Weyl-Conformally Invariant p-Brane Theories

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Abstract

We discuss in some detail the properties of a novel class of Weyl-conformally invariant p-brane theories which describe intrinsically light-like branes for any odd world-volume dimension and whose dynamics significantly differs from that of the ordinary (conformally non-invariant) Nambu-Goto p-branes. We present explicit solutions of the *WILL*-brane (Weyl-Invariant Light-Like brane) equations of motion in various gravitational backgrounds of physical relevance exhibiting the following new phenomena: (i) In spherically symmetric static backgrounds the *WILL*-brane automatically positions itself on (materializes) the event horizon of the corresponding black hole solutions, thus providing an explicit dynamical realization of the membrane paradigm in black hole physics; (ii) In product spaces (of interest in Kaluza-Klein context) the *WILL*-brane wrappes non-trivially around the compact (internal) dimensions and moves as a whole with the speed of light in the non-compact (space-time) dimensions.

1. Introduction

Higher-dimensional extended objects (p-branes, Dp-branes) play an increasingly crucial role in modern non-perturbative string theory of fundamental interactions at ultra-high energies (for a background on string and brane theories, see refs.[1]). Their importance stems primarily from such basic properties as: providing explicit realization of non-perturbative string dualities, microscopic description of black-hole physics, gauge theory/gravity correspondence, large-radius compactifications of extra dimensions, cosmological brane-world scenarios in high-energy particle phenomenology, *etc.*.

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In an independent development new classes of field theory models involving gravity, based on the idea of replacing the standard Riemannian integration measure (Riemannian volume-form) with an alternative non-Riemannian volume-form or, more generally, employing on equal footing both Riemannian and non-Riemannian volume-forms, have been proposed few years ago [2]. Since then, these new models called *two-measure theories* have been a subject of active research and applications [3] [1]. Two-measure theories address various basic problems in cosmology and particle physics, and provide plausible solutions for a broad array of issues, such as: scale invariance and its dynamical breakdown; spontaneous generation of dimensionfull fundamental scales; the cosmological constant problem; the problem of fermionic families; applications to dark energy problem and modern cosmological brane-world scenarios. For a detailed discussion we refer to the series of papers [2, 3].

Subsequently, the idea of employing an alternative non-Riemannian integration measure was applied systematically to string, p-brane and Dp-brane models [5]. The main feature of these new classes of modified string/ brane theories is the appearance of the pertinent string/brane tension as an additional dynamical degree of freedom beyond the usual string/brane physical degrees of freedom, instead of being introduced *ad hoc* as a dimensionfull scale. The dynamical string/brane tension acquires the physical meaning of a world-sheet electric field strength (in the string case) or world-volume $(p+1)$-form field strength (in the p-brane case) and obeys Maxwell (Yang-Mills) equations of motion or their higher-rank antisymmetric tensor gauge field analogues, respectively. As a result of the latter property the modified-measure string model with dynamical tension yields a simple classical mechanism of "color" charge confinement.

One of the drawbacks of modified-measure p-brane and Dp-brane models, similarly to the ordinary Nambu-Goto p-branes, is that Weyl-conformal invariance is lost beyond the simplest string case ($p = 1$). On the other hand, it turns out that the form of the action of the modified-measure string model with dynamical tension suggests a natural way to construct explicitly a radically new class of *Weyl-conformally invariant* p-brane models *for any* p [6]. The most profound property of the latter models is that for any even p they describe the dynamics of inherently *light-like* p-branes which makes them significantly different both from the standard Nambu-Goto (or Dirac-Born-Infeld) branes as well as from their modified versions with dynamical string/brane tensions [5] mentioned above.

Before proceeding to the main exposition, which is the detailed discussion of the properties of the new Weyl-conformally invariant light-like branes, let us briefly recall the standard Polyakov-type formulation of the ordinary (bosonic) Nambu-Goto p-brane action:

$$S = -\frac{T}{2} \int d^{p+1}\sigma \sqrt{-\gamma} \left[\gamma^{ab}\partial_a X^\mu \partial_b X^\nu G_{\mu\nu}(X) - \Lambda(p-1)\right] . \tag{1}$$

Here γ_{ab} is the ordinary Riemannian metric on the $p+1$-dimensional brane

[1] For related ideas, see [4].

