Matrix formulation of Calogero-type models

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Abstract

We study a single matrix oscillator with the quadratic Hamiltonian and deformed commutation relations. It is equivalent to the multispecies Calogero model in one dimension, with inverse-square two-body and three-body interactions. Specially, a new matrix realization of the Calogero model for identical particles have been constructed, without using exchange operators. The critical points at which singular behaviour occurs are briefly discussed. At the end, a connection of a quantum matrix oscillator to a modified matrix Chern-Simons model is established. The quantum mechanics of electrons in the lowest Landau level is shown to be described by the matrix oscillator model with the ground-state wave function of the Laughlin type.

1. Introduction

A class of integrable many-body systems in one spatial dimension is known, referred to as Calogero systems [1]. These systems are formed of N identical particles on the line which interact through an inverse-square two-body interaction and are subjected to a common confining harmonic force. These models are completely integrable in both the classical and the quantum case [2] and are related to a number of mathematical and physical problems, ranging from random matrices [3,4] to gravity, black hole physics [5] and two-dimensional strings [6]. The algebraic structure of the Calogero model has recently been reconsidered by a number of authors in the framework of the exchange operator formalism [7,8] based on a symmetric group. An advantage of this approach is the possibility of an explicit construction of wave functions for an arbitrary number of particles. This approach also emphasizes the interpretation in terms of generalized statistics [9] that allows for the possibility of having particles of different species with a mutual coupling parameter depending on the species coupled.

It is known that a random matrix theory provides a simple relation between the quantum mechanics of the harmonic oscillator and the Calogero

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model. However, this connection is known to be true only for three special values of the coupling parameter ν : $\nu = \frac{1}{2}, 1, 2$. In this approach the Calogero model appears through the calculation of averages in the Gaussian ensembles [3,4]. Also, another remarkable connection between the matrix models and the Calogero models was established in [2,10,11,12]. In Ref's.[2,11,12] a classical matrix system without the quadratic potential was considered by the technique of the Hamiltonian reduction. The quantization is performed through path integral methods [12].

In the present article we introduce matrices whose matrix elements are operators and define the matrix Hamiltonian of the quadratic type. We show that this matrix formulation is in one-to-one correspondence with the Calogero model for an arbitrary value of the coupling parameter that can even depend on the particles coupled. In addition, exchange operators do not appear in our formulation. Finally, the construction presented here has been used for recovering some basic features of the quantum Hall physics obtained within the regularized Chern-Simons matrix model [13].

2. Matrix oscillator and a multispecies Calogero model

Let us consider [14] $N \times N$ matrices $\mathbf{X}$, $\mathbf{P}$ with operator-valued matrix elements and a non-singular mass matrix $\mathcal{M}$. The matrix Hamiltonian, generally non-Hermitean, is given by

$$\mathcal{H} = \frac{1}{2}(\mathbf{P}\mathcal{M}^{-1}\mathbf{P} + \omega^2 \mathbf{X}\mathcal{M}\mathbf{X}), \tag{1}$$

with $\hbar = 1$. It represents a matrix generalization of a single harmonic oscillator.
We assume the following matrix commutation relations:

$$[\mathbf{X}, \mathbf{P}] = \imath \mathcal{V}, \tag{2}$$

where $\mathcal{V}$ is a Hermitean $N \times N$ matrix with constant, real and symmetric matrix elements $\nu_{ij} = \nu_{ji}, \ \ i,j = 1,2,...,N$. We set the diagonal elements to be equal to unity, reflecting the quantum nature of the system. We further assume that the matrix $\mathbf{X}$ is Hermitean and can be represented as a diagonal matrix with real elements $x_i, \ \ i = 1,2,...,N$:

$$\mathbf{X}_{ij} = x_i \delta_{ij} \tag{3}$$

and $\mathcal{M}_{ij} = m_i \delta_{ij}, \ \ m_i > 0$. By introducing the matrix operator $\mathcal{D}$ such that the relation $\mathbf{P} = -\imath \mathcal{D}$ holds, we can rewrite Eq. (2) in the following way :

$$\begin{gathered} \mathcal{D}_{ij} x_j - x_i \mathcal{D}_{ij} = \nu_{ij} \quad \forall i,j; \ \ i \neq j \\ \mathcal{D}_{ii} x_i - x_i \mathcal{D}_{ii} = 1 \quad \forall i. \end{gathered} \tag{4}$$

