the left of the line $Q_1(\phi, \dot{\phi}) = 0$ - zone 1 where $Q_1 > 0$. In the zone 1, there is a node sink $A(-5.7M_p, 0)$. Among trajectories converging to node A there are also trajectories corresponding to a power low inflation of the early universe. Note that the same mechanism of the sudden initial singularity we have discussed in Sec.4, holds also here for solutions whose phase curves are located in zone 1.

In the region to the right of the line $Q_1(\phi, \dot{\phi}) = 0$, all phase curves approach the attractor which in its turn asymptotically (as $\phi \to \infty$) takes the form of the straight line $\dot{\phi} = 0$. This region is divided into two zones by the line $Q_2(\phi, \dot{\phi}) = 0$. In all points of this line $w = -1$. In zone 2, i.e. between the lines $Q_1 = 0$ and $w = -1$, the equation-of-state $w > -1$ and the sound speed $c_s^2 < 0$. Therefore in zone 2 the model is absolutely unstable and has no any physical meaning. In zone 3, i.e. between the line $w = -1$ and the line $\rho = 0$, the equation-of-state $w < -1$ and the sound speed $c_s^2 > 0$. The main features of the solution of the equations of motion are the following (part of them, for a particular choice of the initial data $\phi_{in} = M_p$, $\dot{\phi}_{in} =$

$9M^4/\sqrt{b_g}$, one can see in the figure below): 1) ϕ slowly increases in time; 2) the energy density ρ slowly increases approaching a constant; 3) in zone 3, w becomes less than -1 and after achieving a minimum (for the initial data chosen, this minimum is $w \approx -1.2$) it then increases asymptotically approaching -1 from below. Using the classification of Ref.[9] of conditions for the dark energy to evolve from the state with $w > -1$ to the phantom state, we see that transition of the phase curves from zone 2 (where $w > -1$) to the phantom zone 3 (where $w < -1$) occurs under the conditions $p_{,X} = 0$, $\rho_{,X} \neq 0$, $X \neq 0$. Qualitatively the same behavior one observes for all initial conditions $(\phi_{in}, \dot{\phi}_{in})$ given in the zone 2.



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