The one-loop renormalization of the gauge sector in the θ-expanded noncommutative standard model

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Abstract

In this paper we construct a version of the standard model gauge sector on noncommutative space-time which is one-loop renormalizable to first order in the expansion in the noncommutativity parameter θ. The one-loop renormalizability is obtained by the Seiberg-Witten redefinition of the noncommutative gauge potential for the model containing the usual six representations of matter fields of the first generation.

1. Introduction

The interest to formulate a consistent quantum field theory on noncommutative space comes, besides from string theory, also from mathematics [1] and from phenomenology. There are two main approaches to define gauge theories on the canonical noncommutative space. One possibility, extensively analyzed in the literature [2, 3], is to replace the ordinary product in the Lagrangian by the Moyal-Weyl $\star$-product; it is well defined owing to associativity and the trace property of the $\star$-product. Using this prescription, however, only U(N) gauge theories can be consistently defined and the group representations are restricted to the fundamental and the adjoint. This implies in particular the quantization of the electric charge which takes values in $\{\pm 1, 0\}$. In perturbative quantization, the interaction vertices obtain additional phase factors in comparison with commutative theory, and this leads to the well-know UV/IR mixing.

A slightly different and nonequivalent representation is the so-called θ-expanded approach. A consequence of the requirement that the gauge algebra closes on noncommutative fields is that the fields are enveloping algebra-valued. Using the Seiberg-Witten map, which is also an expansion

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in the noncommutativity parameter θ, noncommutative fields are expressed in terms of their commutative counterparts [4, 5]. The major advantage of this approach is that models with any gauge group and any particle content can be constructed.

There is a number of versions of the noncommutative standard model in the θ- expanded approach [6, 7, 8, 9]. The argument of renormalizability was previously not included in the construction because it was believed that field theories on noncommutative Minkowski space were not renormalizable in general [10, 11]. However, a recent result on the one-loop renormalizability of the θ-expanded noncommutative SU(N) gauge theory opens different perspectives [12, 13]. Of course, renormalizability in linear order does not mean renormalizability of the complete theory, but one can expect that the additional Ward identities, which correspond to the full noncommutative symmetry and relate different orders, might help. In this paper we will follow paper [12]. We show that it is possible to construct a version of the NCSM gauge sector which is one-loop renormalizable to first order in θ. An important point in our consideration is that the SW map is not unique. Therefore, the SW map in fact gives a class of 'truncated' gauge theories, no theory a priori preferred. We show that the requirement of renormalizability pinpoints one of them as 'physical'.

2. Noncommutative standard model

2.1. General considerations

The noncommutative space which we consider is the flat Minkowski space, generated by four hermitian coordinates $\widehat{x}^\mu$ which satisfy the commutation rule

$$[\widehat{x}^\mu, \widehat{x}^\nu] = i\theta^{\mu\nu} = \text{const.} \tag{2..1}$$

The algebra of the functions $\widehat{\phi}(\widehat{x})$, $\widehat{\chi}(\widehat{x})$ on this space can be represented by the algebra of the functions $\widehat{\phi}(x)$, $\widehat{\chi}(x)$ on the commutative $\mathbf{R}^4$ with the Moyal-Weyl multiplication:

$$\widehat{\phi}(x) \star \widehat{\chi}(x) = e^{\frac{i}{2}\theta^{\mu\nu}\frac{\partial}{\partial x^\mu}\frac{\partial}{\partial y^\nu}}\widehat{\phi}(x)\widehat{\chi}(y)|_{y\to x} \ . \tag{2..2}$$