There are many solutions of equations (4) since the addition of a diagonal piece to $\mathcal{D}_{ij}$ depending only on the coordinates does not affect these equations. To express this fact in a more explicit maneer, we can consider a transformation $\mathcal{D}^{(f)} = f^{-1}\mathcal{D}f$ of the operator $\mathcal{D}$ by the arbitrary function f of the coordinates. The corresponding Hamiltonians are connected by non-unitary gauge transformations, i.e. by similarity transformations of the form $\mathcal{H}^{(f)} = f^{-1}\mathcal{H}f$. We shall restrict ourselves to gauge transformations defined by $f = \prod_{i<j}(x_i - x_j)^{\lambda_{ij}}, \quad \lambda_{ij} = \lambda_{ji}$. A corresponding class of solutions of equations (4) is given by

$$\mathcal{D}_{ij} = \delta_{ij}(\frac{\partial}{\partial x_i} + \sum_{k\neq i}\frac{\lambda_{ik}}{x_i - x_k}) - \frac{\nu_{ij}(1-\delta_{ij})}{x_i - x_j}, \tag{5}$$

where λ_{ij} are gauge parameters. Note that the dependence of the Hamiltonian $\mathcal{H}$ on the gauge parameters λ_{ij} enters through the operator $\mathcal{D}$.

The matrix Schroedinger equation is

$$\mathcal{H} * \Psi(\mathbf{X}) \equiv \mathcal{H}\mathcal{J}\Psi(\mathbf{X}) = \mathcal{E}\Psi(\mathbf{X}), \tag{6}$$

where $\Psi(\mathbf{X})$ is a column wave function $(\psi_i(\mathbf{X})), \quad i = 1,...,N$, $\mathcal{J}$ is an $N \times N$ matrix with units at all positions, and multiplication $*$ is defined in the above equation. For example, the ground state in the gauge $\lambda_{ij} = \nu_{ij}, \quad \forall i,j$ is described by the column matrix

$$\|0\rangle \sim \left(e^{-\frac{\omega}{2}\sum_{i=1}^{N} m_i x_i^2}\right) C. \tag{7}$$

Here C is the column matrix with all elements equal to unity. Analogously, one can introduce the left action of the Hamiltonian $\mathcal{H}$ on the row wave function. In this case (in the gauge $\lambda_{ij} = \nu_{ij}, \quad \forall i,j$), the ground state is represented by $\langle 0\| \sim Re^{-\frac{\omega}{2}\sum_{i=1}^{N} m_i x_i^2}$, where R is a transpose of C. Note that $RC = N$, $CR = \mathcal{J}$, $\mathcal{J}\|0\rangle = N\|0\rangle$ and $\langle 0\|0\rangle = 1$. At this point it should be emphasized that $\Psi(\mathbf{X})$ is not the eigenstate of the Hamiltonian $\mathcal{H}$ in the usual sense, but rather it satisfies the "eigenvalue" equation in which the "eigenvalue" $\mathcal{E}$ is a matrix. We can give a well-defined meaning to this equation after performing multiplication of both sides by the row matrix R from the left. In this case, Eq. (6) is reduced to the eigenvalue equation $H\psi = E\psi$, where H is the Hamiltonian corresponding to the matrix Hamiltonian $\mathcal{H}$ by $H = R\mathcal{H}C = tr(\mathcal{H}\mathcal{J})$. We point out that in the special case where the gauge $\lambda_{ij} = \nu_{ij}, \quad \forall i,j$ is chosen, we get the familiar eigenvalue equation $\tilde{H}\psi = E\psi$, where $\tilde{H}$ stands for the transformed Hamiltonian for a multispecies Calogero model [17]. In this case $\mathcal{H}$ is related to $\tilde{H}$ as

$$\tilde{H} = R\mathcal{H}C = tr(\mathcal{H}\mathcal{J})$$

$$= -\frac{1}{2}\sum_{i=1}^{N}\frac{1}{m_i}\frac{\partial^2}{\partial x_i^2} + \frac{\omega^2}{2}\sum_{i=1}^{N} m_i x_i^2$$

$$-\frac{1}{2}\sum_{i\neq j}\frac{\nu_{ij}}{(x_i-x_j)}(\frac{1}{m_i}\frac{\partial}{\partial x_i}-\frac{1}{m_j}\frac{\partial}{\partial x_j})$$

$$\equiv -T_- + \omega^2 T_+, \tag{8}$$

where $T_\pm$ are $SU(1,1)$ generators. After performing the similarity transformation with the inverse Yastrow factor $\prod_{i<j}(x_i - x_j)^{-\nu_{ij}}$, we obtain an original Hamiltonian H_{cal} for the multispecies Calogero model, with inverse-square two-body and three-body interactions [17]. This original Hamiltonian H_{cal} can also be reproduced directly, $H_{cal} = tr(\mathcal{H}\mathcal{J})$ in the gauge $\lambda_{ij} = 0, \quad \forall i,j$. The procedure outlined here differs significantly from that followed in Ref.[12] in the way how the Calogero Hamiltonian appears. Namely, in this paper the Hamiltonian is not just the trace of the matrix Hamiltonian $\mathcal{H}$, but is given by $H = tr(\mathcal{H}\mathcal{J})$, see Eq.(8). In the rest of the paper we restrict to the gauge $\lambda_{ij} = \nu_{ij}, \quad \forall i,j$.

