Dilaton field contribution to noncommutativity *

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Abstract

It is known that the Dp-brane world-volume is non-commutative if open string ends on a Dp-brane with Neveu-Schwarz field $B_{\mu\nu}$. We will show that the dilaton field Φ turns the conformal part of the world-sheet metric to new non-commutative variable and the coordinate in $\partial_\mu\Phi$ direction to commutative one. We apply the canonical method and treat the boundary conditions as constraints.

1. Introduction

Quantization of the open string, ending on Dp-brane with nontrivial antisymmetric tensor field $B_{\mu\nu}$, leads to non-commutativity of Dp-brane world-volume. This result has been obtained in the literature [1]-[3], for constant metric $G_{\mu\nu}$ and antisymmetric tensor $B_{\mu\nu}$.

In this lecture notes, following ref.[4], we include the linear part of the dilaton field Φ, which turns the coordinate in $\partial_\mu\Phi$ direction to commutative one. We preserve the condition for the background fields $G_{\mu\nu}$ and $B_{\mu\nu}$ to be constant, and impose the same assumption for $\partial_\mu\Phi$. This choice is consistent with the space-time field equation, obtained from the quantum world-sheet conformal invariance. The noncommutative properties of this theory, have been studied in refs. [4], [5].

In the above choice of background, conformal part of the world-sheet metric, F, is a dynamical variable. So, beside the known boundary conditon $\gamma_i^{(0)}|_{\partial\Sigma} = 0$, corresponding to the Dp-brane coordinate x^i there is an additional one $\gamma^{(0)}|_{\partial\Sigma} = 0$, corresponding to the variable F. In ref. [4], the conformal part of the metric has been fixed and the additional boundary condition, $\gamma^{(0)}|_{\partial\Sigma} = 0$, has been lost.

Following the paper [4], we apply the canonical method and treat boundary conditions as canonical constraints. We show that they are of the second class. Instead to use the Dirac brackets, as in ref.[3], we explicitly solved the

* Work supported in part by the Serbian Ministry of Science and Environmental Protection, under contract No. 1486.

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constraints in terms of the open string variables: the effective coordinate q^i and the effective conformal part of the world-sheet metric f.

We find the energy-momentum tensor components in terms of the open string variables. They have exactly the same form as original ones, but with different Dp-brane background fields. The explicit dependence on antisymmetric field disappears and it contributes only to the effective metric tensor $\tilde{G}_{ij}$. This metric tensor and the non-commutativity parameters, explicitly depend on the dilaton field. The effective dilaton field is linear in the open string coordinate q^i.

We calculate Poisson brackets of all the variables. We find that the conformal part of the metric does not commute with the Dp-brane coordinates, on the world-sheet boundary. On the other hand, there exists one Dp-brane coordinate, $x \equiv x^\mu \partial_\mu \Phi$, which commutes with all other variables.

In ref.[5] the commutative direction is a consequence of the relation, $\partial_\nu \Phi \, B^\nu{}_\mu = 0$. In particular, this condition reduces our effective metric tensor and non-commutativity parameter to ones of ref.[5], while non-commutativity parameter between coordinates and conformal factor vanishes. Consequently, our results are more general, because they valid without above restriction on background fields. Boundary conditions, which we obtained from the action principle, are enough to fulfill requirement that there is no net flow of energy and momentum from the boundary.

2. The model

We are going to consider the action [6]-[10]

$$S = \kappa \int_\Sigma d^2\xi \sqrt{-g} \left\{ \left[\frac{1}{2} g^{\alpha\beta} G_{\mu\nu}(x) + \frac{\varepsilon^{\alpha\beta}}{\sqrt{-g}} \mathcal{F}_{\mu\nu}(x) \right] \partial_\alpha x^\mu \partial_\beta x^\nu + \Phi(x) R^{(2)} \right\}, \tag{1}$$