world-volume with $\gamma \equiv \det ||\gamma_{ab}||$. The world-volume indices $a, b = 0, 1, .., p$; $G_{\mu\nu}$ denotes the Riemannian metric in the embedding space-time with space-time indices $\mu, \nu = 0, 1, \ldots, D-1$. T is the given *ad hoc* brane tension; the constant Λ can be absorbed by rescaling T (see below Eq.(7)). The equations of motion w.r.t. γ^{ab} and X^μ read:

$$T_{ab} \equiv \left(\partial_a X^\mu \partial_b X^\nu - \frac{1}{2}\gamma_{ab}\gamma^{cd}\partial_c X^\mu \partial_d X^\nu\right) G_{\mu\nu} + \gamma_{ab}\frac{\Lambda}{2}(p-1) = 0 \;, \tag{2}$$

$$\partial_a \left(\sqrt{-\gamma}\gamma^{ab}\partial_b X^\mu\right) + \sqrt{-\gamma}\gamma^{ab}\partial_a X^\nu \partial_b X^\lambda \Gamma^\mu_{\nu\lambda} = 0 \;, \tag{3}$$

where:

$$\Gamma^\mu_{\nu\lambda} = \frac{1}{2}G^{\mu\kappa}\left(\partial_\nu G_{\kappa\lambda} + \partial_\lambda G_{\kappa\nu} - \partial_\kappa G_{\nu\lambda}\right) \tag{4}$$

is the Cristoffel connection for the external metric.

Eqs.(2) when $p \neq 1$ imply:

$$\Lambda\gamma_{ab} = \partial_a X^\mu \partial_b X^\nu G_{\mu\nu} \;, \tag{5}$$

which in turn allows to rewrite Eq.(2) as:

$$T_{ab} \equiv \left(\partial_a X^\mu \partial_b X^\nu - \frac{1}{p+1}\gamma_{ab}\gamma^{cd}\partial_c X^\mu \partial_d X^\nu\right) G_{\mu\nu} = 0 \;. \tag{6}$$

Furthermore, using (5) the Polyakov-type brane action (1) becomes on-shell equivalent to the Nambu-Goto-type brane action:

$$S = -T\Lambda^{-\frac{p-1}{2}} \int d^{p+1}\sigma \sqrt{-\det ||\partial_a X^\mu \partial_b X^\nu G_{\mu\nu}||} \;. \tag{7}$$

Let us note the following properties of standard Nambu-Goto p-branes manifesting their crucial differences w.r.t. the Weyl-conformally invariant branes discussed below. Eq.(5) tells us that: (i) the induced metric on the Nambu-Goto p-brane world-volume is *non-singular*; (ii) standard Nambu-Goto p-branes describe intrinsically *massive* modes.

2. String and Brane Models with a Modified World-Sheet/ World-Volume Integration Measure

Here we briefly recall the construction of modified string and (p- and Dp-) brane models with dynamical tension based on the use of alternative non-Riemannian world-sheet/world-volume volume form (integration measure density) [5].

The modified-measure bosonic string model is given by the following action:

$$\begin{aligned} S \;=\; & -\int d^2\sigma\, \Phi(\varphi)\Big[\frac{1}{2}\gamma^{ab}\partial_a X^\mu \partial_b X^\nu G_{\mu\nu}(X) - \frac{\varepsilon^{ab}}{2\sqrt{-\gamma}}F_{ab}(A)\Big] \\ & + \int d^2\sigma \sqrt{-\gamma} A_a J^a \end{aligned} \tag{8}$$

with the notations:

$$\Phi(\varphi) \equiv \frac{1}{2}\,\varepsilon_{ij}\,\varepsilon^{ab}\,\partial_a\varphi^i\,\partial_b\varphi^j \quad , \quad F_{ab}(A) = \partial_a A_b - \partial_b A_a \; , \tag{9}$$

γ_{ab} denotes the intrinsic Riemannian world-sheet metric with $\gamma = \det\|\gamma_{ab}\|$ and $G_{\mu\nu}(X)$ is the Riemannian metric of the embedding space-time $(a,b = 0,1; i,j = 1,2; \mu,\nu = 0,1,\ldots,D-1)$.
Below is the list of differences w.r.t. the standard Nambu-Goto string (in the Polyakov-like formulation) :