In the same way as we have introduced the matrix Hamiltonian $\mathcal{H}$, we introduce matrix generators with operator- valued matrix elements

$$\begin{array}{l}\mathcal{T}_+ = \frac{1}{2}\mathbf{X}\mathcal{M}\mathbf{X},\\ \mathcal{T}_- = \frac{1}{2}\mathcal{D}\mathcal{M}^{-1}\mathcal{D},\\ \mathcal{T}_0 = \frac{1}{4}(\mathbf{X}\mathcal{D}+\mathcal{D}\mathbf{X}) = \mathbf{\frac{1}{2}}(\mathbf{X}\mathcal{D}+\mathbf{\frac{1}{2}}\mathcal{V}).\end{array} \tag{9}$$

They satisfy the relations

$$R[\mathcal{T}_-,\mathcal{T}_+]_J C = 2R\mathcal{T}_0 C, \tag{10}$$

$$R[\mathcal{T}_0,\mathcal{T}_\pm]_J C = \pm R\mathcal{T}_\pm C, \tag{11}$$

where the $J-$commutator is defined by

$$[A,B]_J = A\mathcal{J}B - B\mathcal{J}A. \tag{12}$$

$\mathcal{T}_\pm, \mathcal{T}_0$ are related to the generators $T_\pm, T_0$ [17] of $SU(1,1)$ algebra in the following way :

$$\begin{array}{l}T_\pm = R\mathcal{T}_\pm C = tr(\mathcal{T}_\pm\mathcal{J}),\\ T_0 = R\mathcal{T}_0 C = tr(\mathcal{T}_0\mathcal{J}).\end{array} \tag{13}$$

The wave functions ψ of the Hamiltonian (8) ($\tilde{H}\psi = E\psi$) are related to the column wave functions defined in (6). As it is readily seen, the connection is simply $\psi(x_1,...,x_N) \sim R\Psi(\mathbf{X})$ and $\psi^*(x_1,...,x_N) \sim \Psi^\dagger(\mathbf{X})C$.

If we further restrict ourselves to the case where $\nu_{ij} = \nu$, $m_i = m$ for all $i,j = 1,...,N$, then we recover the original Calogero model (cf. Eq. (8)). In this case, we introduce the matrix creation and annihilation operators $\mathcal{A}^\pm$ defined by

$$\mathcal{A}^\pm = \frac{1}{\sqrt{2m\omega}}(m\omega\mathbf{X} \mp \mathcal{D}) = \frac{1}{\sqrt{2m\omega}}(m\omega\mathbf{X} \mp \imath\mathbf{P}). \tag{14}$$

Then the following relations hold :

$$[\mathcal{A}^-, \mathcal{A}^+] = (1-\nu)\mathbf{1} + \nu\mathcal{J} \tag{15}$$

and

$$\mathcal{H} = \frac{\omega}{2}\{\mathcal{A}^-, \mathcal{A}^+\}, \quad R\mathcal{H}C = \omega R[\mathcal{A}^{-2}, \mathcal{A}^{+2}]_J C, \tag{16}$$

$$R[\mathcal{A}^{\mp k}, \mathcal{A}^{\mp l}]_J C = 0, \quad R[\mathcal{H}, \mathcal{A}^{\pm k}]_J C = \pm\omega k R\mathcal{A}^{\pm k} C, \tag{17}$$

where k, l can be arbitrary non-negative integers. In the above relations, $[,]$ and $\{,\}$ denote the ordinary commutators and anticommutators, respectively.