It is possible to represent the action of an arbitrary Lie group G (with the generators denoted by T^a) on noncommutative space. In analogy to the ordinary case, one introduces the gauge parameter $\widehat{\Lambda}(x)$ and the vector potential $\widehat{V}_\mu(x)$. The main difference is that the noncommutative $\widehat{\Lambda}$ and $\widehat{V}_\mu$ cannot take values in the Lie algebra $\mathcal{G}$ of the group G: they are enveloping algebra-valued. The noncommutative gauge field strength $\widehat{F}_{\mu\nu}$ is defined in the usual way

$$\widehat{F}_{\mu\nu} = \partial_\mu\widehat{V}_\nu - \partial_\nu\widehat{V}_\mu - i(\widehat{V}_\mu \star \widehat{V}_\nu - \widehat{V}_\nu \star \widehat{V}_\mu). \tag{2..3}$$

There is, however, a relation between the noncommutative gauge symmetry and the commutative one: it is given by the Seiberg-Witten (SW) mapping

[4]. The expansions of the NC vector potential and of the field strength, up to first order in θ, read

$$\begin{aligned}\widehat{V}_\rho(x) &= V_\rho(x) - \frac{1}{4}\,\theta^{\mu\nu}\left\{V_\mu(x), \partial_\nu V_\rho(x) + F_{\nu\rho}(x)\right\}.\\ \widehat{F}_{\rho\sigma} &= F_{\rho\sigma} + \frac{1}{4}\,\theta^{\mu\nu}\Big(2\{F_{\mu\rho}, F_{\nu\sigma}\} - \{V_\mu, (\partial_\nu + D_\nu)F_{\rho\sigma}\}\Big)\end{aligned} \tag{2..4}$$

D_μ is the commutative covariant derivative.

The solution for the SW map given above is not unique. As it was shown in [10, 15], all expressions $\widehat{V}'_\mu$, $\widehat{F}'_{\mu\nu}$ of the form

$$\widehat{V}'_\mu = \widehat{V}_\mu + X_\mu, \qquad \widehat{F}'_{\mu\nu} = \widehat{F}_{\mu\nu} + D_\mu X_\nu - D_\nu X_\mu, \tag{2..5}$$

are solutions to the closing condition to linear order, if X_μ is a gauge covariant expression linear in θ, otherwise arbitrary. One can think of this transformation as of a redefinition of the fields V_μ and $F_{\mu\nu}$.

Taking the action of the noncommutative gauge theory

$$S = -\frac{1}{2}\,\mathrm{Tr}\int d^4x\,\widehat{F}_{\mu\nu}\star\widehat{F}^{\mu\nu}, \tag{2..6}$$

and expanding the fields via SW map and $*$ product we obtain the expression

$$\begin{aligned}S &= -\frac{1}{2}\,\mathrm{Tr}\int d^4x\,F_{\mu\nu}F^{\mu\nu}\\ &\quad + \theta^{\mu\nu}\,\mathrm{Tr}\int d^4x\,\Big(\frac{a}{4}\,F_{\mu\nu}F_{\rho\sigma} - F_{\mu\rho}F_{\nu\sigma}\Big)F^{\rho\sigma}.\end{aligned} \tag{2..7}$$

where SW freedom (2..5) is also used. The coefficient a is going to be fixed by the requirement of renormalizability in the next section.

2.2. $\mathrm{U(1)_Y}\otimes\mathrm{SU(2)_L}\otimes\mathrm{SU(3)_C}$

The discussion given above was a general one, without any specification of the gauge group G or of its representations. In (2..7) we have a factor $\mathrm{Tr}\,\{T^a, T^b\}T^c \sim d^{abc}$. One could perhaps assume that, as the field strength transforms according to the adjoint representation, the symmetric coefficients d^{abc} are given in that representation. However, when the matter fields are included, other representations of G are present too, and therefore the expression (2..7) is ambiguous.