- New non-Riemannian integration measure density $\Phi(\varphi)$ built in terms of auxiliary world-sheet scalar fields φ^i $(i = 1,2)$, independent of the world-sheet metric γ_{ab}, instead of the standard Riemannian one $\sqrt{-\gamma}$;
- Dynamical string tension $T \equiv \frac{\Phi(\varphi)}{\sqrt{-\gamma}}$ instead of *ad hoc* dimensionfull constant;
- Auxiliary world-sheet gauge field A_a in a would-be "topological" term $\int d^2\sigma\,\frac{\Phi(\varphi)}{\sqrt{-\gamma}}\,\frac{1}{2}\,\varepsilon^{ab}F_{ab}(A)$;
- Optional natural coupling of auxiliary A_a to external conserved world-sheet electric current J^a (see last term in (8) and Eq.(11) below).

The modified string model (8) is Weyl-conformally invariant similarly to the ordinary case. Here Weyl-conformal symmetry is given by Weyl rescaling of γ_{ab} supplemented with a special diffeomorphism in φ-target space:

$$\gamma_{ab} \longrightarrow \gamma'_{ab} = \rho\,\gamma_{ab}\,, \quad \varphi^i \longrightarrow \varphi'^{\,i} = \varphi'^{\,i}(\varphi) \;\text{ with }\; \det\Big\|\frac{\partial\varphi'^{\,i}}{\partial\varphi^j}\Big\| = \rho \;. \tag{10}$$

The dynamical string tension appears as a canonically conjugated momentum w.r.t. A_1: $\pi_{A_1} \equiv \frac{\partial\mathcal{L}}{\partial\dot{A}_1} = \frac{\Phi(\varphi)}{\sqrt{-\gamma}} \equiv T$, *i.e.*, T has the meaning of a *world-sheet electric field strength*, and the equations of motion w.r.t. auxiliary gauge field A_a look exactly as $D = 2$ Maxwell eqs.:

$$\frac{\varepsilon^{ab}}{\sqrt{-\gamma}}\,\partial_b T + J^a = 0 \;. \tag{11}$$

In particular, for $J^a = 0$:

$$\varepsilon^{ab}\,\partial_b\Big(\frac{\Phi(\varphi)}{\sqrt{-\gamma}}\Big) = 0\,, \qquad \frac{\Phi(\varphi)}{\sqrt{-\gamma}} \equiv T = \text{const}\;, \tag{12}$$

one gets a *spontaneously induced* constant string tension. Furthermore, when the modified string couples to point-like charges on the world-sheet (*i.e.*, $J^0\sqrt{-\gamma} = \sum_i e_i\delta(\sigma-\sigma_i)$ in (11)) one obtains classical charge *confinement*: $\sum_i e_i = 0$.

The above charge confinement mechanism has also been generalized in [5] to the case of coupling the modified string model with dynamical tension to non-Abelian world-sheet "color" charges. The latter is achieved as follows. Notice the following identity in $2D$ involving Abelian gauge field A_a:

$$\frac{\varepsilon^{ab}}{2\sqrt{-\gamma}} F_{ab}(A) = \sqrt{-\frac{1}{2} F_{ab}(A) F_{cd}(A) \gamma^{ac} \gamma^{bd}} \,. \qquad (13)$$

Then the extension of the action (8) to the non-Abelian case is straightforward:

$$S = -\int d^2\sigma\, \Phi(\varphi) \Big[\frac{1}{2} \gamma^{ab} \partial_a X^\mu \partial_b X^\nu G_{\mu\nu}(X) \qquad (14)$$
$$- \sqrt{-\frac{1}{2} \,\mathrm{T}r\, (F_{ab}(A) F_{cd}(A)) \gamma^{ac} \gamma^{bd}} \Big] + \int d^2\sigma \,\mathrm{T}r\, (A_a j^a)$$

with $F_{ab}(A) = \partial_a A_b - \partial_b A_c + i\,[A_a,\, A_b]$, sharing the same principal property – dynamical generation of string tension as an additional degree of freedom.

Similar construction has also been proposed for higher-dimensional modified-measure p- and Dp-brane models whose brane tension appears as an additional dynamical degree of freedom. On the other hand, like the standard Nambu-Goto branes, they are Weyl-conformally *non*-invariant and describe dynamics of *massive* modes.