Starting from the column vacuum state (7) (now with all m_i equal to m), which satisfies the condition $\mathcal{A}^-\|0\rangle = 0$, the complete matrix Fock space can be constructed. The general state of the full Fock space is generated by the operators (14) and can be written as

$$\|n_1, ..., n_N\rangle \sim \prod_{k=1}^{N} (\mathcal{A}^{+k}\mathcal{J})^{n_k}\|0\rangle, \tag{18}$$

where $n_k = 0, 1, 2, ...$ and

$$\mathcal{A}^-\mathcal{J}\mathcal{A}^{+n}\|0\rangle = n\mathcal{A}^{+n-1}\|0\rangle. \tag{19}$$

The general state, Eq.(18), satisfies the equation

$$\mathcal{H} * \|n_1, ..., n_N\rangle \equiv \mathcal{H}\mathcal{J}\|n_1, ..., n_N\rangle = \mathcal{E}_{\{n\}}\|n_1, ..., n_N\rangle. \tag{20}$$

Here again, $\mathcal{E}_{\{n\}}$ is the matrix that satisfies the relation

$$\frac{1}{\sqrt{N}} R\mathcal{E}_{\{n\}}\|n_1, ..., n_N\rangle = \frac{1}{\sqrt{N}} E_{\{n\}} R\|n_1, ..., n_N\rangle$$

$$= E_{\{n\}}|n_1, ..., n_N\rangle = ((\sum_{k=1}^{N} kn_k + \varepsilon_0)\omega)|n_1, ..., n_N\rangle, \tag{21}$$

where $\varepsilon_0 = \frac{1}{2}[1 + \nu(N-1)]N$ and $E_{\{n\}}$ and $|n_1, ..., n_N\rangle$ are eigenvalues and eigenstates of the corresponding Calogero problem. For example, we explicitly show how the Hamiltonian $\mathcal{H}$ acts on the state $\mathcal{A}^{+n}\|0\rangle$:

$$\mathcal{H} * \mathcal{A}^{+n}\|0\rangle = \omega(n\mathbf{1} + \frac{1}{2}\mathcal{V}\mathcal{J})\mathcal{A}^{+n}\|0\rangle. \tag{22}$$

Note that after multiplying both sides of this equation by R from the left, we get the familiar result. Now we make complete correspondence with the ordinary Calogero model for identical particles.

The Fock space for the ordinary single-species Calogero model is generated by totally symmetric combinations $A_k^{\pm} = \sum_{i=1}^{N}(a_i^{\pm})^k$, $(k =$

$1, 2, ..., N)$ of auxiliary creation and annihilation operators $a_i^{\pm} = \frac{1}{\sqrt{2\omega}}(\omega x_i \mp \frac{\partial}{\partial x_i} \mp \nu \sum_{i \neq j} \frac{1}{x_i - x_j}(1 - K_{ij}))$, where the operators K_{ij} exchange labels i and j in all quantities [7,8].

To be more precise, the complete Fock space is spanned by the states of the form $\prod_{k=1}^{N}(A_k^{+})^{n_k}|0\rangle$, where $|0\rangle \sim e^{-\frac{m\omega}{2}\sum_{i=1}^{N} x_i^2}$ is the ordinary vacuum, $a_i^{-}|0\rangle = 0$ for all $i = 1, ..., N$.

At this point we emphasize that the following crucial relations hold:

$$(a_i^{+})^k_{/symm} = (\mathcal{A}^{+k}C)_i, \quad (a_i^{-})^k_{/symm} = (R\mathcal{A}^{-k})_i \tag{23}$$

for every $k = 1, 2, ..., N$, which can be proved by induction. From this relation we immediately obtain

$$A_k^{\pm} = \sum_{i=1}^{N}(a_i^{\pm})^k_{/symm} = R\mathcal{A}^{\pm k}C = tr(\mathcal{A}^{\pm k}\mathcal{J}). \tag{24}$$

The label *symm* designates that the action is restricted to the symmetric states only. In this case, the exchange operators K_{ij} coincide with the identity operator $(K_{ij}\psi = \psi)$. Equations (23) provide one-to-one correspondence between our matrix oscillator and the ordinary single-species Calogero model. Note that in the matrix oscillator approach the exchange operators K_{ij} do not appear. The approach considered in this paper can be repeated in an arbitrary gauge, particularly in the gauge $\lambda = 0$.

3. Matrix oscillator and action

We now construct an action for the matrix oscillator (1) for identical particles ($m_i = m, i = 1, ..., N;\ \nu_{ij} = \nu, i, j = 1, ..., N$) in order to describe two-dimensional systems of N charged particles with charge e in a magnetic field B. For this purpose, it is convenient to introduce the matrix $X_1 = \mathbf{X}$, and a second matrix X_2 expressed in terms of $\mathbf{P}$ as

$$X_2 = -\frac{1}{eB}\mathbf{P} = -\frac{1}{m\omega}\mathbf{P}, \tag{25}$$

where $\omega = \frac{eB}{m}$. Note that the trace $Tr[X_1, X_2]$ is equal to $\frac{N}{\imath eB}$, in accordance with the relation (2).