To start the discussion of the gauge field action-dependence on the gauge group and/or on its representation, we use the most general form of the action, [7]:

$$S_{cl} = -\frac{1}{2}\int d^4x\,\sum_{\mathcal{R}} C_{\mathcal{R}}\,\mathrm{Tr}\Big(\mathcal{R}(\widehat{F}_{\mu\nu}) * \mathcal{R}(\widehat{F}^{\mu\nu})\Big). \tag{2..8}$$

The sum is, in principle, taken over all irreducible representations $\mathcal{R}$ of G with arbitrary weights $C_{\mathcal{R}}$. Of course, for the gauge group G we take $\mathrm{U(1)_Y} \otimes \mathrm{SU(2)_L} \otimes \mathrm{SU(3)_C}$. To relate the action (2..7) to the usual action of the commutative standard model, we make the decompositions

$$\begin{aligned} V_\mu &= g'\mathcal{A}_\mu \mathcal{R}(Y) + gB^i_\mu \mathcal{R}(T^i_L) + g_S G^a_\mu \mathcal{R}(T^a_S), \\ F_{\mu\nu} &= g' f_{\mu\nu} \mathcal{R}(Y) + gB^i_{\mu\nu} \mathcal{R}(T^i_L) + g_S G^a_{\mu\nu} \mathcal{R}(T^a_S). \end{aligned}$$

The $\mathcal{R}(Y)$, $\mathcal{R}(T^i_L)$, $\mathcal{R}(T^a_S)$ denote the representations of the group generators Y, T^i_L and T^a_S of $\mathrm{U(1)_Y}$, $\mathrm{SU(2)_L}$ and $\mathrm{SU(3)_C}$, respectively; the group indices run as $i, j = 1, \dots 3$ and $a, b = 1, \dots 8$. According to [7], we take that $C_{\mathcal{R}}$ are nonzero only for the particle representations which are present in the standard model. Then from (2..8) we obtain the expression for the θ-independent part of the Lagrangian

$$\begin{aligned} \mathcal{L}_{SM} &= -\frac{g'^2}{2} \sum_{\mathcal{R}} C_{\mathcal{R}} d(\mathcal{R}_2) d(\mathcal{R}_3) \mathcal{R}_1(Y) \mathcal{R}_1(Y)\, f_{\mu\nu} f^{\mu\nu} \\ &- \frac{g^2}{2} \sum_{\mathcal{R}} C_{\mathcal{R}} d(\mathcal{R}_3) \mathrm{Tr}\,(\mathcal{R}(T^i_L)\mathcal{R}(T^j_L))\, B^i_{\mu\nu} B^{\mu\nu j} \\ &- \frac{g^2}{2}{}_S \sum_{\mathcal{R}} C_{\mathcal{R}} d(\mathcal{R}_2) \mathrm{Tr}\,(\mathcal{R}(T^a_S)\mathcal{R}(T^b_S))\, G^a_{\mu\nu} G^{\mu\nu b}, \end{aligned} \tag{2..9}$$

where $d(\mathcal{R})$ denotes the dimension of the representation $\mathcal{R}$. The noncommutative correction, that is the θ-linear part of the Lagrangian, reads

$$\begin{aligned} \mathcal{L}^\theta &= \sum \mathcal{L}^\theta_i = g'^3 \kappa_1 \theta^{\mu\nu} \left(\frac{a}{4} f_{\mu\nu} f_{\rho\sigma} f^{\rho\sigma} - f_{\mu\rho} f_{\nu\sigma} f^{\rho\sigma} \right) \\ &+ g^3 \kappa_4^{ijk} \theta^{\mu\nu} \left(\frac{a}{4} B^i_{\mu\nu} B^j_{\rho\sigma} B^{\rho\sigma k} - B^i_{\mu\rho} B^j_{\nu\sigma} B^{\rho\sigma k} \right) \\ &+ g_S^3 \kappa_5^{abc} \theta^{\mu\nu} \left(\frac{a}{4} G^a_{\mu\nu} G^b_{\rho\sigma} G^{\rho\sigma c} - G^a_{\mu\rho} G^b_{\nu\sigma} G^{\rho\sigma c} \right) \\ &+ g' g^2 \kappa_2 \theta^{\mu\nu} \left(\frac{a}{4} f_{\mu\nu} B^i_{\rho\sigma} B^{\rho\sigma i} - f_{\mu\rho} B^i_{\nu\sigma} B^{\rho\sigma i} + c.p. \right) \\ &+ g' g_S^2 \kappa_3 \theta^{\mu\nu} \left(\frac{a}{4} f_{\mu\nu} G^a_{\rho\sigma} G^{\rho\sigma a} - f_{\mu\rho} G^a_{\nu\sigma} G^{\rho\sigma a} + c.p. \right), \end{aligned} \tag{2..10}$$