which describes propagation of the bosonic open string. By $x^\mu(\xi)\,, (\mu = 0, 1, ..., D-1)$, we denote the D dimensional space-time coordinates, and chose the gauge so that $x^i \, (i = 0, 1, ...p)$ are the Dp-bane coordinates. Here, $\xi^\alpha \, (\alpha = 0, 1)$ are coordinates of the two dimensional world-sheet Σ, while $g_{\alpha\beta}$ is the intrinsic world-sheet metric and $R^{(2)}$ is the corresponding scalar curvature. The string propagates in the non-trivial, x^μ dependent, background described by: metric tensor $G_{\mu\nu}$, antisymmetric tensor field $B_{\mu\nu} = -B_{\nu\mu}$, dilaton field Φ and $U(1)$ gauge field A_i living on the Dp-brane. The modified Born-Infeld field strength

$$\mathcal{F}_{\mu\nu} = B_{\mu\nu} + (\partial_i A_j - \partial_j A_i) \delta^i_\mu \delta^j_\nu \,, \tag{2}$$

incorporate the antisymmetric field with the field strength of the vector field. We will use the notation $\partial_\alpha \equiv \frac{\partial}{\partial \xi^\alpha}$, $\partial_\mu \equiv \frac{\partial}{\partial x^\mu}$ and $\partial_i \equiv \frac{\partial}{\partial x^i}$.

In the conformal gauge

$$g_{\alpha\beta} = e^{2F} \eta_{\alpha\beta} \,, \tag{3}$$

we have $R^{(2)} = 2\Delta F$, and the action takes the form

$$S = \kappa \int_\Sigma d^2\xi \left\{ \left[\frac{1}{2}\eta^{\alpha\beta} G_{\mu\nu}(x) + \varepsilon^{\alpha\beta}\mathcal{F}_{\mu\nu}(x) \right] \partial_\alpha x^\mu \partial_\beta x^\nu + 2\Phi(x) e^{2F} \Delta F \right\} . \tag{4}$$

As a consequence of nontrivial dilaton field, the action explicitly depends on the metric tensor component F and looses the conformal invariance.

2.1. Solution of the space-time field equations

The condition for the world-sheet conformal invariance produces the space-time field equations [7]

$$\beta^G_{\mu\nu} \equiv R_{\mu\nu} - \tfrac{1}{4}\mathcal{F}_{\mu\rho\sigma}\mathcal{F}_\mu{}^{\rho\sigma} + 2D_\mu a_\nu = 0 \,, \tag{5}$$

$$\beta^{\mathcal{F}}_{\mu\nu} \equiv D_\rho \mathcal{F}^\rho_{\mu\nu} - 2a_\rho \mathcal{F}^\rho_{\mu\nu} = 0 \,, \tag{6}$$

$$\beta^\Phi \equiv 4\pi\kappa \frac{D-26}{3} - R + \tfrac{1}{12}\mathcal{F}_{\mu\rho\sigma}\mathcal{F}^{\mu\rho\sigma} - 4D_\mu a^\mu + 4a^2 = 0 \,, \tag{7}$$

were $a_\mu = \partial_\mu \Phi$ is gradient of the dilaton field, $\mathcal{F}_{\mu\rho\sigma}$ is field strength of the field $\mathcal{F}_{\mu\nu}$ and $R_{\mu\nu}$, R and D_μ are Ricci tensor, scalar curvature and covariant derivative with respect to the space-time metric. For

$$a^2 = \kappa\pi \frac{26-D}{3} \,, \tag{8}$$

there exists the exact solution of the form (see ref.[10])

$$\begin{aligned} G_{\mu\nu}(x) &= G_{\mu\nu} = const \,, \qquad \mathcal{F}_{\mu\nu}(x) = \mathcal{F}_{\mu\nu} = const \,, \\ \Phi(x) &= \Phi_0 + a_\mu x^\mu \,. \qquad (a_\mu = const) \end{aligned} \tag{9}$$

As usual, the central charge, $c = D + \frac{3}{\kappa\pi}a^2$, is equal to 26. We require $a^2 \neq 0$, so that in the present example, the number of space-time dimensions is smaller then 26.