3. Weyl-Invariant Branes: Action and Equations of Motion

The identity (13) suggests how to construct **Weyl-invariant** p-brane models for any p. Namely, we consider the following novel class of p-brane actions:

$$S = -\int d^{p+1}\sigma\, \Phi(\varphi) \Big[\frac{1}{2} \gamma^{ab} \partial_a X^\mu \partial_b X^\nu G_{\mu\nu}(X) - \sqrt{F_{ab}(A) F_{cd}(A) \gamma^{ac} \gamma^{bd}} \Big] \qquad (15)$$

$$\Phi(\varphi) \equiv \frac{1}{(p+1)!} \varepsilon_{i_1 \ldots i_{p+1}} \varepsilon^{a_1 \ldots a_{p+1}} \partial_{a_1} \varphi^{i_1} \ldots \partial_{a_{p+1}} \varphi^{i_{p+1}} \,, \qquad (16)$$

where notations similar to those in (8) are used (here $a, b = 0, 1, \ldots, p$; $i, j = 1, \ldots, p+1$).

The above action (15) is invariant under Weyl-conformal symmetry (the same as in the dynamical-tension string model (8)):

$$\gamma_{ab} \longrightarrow \gamma'_{ab} = \rho\, \gamma_{ab} \,, \quad \varphi^i \longrightarrow \varphi'^{\,i} = \varphi'^{\,i}(\varphi) \;\text{ with }\; \det \Big\| \frac{\partial \varphi'^{\,i}}{\partial \varphi^j} \Big\| = \rho \,. \qquad (17)$$

Let us note the following significant differences of (15) w.r.t. the standard Nambu-Goto p-branes (in the Polyakov-like formulation) :

- New non-Riemannian integration measure density $\Phi(\varphi)$ instead of $\sqrt{-\gamma}$, and *no* "cosmological-constant" term $((p-1)\sqrt{-\gamma})$;
- Variable brane tension $\chi \equiv \frac{\Phi(\varphi)}{\sqrt{-\gamma}}$ which is Weyl-conformal *gauge dependent*: $\chi \to \rho^{\frac{1}{2}(1-p)}\chi$;
- Auxiliary world-volume gauge field A_a in a "square-root" Maxwell (Yang-Mills) term[2]; the latter is straightforwardly generalized to the non-Abelian case – $\sqrt{-\,\mathrm{T}r\,(F_{ab}(A)\,F_{cd}(A))\,\gamma^{ac}\,\gamma^{bd}}$ similarly to (15);
- Natural optional couplings of the auxiliary gauge field A_a to external world-volume "color" charge currents j^a;
- The action (15) is manifestly Weyl-conformal invariant for *any* p; it describes *intrinsically light-like* p-branes for any even p, as it will be shown below.

In what follows we shall frequently use the short-hand notations:

$$(\partial_a X\, \partial_b X) \equiv \partial_a X^\mu\, \partial_b X^\nu G_{\mu\nu}\,, \qquad \sqrt{FF\gamma\gamma} \equiv \sqrt{F_{ab}\, F_{cd}\, \gamma^{ac}\, \gamma^{bd}}\,. \tag{18}$$

Employing (18) the equations of motion w.r.t. measure-building auxiliary scalars φ^i and w.r.t. γ^{ab} read, respectively:

$$\frac{1}{2}\,\gamma^{cd}\,(\partial_c X\, \partial_d X) - \sqrt{FF\gamma\gamma} = M\, \Bigl(= \text{const}\Bigr)\,, \tag{19}$$

$$\frac{1}{2}\,\left(\partial_a X\, \partial_b X\right) + \frac{F_{ac}\,\gamma^{cd}\,F_{db}}{\sqrt{FF\gamma\gamma}} = 0\,, \tag{20}$$

Taking the trace in (20) implies $M=0$ in Eq.(19).

Next we have the following equations of motion w.r.t. auxiliary gauge field A_a and w.r.t. X^μ, respectively:

$$\partial_b \left(\frac{F_{cd}\,\gamma^{ac}\,\gamma^{bd}}{\sqrt{FF\gamma\gamma}}\Phi(\varphi)\right) = 0\,, \tag{21}$$

$$\partial_a \Bigl(\Phi(\varphi)\,\gamma^{ab}\,\partial_b X^\mu\Bigr) + \Phi(\varphi)\,\gamma^{ab}\,\partial_a X^\nu\,\partial_b X^\lambda \Gamma^\mu_{\nu\lambda} = 0\,, \tag{22}$$

where $\Gamma^\mu_{\nu\lambda}$ is the Cristoffel connection corresponding to the external space-time metric $G_{\mu\nu}$ as in (4).