where the *c.p.* in (2..10) denotes the addition of the terms obtained by a cyclic permutation of fields without changing the positions of indices. The couplings in (2..10) are defined as follows:

$$\kappa_1 = \sum_{\mathcal{R}} C_{\mathcal{R}} d(\mathcal{R}_2) d(\mathcal{R}_3) \mathcal{R}_1(Y)^3,$$

$$
\begin{aligned}
\kappa_2\delta^{ij} &= \sum_{\mathcal{R}} C_{\mathcal{R}} d(\mathcal{R}_3)\mathcal{R}_1(Y)\mathrm{Tr}\,(\mathcal{R}_2(T_L^i)\mathcal{R}_2(T_L^j)),\\
\kappa_3\delta^{ab} &= \sum_{\mathcal{R}} C_{\mathcal{R}} d(\mathcal{R}_2)\mathcal{R}_1(Y)\mathrm{Tr}\,(\mathcal{R}_3(T_S^a)\mathcal{R}_3(T_S^b)),\\
\kappa_4^{ijk} &= \frac{1}{2}\sum_{\mathcal{R}} C_{\mathcal{R}} d(\mathcal{R}_3)\mathrm{Tr}\,(\{\mathcal{R}_2(T_L^i),\mathcal{R}_2(T_L^j)\}\mathcal{R}_2(T_L^k)),\\
\kappa_5^{abc} &= \frac{1}{2}\sum_{\mathcal{R}} C_{\mathcal{R}} d(\mathcal{R}_2)\mathrm{Tr}\,(\{\mathcal{R}_3(T_S^a),\mathcal{R}_3(T_S^b)\}\mathcal{R}_3(T_S^c)).
\end{aligned}
$$

Let us discuss the dependence of $\kappa_1,\dots,\kappa_5$ on the representations of matter fields. For the first generation of the standard model there are six such representations ; they produce six independent constants $C_{\mathcal{R}}$[1]. These constants are constrained by the three relations which defined g', g, g_S. One can immediately verify that $\kappa_4^{ijk}=0$. We shall in addition take that $\kappa_5^{abc}=0$. The argument for this assumption is related to the invariance of the colour sector of the SM under charge conjugation. Although apparently one has only the fundamental representation $\mathbf{3}$ of $\mathrm{SU(3)_C}$, there are in fact both $\mathbf{3}$ and $\overline{\mathbf{3}}$ representations with the same weights, $C_{\mathbf{3}} = C_{\overline{\mathbf{3}}}$. Since the symmetric coefficients for the $\mathbf{3}$ and $\overline{\mathbf{3}}$ representations satisfy $d_{\overline{\mathbf{3}}}^{abc} = -d_{\mathbf{3}}^{abc}$, we obtain

$$
\kappa_5^{abc} = C_{\mathbf{3}} d_{\mathbf{3}}^{abc} + C_{\overline{\mathbf{3}}} d_{\overline{\mathbf{3}}}^{abc} = 0. \tag{2..11}
$$