For simplicity, we suppose that antisymmetric tensor and the gradient of dilaton field are nontrivial only along directions of the Dp-brane world-volume, so that $\mathcal{F}_{\mu\nu} \to \mathcal{F}_{ij}$ and $a_\mu \to a_i$. We also chose coordinates so that $G_{\mu\nu} = 0$ for $\mu = i \in \{0, 1, .., p\}$ and $\nu = a \in \{p+1, ..., D-1\}$.

2.2. Canonical analyzes

The canonical analysis of the above action has been done in ref.[11]. The canonical variables of the theory are x^i, π_i, F and π. The Hamiltonian density, which corresponds to Dp-brane part, $\mathcal{H}_c = T_- - T_+$, is defined in terms of energy momentum tensor components

$$T_\pm = \mp\frac{1}{4\kappa}\left(G^{ij} J_{\pm i} J_{\pm j} + \frac{j}{a^2} i^\Phi_\pm \right) + \frac{1}{2}(i^{\Phi\prime}_\pm - F' i^\Phi_\pm) \,. \tag{10}$$

They satisfy two independent Virasoro algebras

$$\{T_\pm, T_\pm\} = -[T_\pm(\sigma) + T_\pm(\bar\sigma)]\delta' , \qquad \{T_\pm, T_\mp\} = 0 .$$

The currents on the Dp-brane have the form

$$J^i_\pm = P^{Tij} j_{\pm j} + \frac{a^i}{2a^2} i^\Phi_\pm , \qquad i^\Phi_\pm = \pi \pm 2\kappa a_i x^{i\prime} , \tag{11}$$

$$j_{\pm i} = \pi_i + 2\kappa \Pi_{\pm ij} x^{j\prime} , \qquad j = a^i j_{\pm i} - \frac{1}{2} i^\Phi_\pm , \tag{12}$$

$$\left(\Pi_{\pm ij} \equiv \mathcal{F}_{ij} \pm \frac{1}{2} G_{ij}\right) . \tag{13}$$

We also introduced the projection operators

$$P^L_{ij} = \frac{a_i a_j}{a^2} , \qquad P^T_{ij} = G_{ij} - \frac{a_i a_j}{a^2} . \tag{14}$$

3. Canonical treatment of the boundary conditions

3.1. Derivation of the boundary conditions

The evolution of the open string is described by both the equations of motion and the boundary conditions. The field equations are standard $\Delta x^\mu = 0$ and $\Delta F = 0$. Generally, the boundary conditions are of the form

$$\left(\frac{\partial S}{\partial x'^\mu} \delta x^\mu + \frac{\partial S}{\partial F'} \delta F\right) |_{\partial\Sigma} = 0 . \tag{15}$$

We use Neumann boundary conditions for Dp-brane coordinates x^i and for conformal part of the world-sheet metric F, allowing arbitrary variations δx^i and δF on the string end points. It means that

$$\gamma^{(0)}_i |_{\partial\Sigma} = 0 , \qquad \gamma^{(0)} |_{\partial\Sigma} = 0 , \tag{16}$$

where we introduced the variables

$$\gamma^{(0)}_i \equiv \frac{\delta S}{\delta x'^i} = \kappa(-G_{ij} x^{j\prime} + 2\mathcal{F}_{ij} \dot x^j - 2a_i F') , \qquad \gamma^{(0)} \equiv \frac{\delta S}{\delta F'} = -2\kappa a_i x^{i\prime} . \tag{17}$$

Compared with the dilaton free case, the second condition, relating to the additional variable F, is a new one.

For the other coordinates we use the Dirichlet boundary conditions, requiring the edges of the string to be fixed, $\delta x^a|_{\partial\Sigma} = 0$.