The A_a-equations of motion (21) can be solved in terms of $(p-2)$-form gauge potentials $\Lambda_{a_1 \ldots a_{p-2}}$ dual w.r.t. A_a. The respective field-strengths

[2] "Square-root" Maxwell (Yang-Mills) action in $D=4$ was originally introduced in the first ref.[7] and later generalized to "square-root" actions of higher-rank antisymmetric tensor gauge fields in $D \geq 4$ in the second and third refs.[7].

are related as follows:

$$F_{ab}(A) = -\frac{1}{\chi}\frac{\sqrt{-\gamma}\,\varepsilon_{abc_1\ldots c_{p-1}}}{2\,(p-1)}\,\gamma^{c_1d_1}\ \ldots\ \gamma^{c_{p-1}d_{p-1}} \\ \times\ F_{d_1\ldots d_{p-1}}(\Lambda)\,\gamma^{cd}\,(\partial_c X\partial_d X)\ , \tag{23}$$

$$\chi^2 = -\frac{2}{(p-1)^2}\,\gamma^{a_1b_1}\ldots\gamma^{a_{p-1}b_{p-1}}F_{a_1\ldots a_{p-1}}(\Lambda)\,F_{b_1\ldots b_{p-1}}(\Lambda)\,, \tag{24}$$

where $\chi \equiv \frac{\Phi(\varphi)}{\sqrt{-\gamma}}$ is the variable brane tension, and:

$$F_{a_1\ldots a_{p-1}}(\Lambda) = (p-1)\partial_{[a_1}\Lambda_{a_2\ldots a_{p-1}]} \tag{25}$$

is the $(p-1)$-form dual field-strength.

All equations of motion can be equivalently derived from the following *dual WILL*-brane action:

$$S_{\rm dual}[X,\gamma,\Lambda] = -\frac{1}{2}\int d^{p+1}\sigma\,\chi(\gamma,\Lambda)\sqrt{-\gamma}\,\gamma^{ab}\,\partial_a X^\mu\,\partial_b X^\nu\,G_{\mu\nu}(X) \tag{26}$$

with $\chi(\gamma,\Lambda)$ given in (24) above.

4. Intrinsically Light-Like Branes. *WILL*-Membrane

Let us consider the γ^{ab}-equations of motion (20). F_{ab} is an anti-symmetric $(p+1)\times(p+1)$ matrix, therefore, F_{ab} is *not invertible* in any odd $(p+1)$ – it has at least one zero-eigenvalue vector V^a ($F_{ab}V^b = 0$). Therefore, for any odd $(p+1)$ the induced metric

$$g_{ab} \equiv (\partial_a X\,\partial_b X) \equiv \partial_a X^\mu\,\partial_b X^\nu G_{\mu\nu}(X) \tag{27}$$

on the world-volume of the Weyl-invariant brane (15) is *singular* as *opposed* to the ordinary Nambu-Goto brane (where the induced metric is proportional to the intrinsic Riemannian world-volume metric, cf. Eq.(5)):

$$(\partial_a X\partial_b X)\,V^b = 0\,, \qquad \text{i.e.}\ \ (\partial_V X\,\partial_V X) = 0\,, \quad (\partial_\perp X\,\partial_V X) = 0\,, \tag{28}$$

where $\partial_V \equiv V^a\partial_a$ and $\partial_\perp$ are derivates along the tangent vectors in the complement of the tangent vector field V^a.

Thus, we arrive at the following important conclusion: every point on the world-surface of the Weyl-invariant p-brane (15) (for odd $(p+1)$) moves with the speed of light in a time-evolution along the zero-eigenvalue vector-field V^a of F_{ab}. Therefore, we will name (15) (for odd $(p+1)$) by the acronym *WILL-brane* (Weyl-Invariant Light-Like-brane) model.