We are left only with three non vanishing couplings, κ_1, κ_2 and κ_3, depending on six constants $C_1,\dots,C_6$. Our classical noncommutative action reads [12]

$$
S_{cl} = S_{SM} + S^\theta, \tag{2..12}
$$

with

$$
\begin{aligned}
S^\theta &= g'^3\kappa_1\theta^{\mu\nu}\int d^4x\left(\frac{a}{4}f_{\mu\nu}f_{\rho\sigma}f^{\rho\sigma} - f_{\mu\rho}f_{\nu\sigma}f^{\rho\sigma}\right)\\
&\quad + g'g^2\kappa_2\theta^{\mu\nu}\int d^4x\left(\frac{a}{4}f_{\mu\nu}B^i_{\rho\sigma}B^{\rho\sigma i} - f_{\mu\rho}B^i_{\nu\sigma}B^{\rho\sigma i} + c.p.\right)\\
&\quad + g'g_S^2\kappa_3\theta^{\mu\nu}\int d^4x\left(\frac{a}{4}f_{\mu\nu}G^a_{\rho\sigma}G^{\rho\sigma a} - f_{\mu\rho}G^a_{\nu\sigma}G^{\rho\sigma a} + c.p.\right).
\end{aligned} \tag{2..13}
$$

The first term in (2..13) is one-loop renormalizable to linear order in θ [13] since the one-loop correction is of the second order in θ. We need to investigate only the renormalizability of remaining parts of the action (2..13).

[1] We assume that $C_{\mathcal{R}} > 0$; therefore the six $C_{\mathcal{R}}$'s were denoted by $\frac{1}{g_i^2}, i = 1, ..., 6$, in [6, 8].

3. One-loop renormalizability

We compute the divergences in the one-loop effective action using the background-field method. Here we will give only main results, the details are given in [12]. For the action (2..7), the classical Lagrangian reads

$$
\begin{aligned}
\mathcal{L}_{cl} &= -\frac{1}{4} f_{\mu\nu} f^{\mu\nu} - \frac{1}{4} B^i_{\mu\nu} B^{\mu\nu i} - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu a} \\
&+ g'^3 \kappa_1 \theta^{\mu\nu} \left(\frac{a}{4} f_{\mu\nu} f_{\rho\sigma} f^{\rho\sigma} - f_{\mu\rho} f_{\nu\sigma} f^{\rho\sigma} \right) \\
&+ g' g^2 \kappa_2 \theta^{\mu\nu} \Big(\frac{a}{4} f_{\mu\nu} B^i_{\rho\sigma} B^{\rho\sigma i} - 2 f_{\mu\rho} B^i_{\nu\sigma} B^{\rho\sigma i} \\
&+ \frac{a}{2} f_{\rho\sigma} B^i_{\mu\nu} B^{\rho\sigma i} - f_{\rho\sigma} B^i_{\mu\rho} B^{\nu\sigma i} \Big) \\
&+ g' g_S^2 \kappa_3 \theta^{\mu\nu} \Big(\frac{a}{4} f_{\mu\nu} G^a_{\rho\sigma} G^{\rho\sigma a} - 2 f_{\mu\rho} G^a_{\nu\sigma} G^{\rho\sigma a} \\
&+ \frac{a}{2} f_{\rho\sigma} G^a_{\mu\nu} G^{\rho\sigma a} - f_{\rho\sigma} G^a_{\mu\rho} G^{\nu\sigma a} \Big) .
\end{aligned}
\tag{3..14}
$$

After a very long and straightforward calculation [12] we get the divergent part of one-loop effective action

$$
\begin{aligned}
\Gamma_{div} &= \frac{1}{3(4\pi)^2 \epsilon} 11 \Big[\int d^4x B^i_{\mu\nu} B^{\mu\nu i} + \frac{33}{2} \int d^4x G^a_{\mu\nu} G^{\mu\nu a} \\
&+ 4(3-a) g' g^2 \kappa_2 \theta^{\mu\nu} \int d^4x \Big(\frac{1}{4} f_{\mu\nu} B^i_{\rho\sigma} B^{\rho\sigma i} - f_{\mu\rho} B^i_{\nu\sigma} B^{\rho\sigma i} \Big) \\
&+ 6(3-a) g' g_S^2 \kappa_3 \theta^{\mu\nu} \int d^4x \Big(\frac{1}{4} f_{\mu\nu} G^a_{\rho\sigma} G^{\rho\sigma a} - f_{\mu\rho} G^a_{\nu\sigma} G^{\rho\sigma a} \Big) \Big] .
\end{aligned}
\tag{3..15}
$$