In terms of the currents the variables (17) obtain the form

$$\gamma^{(0)}_i = \gamma_{i-} + \gamma_{i+} , \qquad \gamma_{i\pm} \equiv \Pi_{\mp ij} J^j_\pm \mp \frac{a_i}{2} i^F_\pm , \tag{18}$$

$$\gamma^{(0)} = \gamma_- + \gamma_+ , \qquad \gamma_\pm \equiv \mp \frac{1}{2} i^\Phi_\pm . \tag{19}$$

3.2. Canonical constraints and consistency conditions

We will consider expressions, $\gamma_i^{(0)}|_{\partial\Sigma}$ and $\gamma^{(0)}|_{\partial\Sigma}$, as canonical constraints. The constant background fields $G_{ij}, \mathcal{F}_{ij}$ and a_i simplify Poisson brackets

$$\{H_c, J_{\pm A}\} = \mp J'_{\pm A}, \quad J_{\pm A} = \{J_{\pm i}, \, i_\pm^F, \, i_\pm^\Phi, \, \gamma_{\pm i}, \, \gamma_\pm\}\,. \tag{20}$$

The Diarc consistency procedure generate two infinity sets of new conditions $\gamma_i^{(n)}|_{\partial\Sigma} = 0$ and $\gamma^{(n)}|_{\partial\Sigma} = 0$, $(n \geq 1)$, where

$$\gamma_i^{(n)} \equiv \{H_c, \gamma_i^{(n-1)}\} = \partial_\sigma^n \left\{\gamma_{i-} + (-1)^n \gamma_{i+}\right\}\,, \tag{21}$$

$$\gamma^{(n)} \equiv \{H_c, \gamma^{(n-1)}\} = \partial_\sigma^n \left[\gamma_- + (-1)^n \gamma_+\right]\,. \tag{22}$$

We can rewrite all the conditions in the compact form

$$\Gamma_i(\sigma) \equiv \sum_{n\geq 0} \frac{\sigma^n}{n!} \gamma_i^{(n)}(0) = \gamma_{i-}(\sigma) + \gamma_{i+}(-\sigma)\,, \tag{23}$$

$$\Gamma(\sigma) \equiv \sum_{n\geq 0} \frac{\sigma^n}{n!} \gamma^{(n)}(0) = \gamma_-(\sigma) + \gamma_+(-\sigma)\,, \tag{24}$$

and similarly on the other string endpoint

$$\bar\Gamma_i(\sigma) \equiv \sum_{n\geq 0} \frac{(\sigma-\pi)^n}{n!} \gamma_i^{(n)}(\pi) = \gamma_{i-}(\sigma) + \gamma_{i+}(2\pi - \sigma)\,, \tag{25}$$

$$\bar\Gamma(\sigma) \equiv \sum_{n\geq 0} \frac{(\sigma-\pi)^n}{n!} \gamma^{(n)}(\pi) = \gamma_-(\sigma) + \gamma_+(2\pi - \sigma)\,. \tag{26}$$

These expressions differ from the boundary conditions, (18)-(19), only in the arguments of the positive chirality currents. From (23)-(26) we can conclude that all positive chirality currents and consequently, all variables x^i, π_i, F and π are periodic, for $\sigma \to \sigma + 2\pi$.

As a consequence of (20) we have

$$\{H_c, \Gamma_i(\sigma)\} = \Gamma'_i(\sigma)\,, \qquad \{H_c, \Gamma(\sigma)\} = \Gamma'(\sigma)\,, \tag{27}$$

and all constraints weakly commute with hamiltonian. Therefore, there are no more constraints.

The constraint algebra has the form

$$\{\Gamma_i(\sigma), \Gamma_j(\bar\sigma)\} = -\kappa \tilde G_{ij} \delta'(\sigma - \bar\sigma)\,, \qquad \{\Gamma(\sigma), \Gamma(\bar\sigma)\} = 0\,, \tag{28}$$

$$\{\Gamma_i(\sigma), \Gamma(\bar\sigma)\} = -2\kappa a_i \delta'(\sigma - \bar\sigma)\,, \tag{29}$$

were we introduced the effective metric tensor

$$\tilde G_{ij} \equiv G_{ij} - 4\mathcal{F}_{ik} P^{Tkq} \mathcal{F}_{qj}\,. \tag{30}$$

In agreement with ref.[2] we will refer to it as the open string metric tensor, — the metric tensor seen by the open string.