Henceforth we will explicitly consider the special case $p = 2$ of (15), *i.e.*, the Weyl-invariant light-like membrane model. The associated *WILL*-membrane dual action (particular case of (26) for $p = 2$) reads:

$$S_{\rm dual} = -\frac{1}{2}\int d^3\sigma\, \chi(\gamma, u)\, \sqrt{-\gamma}\, \gamma^{ab}\, (\partial_a X \partial_b X)\ ,\ \ \chi(\gamma, u) \equiv \sqrt{-2\, \gamma^{cd}\, \partial_c u\, \partial_d u}\,, \tag{29}$$

where u is the dual "gauge" potential w.r.t. A_a:

$$F_{ab}(A) = -\frac{1}{2\,\chi(\gamma, u)}\, \sqrt{-\gamma}\, \varepsilon_{abc}\, \gamma^{cd}\, \partial_d u\, \gamma^{ef}\, (\partial_e X\, \partial_f X)\ . \tag{30}$$

$S_{\rm dual}$ is manifestly Weyl-invariant (under $\gamma_{ab} \to \rho\gamma_{ab}$).
The equations of motion w.r.t. γ^{ab}, u (or A_a), and X^μ read accordingly:

$$(\partial_a X\, \partial_b X) + \frac{1}{2}\, \gamma^{cd}\, (\partial_c X\, \partial_d X) \left(\frac{\partial_a u\, \partial_b u}{\gamma^{ef}\, \partial_e u\, \partial_f u} - \gamma_{ab}\right) = 0\,, \tag{31}$$

$$\partial_a \left(\frac{\sqrt{-\gamma}\, \gamma^{ab}\, \partial_b u}{\chi(\gamma, u)}\, \gamma^{cd}\, (\partial_c X\, \partial_d X)\right) = 0\,, \tag{32}$$

$$\partial_a \Big(\chi(\gamma, u)\, \sqrt{-\gamma}\, \gamma^{ab}\, \partial_b X^\mu\Big) + \chi(\gamma, u)\, \sqrt{-\gamma}\, \gamma^{ab}\, \partial_a X^\nu\, \partial_b X^\lambda\, \Gamma^\mu_{\nu\lambda} = 0\,. \tag{33}$$

The last factor in brackets on the l.h.s. of Eq.(31) is a projector implying that the induced metric $g_{ab} \equiv (\partial_a X \partial_b X)$ has zero-mode eigenvector $V^a = \gamma^{ab}\partial_b u$.
The invariance under world-volume reparametrizations allows to introduce the following standard (synchronous) gauge-fixing conditions:

$$\gamma^{0i} = 0\ \ (i = 1, 2)\,, \qquad \gamma^{00} = -1\,. \tag{34}$$

Using (34) we can easily find solutions of Eq.(32) for the dual "gauge potential" u in spite of its high non-linearity by taking the following ansatz:

$$u(\tau, \sigma^1, \sigma^2) = \frac{T_0}{\sqrt{2}}\, \tau\,. \tag{35}$$

Here T_0 is an arbitrary integration constant with the dimension of membrane tension. In particular:

$$\chi \equiv \sqrt{-2\, \gamma^{ab}\, \partial_a u\, \partial_b u} = T_0\,. \tag{36}$$

The ansatz (35) means that we take $\tau \equiv \sigma^0$ to be evolution parameter along the zero-eigenvalue vector-field of the induced metric on the brane ($V^a = \gamma^{ab}\partial_b u = {\rm const}\,(1, 0, 0)$).

The ansatz for u (35) together with the gauge choice for γ_{ab} (34) brings the equations of motion w.r.t. γ^{ab}, u (or A_a) and X^μ in the following form (recall $(\partial_a X\, \partial_b X) \equiv \partial_a X^\mu\, \partial_b X^\nu\, G_{\mu\nu}$):

$$(\partial_0 X\, \partial_0 X) = 0\,, \qquad (\partial_0 X\, \partial_i X) = 0\,, \tag{37}$$

$$(\partial_i X\, \partial_j X) - \frac{1}{2}\,\gamma_{ij}\,\gamma^{kl}\,(\partial_k X\, \partial_l X) = 0\,, \tag{38}$$