The divergent contribution due to $\mathrm{U(1)_Y}$ solely vanishes, both the commutative and the noncommutative one. It is clear from (3..15) that the divergences in the noncommutative sector vanish for the choice $a = 3$. Therefore one obtains that the noncommutative gauge sector interaction is not only renormalizable but finite. The renormalization is performed by adding counter terms to the Lagrangian. We obtain

$$
\begin{aligned}
\mathcal{L} + \mathcal{L}_{ct} &= -\frac{1}{4} f_{0\mu\nu} f_0{}^{\mu\nu} - \frac{1}{4} B_0{}^i_{\mu\nu} B_0{}^{\mu\nu i} - \frac{1}{4} G_0{}^a_{\mu\nu} G_0{}^{\mu\nu a} \\
&+ g'^3 \kappa_1 \theta^{\mu\nu} \left(\frac{3}{4} f_{0\mu\nu} f_{0\rho\sigma} f_0{}^{\rho\sigma} - f_{0\mu\rho} f_{0\nu\sigma} f_0{}^{\rho\sigma} \right) \\
&+ g'_0 g_0^2 \kappa_2 \theta^{\mu\nu} \left(\frac{3}{4} f_{0\mu\nu} B_0{}^i_{\rho\sigma} B_0^{\rho\sigma i} - f_{0\mu\rho} B_0{}^i_{\nu\sigma} B_0^{\rho\sigma i} + c.p. \right) \\
&+ g'_0 (g_S)_0^2 \kappa_3 \theta^{\mu\nu} \left(\frac{3}{4} f_{0\mu\nu} G_0{}^a_{\rho\sigma} G_0^{\rho\sigma a} - f_{0\mu\rho} G_0{}^a_{\nu\sigma} G_0^{\rho\sigma a} + c.p. \right) ,
\end{aligned}
\tag{3..16}
$$

where the bare quantities are given as follows:

$$
\mathcal{A}_0{}^\mu = \mathcal{A}^\mu , \qquad g'_0 = g' , \tag{3..17}
$$

$$B_0{}^{\mu i} = B^{\mu i}\sqrt{1+\frac{44g^2}{3(4\pi)^2\epsilon}} \tag{3..18}$$

$$g_0 = \frac{g\,\mu^{\epsilon/2}}{\sqrt{1+\frac{44g^2}{3(4\pi)^2\epsilon}}}\,, \tag{3..19}$$

$$G_0{}^{\mu a} = G^{\mu a}\sqrt{1+\frac{22g_S^2}{(4\pi)^2\epsilon}} \tag{3..20}$$

$$(g_S)_0 = \frac{g_S\,\mu^{\epsilon/2}}{\sqrt{1+\frac{22g_S^2}{(4\pi)^2\epsilon}}}\,. \tag{3..21}$$

Finally, an important point is that the noncommutativity parameter θ need not be renormalized.

4. Discussion and conclusion

We have constructed a version of the standard model on the noncommutative Minkowski space which is one-loop renormalizable and finite in the gauge sector and in first order in the θ parameter. The renormalizability in the model was obtained by choosing six particle representations of the matter fields for the first generation of the SM, and by fixing the arbitrariness in the θ-linear expansion terms in the Seiberg-Witten map.

The one-loop renormalizability of the NCSM gauge sector is certainly a very encouraging result from both theoretical and experimental perspectives. So far fermions have not been successfully included: the results on the renormalizability of noncommutative gauge theories with Dirac fermions are negative [10, 11] as a 4ψ-divergence always appears. In the case of SU(N) or SU(3)$\otimes$SU(2)$\otimes$U(1) the unexpanded gauge theory cannot be consistently defined. Furthermore, our results show that the requirement of renormalizability fixes the SW freedom i.e. the corresponding parameter to $a = 1$ or $a = 3$ [16]. We hope that a similar procedure could be applicable to the fermionic sector of the theory.

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