The variables possessing indices raised with inverse effective metric tensor, $\tilde{G}^{ij}$, we marked by tilde: $\tilde{V}^i = \tilde{G}^{ij}V_j$, and $\tilde{V}^2 = \tilde{G}^{ij}V_iV_j$. We also preserve standard notation, $V^i = G^{ij}V_j$ and $V^2 = G^{ij}V_iV_j$.

Direct calculation yields

$$\{\Gamma_A(\sigma), \Gamma_B(\bar\sigma)\} = -\kappa \begin{vmatrix} \tilde{G}_{ij} & 2a_i \\ 2a_j & 0 \end{vmatrix} \delta'(\sigma - \bar\sigma) \equiv \Delta_{AB}\delta'(\sigma - \bar\sigma)\,, \tag{31}$$

and

$$\triangle \equiv \det \triangle_{AB} = -4(-\kappa)^{p+2}\tilde{a}^2 \det \tilde{G}_{ij}\,, \tag{32}$$

where $\Gamma_A = \{\Gamma_i\,, \Gamma\}$. We assume $\tilde{a}^2 \neq 0$, so that $rank\triangle_{AB} = p+2$ and all constraints are of the second class (except the zero mode, see [12]).

3.3. Solution of the boundary conditions

The periodicity condition solves the second set of constraints (25)-(26). To solve the first one (23)-(24) we introduce the open string variables

$$q^i(\sigma) = \frac{1}{2}\left[x^i(\sigma) + x^i(-\sigma)\right]\,, \quad \bar{q}^i(\sigma) = \frac{1}{2}\left[x^i(\sigma) - x^i(-\sigma)\right]\,, \tag{33}$$

$$p_i(\sigma) = \frac{1}{2}\left[\pi_i(\sigma) + \pi_i(-\sigma)\right]\,, \quad \bar{p}_i(\sigma) = \frac{1}{2}\left[\pi_i(\sigma) - \pi_i(-\sigma)\right]\,, \tag{34}$$

$$f(\sigma) = \frac{1}{2}\left[F(\sigma) + F(-\sigma)\right]\,, \quad \bar{f}(\sigma) = \frac{1}{2}\left[F(\sigma) - F(-\sigma)\right]\,, \tag{35}$$

$$p(\sigma) = \frac{1}{2}\left[\pi(\sigma) + \pi(-\sigma)\right]\,, \quad \bar{p}(\sigma) = \frac{1}{2}\left[\pi(\sigma) - \pi(-\sigma)\right]\,. \tag{36}$$

In terms of new variables the constraints obtain the form

$$\Gamma_i(\sigma) = 2(\mathcal{F}P^T)_i{}^j p_j + \bar{p}_i + \frac{1}{a^2}\mathcal{F}_{ij}a^j p - \kappa\tilde{G}_{ij}\bar{q}^{j\prime} - 2\kappa a_i \bar{f}'\,, \tag{37}$$

$$\Gamma(\sigma) = \bar{p} - 2\kappa a_i \bar{q}^{i\prime}\,. \tag{38}$$

The symmetric and antisymmetric parts of expressions $\Gamma_i(\sigma) = 0$ and $\Gamma(\sigma) = 0$ separately vanish and we have

$$\bar{p}_i = 0\,, \qquad \bar{q}^{i\prime} = -2(\Theta^{ij}p_j + \Theta^i p)\,, \tag{39}$$

$$\bar{p} = 0\,, \qquad \bar{f}' = 2\Theta^i p_i\,. \tag{40}$$

Here

$$\Theta^{ij} = \frac{-1}{\kappa}\tilde{P}^{Tik}\mathcal{F}_{kq}P^{Tqj}\,, \qquad (\Theta^{ij} = -\Theta^{ji}) \tag{41}$$