(notice that Eqs.(38) look exactly like the classical (Virasoro) constraints for an Euclidean string theory with world-sheet parameters (σ^1, σ^2));

$$\partial_0 \left(\sqrt{\gamma_{(2)}}\,\gamma^{kl}\,(\partial_k X\, \partial_l X)\right) = 0\,, \tag{39}$$

where $\gamma_{(2)} = \det \|\gamma_{ij}\|$ (the above equation is the only remnant from the A_a-equations of motion (21));

$$\Box^{(3)} X^\mu + \left(-\partial_0 X^\nu\, \partial_0 X^\lambda + \gamma^{kl}\,\partial_k X^\nu\, \partial_l X^\lambda\right)\Gamma^\mu_{\nu\lambda} = 0\,, \tag{40}$$

where:

$$\Box^{(3)} \equiv -\frac{1}{\sqrt{\gamma^{(2)}}}\,\partial_0\left(\sqrt{\gamma^{(2)}}\,\partial_0\right) + \frac{1}{\sqrt{\gamma^{(2)}}}\,\partial_i\left(\sqrt{\gamma^{(2)}}\,\gamma^{ij}\,\partial_j\right)\,. \tag{41}$$

We can also extend the *WILL*-brane model (15) via a coupling to external space-time electromagnetic field $\mathcal{A}_\mu$. The natural Weyl-conformal invariant candidate action reads (for $p = 2$):

$$\begin{aligned} S \;=\; & -\int d^3\sigma\, \Phi(\varphi)\Big[\,\frac{1}{2}\,\gamma^{ab}\,\partial_a X^\mu\, \partial_b X^\nu\, G_{\mu\nu} - \sqrt{F_{ab}\,F_{cd}\,\gamma^{ac}\,\gamma^{bd}}\,\Big] \\ & + q\int d^3\sigma\, \varepsilon^{abc}\mathcal{A}_\mu\, \partial_a X^\mu F_{bc}\,. \end{aligned} \tag{42}$$

The last Chern-Simmons-like term is a special case of a class of Chern-Simmons-like couplings of extended objects to external electromagnetic fields proposed in ref.[8].

Instead of the action (42) we can use its dual one (similar to the simpler case Eq.(15) versus Eq.(29)):

$$S^{\rm dual}_{\rm WILL-brane} = -\frac{1}{2}\int d^3\sigma\, \chi(\gamma, u, \mathcal{A})\,\sqrt{-\gamma}\,\gamma^{ab}\,(\partial_a X\, \partial_b X)\,, \tag{43}$$

where the variable brane tension $\chi \equiv \frac{\Phi(\varphi)}{\sqrt{-\gamma}}$ is given by:

$$\chi(\gamma, u, \mathcal{A}) \equiv \sqrt{-2\,\gamma^{cd}\,(\partial_c u - q\,\mathcal{A}_c)\,(\partial_d u - q\,\mathcal{A}_d)}\,, \quad \mathcal{A}_a \equiv \mathcal{A}_\mu\, \partial_a X^\mu\,. \tag{44}$$

Here u is the dual "gauge" potential w.r.t. A_a and the corresponding field-strength and dual field-strength are related as (cf. Eq.(30)) :

$$F_{ab}(A) = -\frac{1}{2\,\chi(\gamma, u, \mathcal{A})}\,\sqrt{-\gamma}\,\varepsilon_{abc}\,\gamma^{cd}\,(\partial_d u - \mathcal{A}_d)\,\gamma^{ef}(\partial_e X\,\partial_f X)\;. \tag{45}$$

The corresponding equations of motion w.r.t. γ^{ab}, u (or A_a), and X^μ read accordingly:

$$(\partial_a X\,\partial_b X) + \frac{1}{2}\,\gamma^{cd}\,(\partial_c X\,\partial_d X)\left(\frac{(\partial_a u - q\mathcal{A}_a)\,(\partial_b u - q\mathcal{A}_b)}{\gamma^{ef}\,(\partial_e u - q\mathcal{A}_e)\,(\partial_f u - q\mathcal{A}_f)} - \gamma_{ab}\right) = 0\,, \tag{46}$$

$$\partial_a\left(\frac{\sqrt{-\gamma}\,\gamma^{ab}\,(\partial_b u - q\,\mathcal{A}_b)}{\chi(\gamma, u, \mathcal{A})}\,\gamma^{cd}\,(\partial_c X\,\partial_d X)\right) = 0\,, \tag{47}$$

$$\partial_a\left(\chi(\gamma, u, \mathcal{A})\,\sqrt{-\gamma}\,\gamma^{ab}\,\partial_b X^\mu\right) + \chi(\gamma, u, \mathcal{A})\,\sqrt{-\gamma}\,\gamma^{ab}\,\partial_a X^\nu\,\partial_b X^\lambda\,\Gamma^\mu_{\nu\lambda} + q\,\varepsilon^{abc}\,F_{bc}\,\partial_a X^\nu\,(\partial_\lambda\mathcal{A}_\nu - \partial_\nu\mathcal{A}_\lambda)\,G^{\lambda\mu} = 0\,. \tag{48}$$