The coordinates of the electrons can be globally parametrized in a fuzzy way by introducing two $N \times N$ Hermitian matrices $X_a;\ a = 1, 2$. The action leading to the quantum matrix oscillator is then given by

$$S_M = \frac{eB}{2}\int dtTr[\varepsilon_{ab}X_a(\dot{X}_b - \imath[A_0, X_b]) + 2\theta A_0]$$
$$-\frac{\omega eBN}{2\bar{\psi}\psi}\int dt\bar{\psi}X_aX_a\psi - \int dt\bar{\psi}A_0\psi, \tag{26}$$

where $eB\theta = k$, A_0 is a matrix entering into the above action only linearly and ψ ($\bar{\psi} = \psi^{*T}$) is a boundary vector field. The action (26) is invariant under the transformations $X_a \to UX_aU^{-1}$, $\psi \to U\psi$, $\bar{\psi} \to \bar{\psi}U^{-1}$, where U is a unitary matrix, $U \in U(N)$. The term with ω serves as a potential box that keeps particles near the origin and also provides a Hamiltonian for the theory that chooses a unique ground state, while the last term in the action can be interpreted as a boundary term. The boundary fields here are not dynamical variables, but are frozen from the beginning because the action (26) does not include a corresponding dynamical term. The parts $Tr(X_a)^2$ and $\bar{\psi}(X_a)^2\psi$ in the action are distinct, but, when acting on the singlet sector of the $U(N)$ group, they will yield the same spectrum.

The variation of the action S_M in the field variable A_0 gives the equation of motion for the time component A_0 of the gauge field. This equation has the form

$$\imath eB[X_1, X_2] + k\mathbf{1} - \psi\bar{\psi} = 0 \tag{27}$$

and can be interpreted as the Gauss law. Now we pronounce the Gauss law (27) as a quantization condition imposed on the matrices X_1 and X_2, after which their matrix elements become operators.

In this way we have recovered the quantisation condition (2), in the case of identical particles, with the solution

$$-\imath eB(X_2)_{ij} = (\frac{\partial}{\partial x_i} + \sum_{k\neq i}\frac{\lambda_{ik}}{x_i - x_k})\delta_{ij} - \frac{1-\delta_{ij}}{x_i - x_j}\psi_i\bar{\psi}_j, \tag{28}$$

It can be shown [15] that after the transformation to the Bargmann representation is made by the operator $S = e^{-\omega T_+}e^{-\frac{1}{2\omega}T_-}$, the ground state $\| 0\rangle_\nu^B$ of the transformed Hamiltonian has the form of the Laughlin wave function

$$\| 0\rangle_\nu^B = S^{-1}\| 0\rangle_\nu \sim C\prod_{i<j}(x_i - x_j)^\nu \equiv C|0\rangle_\nu^B, \quad \nu \geq 0, \tag{29}$$

and all eigenstates can be represented as

$$C\prod_n(\sum_i {x_i}^n)^{m_n}\prod_{i<j}(x_i - x_j)^\nu. \tag{30}$$

The lowest state in a given tower with fixed ν is just a Laughlin wave function. Generally, the Laughlin wave function exponent $\nu = k+1$ is an integer number and, particularly, if ν is even, the system behaves as the Bose system, and for ν odd, it behaves as the Fermi system.

4. Conclusion

In conclusion, by introducing a pair of $N \times N$ matrices, we have constructed a Hamiltonian for a single quadratic matrix oscillator that is required to obey deformed commutation relations. This is a generalization of a single deformed oscillator. Straightforward analysis has shown that this oscillator is equivalent to the multispecies Calogero model [16,17] in one dimension. Specially, in the case of identical particles, an alternative formulation of ladder operators has been proposed in which exchange operators do not appear. In terms of these operators, the complete Fock space has been constructed. Also, it was outlined that by transforming to a conveniently chosen representation (Bargmann representation), the quantum matrix oscillator (1) leads to Laughlin ground states. The equation of motion following from the modified Chern-Simons action has emerged as the deformed matrix quantisation condition, and the Laughlin wave function exponent $\nu = k+1$ is an integer number. The restriction on the Laughlin wave function exponent $\nu > -\frac{1}{N}$ is dictated by the existence of the ground state and because ν is an integer, it reduces to the condition $\nu \geq 0$. These considerations can be extended to all Calogero-like models, such as models in arbitrary dimensions,the Sutherland model (on a circle) and models defined on root systems and the supersymmetric Calogero model.

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