$$\Theta^i = \frac{(\tilde{a}\mathcal{F})^i}{2\kappa\tilde{a}^2} = \frac{(a\mathcal{F}\tilde{G}^{-1})^i}{2\kappa a^2}\,, \tag{42}$$

where in analogy with (14) we introduced tilde projectors

$$\tilde{P}^{Lij} = \frac{\tilde{a}^i\tilde{a}^j}{\tilde{a}^2}\,, \qquad \tilde{P}^{Tij} = \tilde{G}^{ij} - \frac{\tilde{a}^i\tilde{a}^j}{\tilde{a}^2}\,. \tag{43}$$

Using (33)- (36) and (39)-(40), we can express the original variables in terms of new ones

$$x^i = q^i - 2\int^\sigma d\sigma_1 \left(\Theta^{ij}p_j + \Theta^i p\right)\,, \qquad \pi_i = p_i\,, \tag{44}$$

$$F = f + 2\,\Theta^i \int^\sigma d\sigma_1\; p_i\,, \qquad \pi = p\,. \tag{45}$$

4. The effective theory

4.1. The open string background

The original theory is completely described by the energy-momentum tensor $T_\pm$, eq.(10). We are going to find the effective energy-momentum tensor $\tilde{T}_\pm$, in terms of new variables defined by the relation

$$T_\pm[x^i(q^i,p_i,p),\pi_i(p_i),F(f,p_i),\pi(p)] = \tilde{T}_\pm(q^i,p_i,f,p)\,. \tag{46}$$

Let us first express the currents in terms of new variables. In analogy with equations (11)-(12) we introduce new, open string currents

$$\tilde{J}^i_\pm = \tilde{P}^{Tij}\tilde{j}_{\pm j} + \frac{\tilde{a}^i}{2\tilde{a}^2}\tilde{i}^\Phi_\pm\,, \qquad \tilde{i}^\Phi_\pm = p \pm 2\kappa a_i q^{i\prime}\,, \tag{47}$$

$$\tilde{j}_{\pm i} = p_i \pm \kappa\tilde{G}_{ij}q^{j\prime}\,, \qquad \tilde{j} = \tilde{a}^i\tilde{j}_{\pm i} - \frac{1}{2}\tilde{i}^\Phi_\pm\,, \tag{48}$$

so that

$$\tilde{T}_\pm = \mp\frac{1}{4\kappa}\left(\tilde{G}^{ij}\tilde{J}_{\pm i}\tilde{J}_{\pm j} + \frac{\tilde{j}}{\tilde{a}^2}\tilde{i}^\Phi_\pm\right) + \frac{1}{2}(\tilde{i}^{\Phi\prime}_\pm - f'\tilde{i}^\Phi_\pm)\,. \tag{49}$$

We can conclude that the effective energy-momentum tensor depends on the open string currents in exactly the same way as the original energy-momentum tensor depends on the original currents. Consequently, the complete effective theory is equivalent to the original one, except that it is defined in new, open string background

$$G_{ij} \to \tilde{G}_{ij} = G_{ij} - 4\mathcal{F}_{ik}P^{Tkq}\mathcal{F}_{qj}\,, \quad \mathcal{F}_{ij} \to \tilde{\mathcal{F}}_{ij} = 0\,, \quad \Phi \to \tilde{\Phi} = \Phi_0 + a_i q^i\,, \tag{50}$$

using the symmetries $\sigma \to \sigma + 2\pi$ and $\sigma \to -\sigma$, as orbifold conditions.

4.2. Non-commutativity in presence of the dilaton field

The standard Poisson brackets

$$\{x^i(\sigma), \pi_j(\bar\sigma)\} = \delta^i_j \delta(\sigma - \bar\sigma)\,, \qquad \{F(\sigma), \pi(\bar\sigma)\} = \delta(\sigma - \bar\sigma)\,, \tag{51}$$

produce

$$\{q^i(\sigma), p_j(\bar\sigma)\} = \delta^i_j \delta_s(\sigma, \bar\sigma)\,, \qquad \{f(\sigma), p(\bar\sigma)\} = \delta_s(\sigma, \bar\sigma)\,, \tag{52}$$

where

$$\delta_s(\sigma, \bar\sigma) = \frac{1}{2}\left[\delta(\sigma - \bar\sigma) + \delta(\sigma + \bar\sigma)\right]\,, \qquad (\sigma, \bar\sigma \in [0, \pi]) \tag{53}$$

is symmetric delta-function. So, q^i and p_i, as well as f and p, are canonically conjugate variables on symmetric subspace.

If we separate the center of mass, $x^i_{cm} = \frac{1}{\pi}\int_0^\pi d\sigma x^i(\sigma)$, in the form $x^i(\sigma) = x^i_{cm} + X^i(\sigma)$, the Poisson brackets between dynamical variables obtain the form

$$\{X^i(\sigma), X^j(\bar\sigma)\} = \Theta^{ij} \begin{cases} -1 & \sigma = 0 = \bar\sigma \\ 1 & \sigma = \pi = \bar\sigma \\ 0 & \text{otherwise} \end{cases}\,, \tag{54}$$

$$\{X^i(\sigma), F(\bar\sigma)\} = \Theta^{i} \begin{cases} -1 & \sigma = 0 = \bar\sigma \\ 1 & \sigma = \pi = \bar\sigma \\ 0 & \text{otherwise} \end{cases}\,, \tag{55}$$

where Θ^{ij} and Θ^i have been defined in (41) and (42) respectively.

The relation (55) shows that in the linear dilaton field background, the non-commutativity between the coordinates and the conformal part of the world-sheet metric appears on the world-sheet boundary. The expression for this new non-commutativity parameter, Θ^i, is proportional to Born-Infeld field, $\mathcal{F}_{ij}$.

The relation (54) has the same form as in the absence of dilaton field [1]-[3], but there are some significant differences. Let us first explain geometrical meaning of the projectors $P^{T\,ij}$ and $\tilde P^{T\,ij}$. The vector a_i is normal to the Dp-brane p dimensional submanifold M_p, defined by the condition $\Phi(x) = const$. For $a^2 \neq 0$ ($\tilde a^2 \neq 0$), the corresponding unit vectors for the closed and the open string respectively are $n_i = \frac{a_i}{\sqrt{\varepsilon a^2}}$ and $\tilde n_i = \frac{a_i}{\sqrt{\tilde\varepsilon \tilde a^2}}$. Here $\varepsilon = 1$ ($\tilde\varepsilon = 1$) if a_i is time like vector, and $\varepsilon = -1$ ($\tilde\varepsilon = -1$) if a_i is space like vector with respect to metrics $G_{ij}(\tilde G_{ij})$. Therefore, we can rewrite (41) in the form

$$\Theta^{ij} = \frac{-1}{\kappa}\tilde G^{(p)ik}\mathcal{F}_{kq}G^{(p)qj}\,, \tag{56}$$

because the projectors, in fact, are the induced metrics on M_p

$$P^T{}_{ij} = G_{ij} - \varepsilon n_i n_j \equiv G^{(p)}_{ij}\,, \qquad \tilde P^T{}_{ij} = \tilde G_{ij} - \varepsilon \tilde n_i \tilde n_j \equiv \tilde G^{(p)}_{ij}\,. \tag{57}$$

The expression for the non-commutativity parameter is similar to the corresponding one in absence of dilaton field. The essential part again is the Born-Infeld field strength $\mathcal{F}_{kq}$, but in the present case we raised the indices with the metrics of submanifold M_p: $\tilde{G}^{(p)ij}$ and $G^{(p)ij}$ instead of the Dp-brane metrics: $G^{ij}_{eff} = (G - 4\mathcal{F}G^{-1}\mathcal{F})^{-1ij}$ and G^{ij}.

4.3. A commutative Dp-brane direction

From the relations $a_i P^{Tij} = 0$ and $\tilde{a}\mathcal{F}a = 0$, it follow that $a_i\Theta^{ij} = 0$ and $a_i\Theta^i = 0$. Consequently, the component $x \equiv a_i x^i$ commutes with all other coordinates as with the conformal part of the metric

$$\{x(\sigma), x^j(\bar{\sigma})\} = 0\,, \qquad \{x(\sigma), F(\bar{\sigma})\} = 0\,. \tag{58}$$

This is an example of Dp-brane with one commutative coordinate in a_i direction (proportional to gradient of the dilaton field).

5. Concluding remarks

We investigated the contribution of the dilaton field to non-commutativity of the Dp-brane world-volume. We considered the case, with dilaton field linear in coordinate and constant metric and antisymmetric fields. This choice preserves the world-sheet conformal symmetry.

Using the canonical method, we obtain the effective theory in terms of the open string variables q^j and f. It has precisely the same form as the original theory in terms of the closed string variables x^j and F, but with different background. The closed string background G_{ij}, $\mathcal{F}_{ij} = B_{ij} + \partial_i A_j - \partial_j A_i$ and $\Phi = \Phi_0 + a_i x^i$ should be substituted by the open string one

$$\tilde{G}_{ij} = G_{ij} - 4\mathcal{F}_{ik}P^{Tkq}\mathcal{F}_{qj}\,, \qquad \tilde{\mathcal{F}}_{ij} = 0\,, \qquad \tilde{\Phi} = \Phi_0 + a_i q^i\,. \tag{59}$$

Instead of the boundary conditions $\gamma_i^{(0)}|_{\partial\Sigma} = 0$ and $\gamma^{(0)}|_{\partial\Sigma} = 0$, new variables q^i and f satisfy the orbifold conditions: the symmetries under $\sigma \to \sigma + 2\pi$ and $\sigma \to -\sigma$.

The relation between the closed and the open string variables clarify the origin of non-commutativity. The closed string variables depend on the open string ones, but also on the corresponding momenta. So, the Poisson brackets between the variables are nontrivial on the world-sheet boundary.

Beside known coordinate non-commutativity, the non-commutativity relation between the coordinates and the conformal part of the world-sheet metric has been established in ref.[4], with explicit expressions for non-commutativity parameters Θ^{ij} (41) and Θ^i (42). For the linear dilaton field we have $a_i\Theta^{ij} = 0$ and $a_i\Theta^i = 0$, so that the coordinate in a_i direction is commutative.

Let us compare the symmetric and antisymmetric string parameters, in three different cases. In the closed string case, the metric tensor and the Born-Infeld field strength satisfies

$$\mathcal{F}^{ij} \pm \tfrac{1}{2}G^{ij} = (G^{-1}\Pi_\pm G^{-1})^{ij}\,. \tag{60}$$

The open string is sensitive to the effective metric tensor and to the non-commutative parameter. In the dilaton free case they produce

$$\kappa\theta^{ij} \pm \tfrac{1}{2}G^{ij}_{eff} = (G^{-1}\Pi_\pm G^{-1}_{eff})^{ij}\,, \tag{61}$$

while in the linear dilaton case, corresponding relation obtains the form

$$\kappa\Theta^{ij} \pm \tfrac{1}{2}\tilde{G}^{ij}_{(p)} = (G^{-1}_{(p)}\Pi_\pm \tilde{G}^{-1}_{(p)})^{ij}\,. \tag{62}$$

Here, $G^{eff}_{ij} = (G - 4\mathcal{F}G^{-1}\mathcal{F})_{ij}$ and $\tilde{G}_{ij} = (G - 4\mathcal{F}G^{-1}_{(p)}\mathcal{F})_{ij}$ are the effective metric tensors in absence and in presence of the dilaton field, respectively. Therefore, the addition of dilaton field just turns the metric G_{ij}, of the Dp-brane world-volume, to the metric $G^{(p)}_{ij}$ of its submanifold orthogonal to a_i.

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