5. *WILL*-Membrane Solutions in Various Gravitational Backgrounds

5.1. *WILL*-Membrane in a PP-Wave Background

As a first non-trivial example let us consider *WILL*-membrane dynamics in an external background generalizing the plane-polarized gravitational wave (*pp-wave*):

$$(ds)^2 = -dx^+dx^- - F(x^+, x^I)\,(dx^+)^2 + h_{IJ}(x^K)\,dx^I dx^J\,, \tag{49}$$

(for the ordinary pp-wave $h_{IJ}(x^K) = \delta_{IJ}$), and let us employ in (37)–(41) the following natural ansatz for X^μ (here $\sigma^0 \equiv \tau$; $I = 1, \ldots, D-2$) :

$$X^- = \tau\,, \qquad X^+ = X^+(\tau, \sigma^1, \sigma^2)\,, \qquad X^I = X^I(\sigma^1, \sigma^2)\,. \tag{50}$$

The non-zero affine connection symbols for the generalized pp-wave metric (49) are: $\Gamma^-_{++} = \partial_+ F$, $\Gamma^-_{+I} = \partial_I F$, $\Gamma^I_{++} = \frac{1}{2}\,h^{IJ}\partial_J F$, and Γ^I_{JK} – the ordinary Cristoffel symbols for the metric h_{IJ} in the transverse dimensions. It is straightforward to show that the solution does not depend on the form of the pp-wave front $F(x^+, x^I)$ and reads:

$$X^+ = X^+_0 = \text{const}\,, \qquad \gamma_{ij} \;\text{ are } \tau-\text{independent}\,; \tag{51}$$

$$\left(\partial_i X^I\,\partial_j X^J - \frac{1}{2}\,\gamma_{ij}\,\gamma^{kl}\,\partial_k X^I\,\partial_l X^J\right) h_{IJ} = 0\,, \tag{52}$$

$$\frac{1}{\sqrt{\gamma^{(2)}}}\,\partial_i\left(\sqrt{\gamma^{(2)}}\,\gamma^{ij}\,\partial_j X^I\right) + \gamma^{kl}\,\partial_k X^J\,\partial_l X^K\,\Gamma^I_{JK} = 0\,. \tag{53}$$

The latter two equations for the transverse brane coordinates describe a string moving in the $(D-2)$-dimensional Euclidean-signature transverse space.

5.2. *WILL*-Membrane in a Product-Space Background

Here we consider *WILL*-membrane moving in a general product-space $D=(d+2)$-dimensional gravitational background $\mathcal{M}^d \times \Sigma^2$ with coordinates (x^μ, y^m) $(\mu=0,1,\ldots,d-1,\ m=1,2)$ and Riemannian metric $(ds)^2 = f(y)\, g_{\mu\nu}(x)\, dx^\mu\, dx^\nu + g_{mn}(y)\, dy^m\, dy^n$. The metric on $\mathcal{M}^d$ is of Lorentzian signature and Σ^2 will be taken as a sphere for simplicity.

We assume that the *WILL*-brane wraps around the "internal" space Σ^2 and use the following ansatz (recall $\tau \equiv \sigma^0$):

$$X^\mu = X^\mu(\tau)\,, \qquad Y^m = \sigma^m\,, \qquad \gamma_{mn} = a(\tau)\, g_{mn}(\sigma^1,\sigma^2)\,. \tag{54}$$

Then the equations of motion and constraints (37)–(41) reduce to:

$$\partial_\tau X^\mu\, \partial_\tau X^\nu\, g_{\mu\nu}(X) = 0\,, \quad \frac{1}{a(\tau)}\, \partial_\tau \Big(a(\tau)\, \partial_\tau X^\mu\Big) + \partial_\tau X^\nu\, \partial_\tau X^\lambda\, \Gamma^\mu_{\nu\lambda} = 0 \tag{55}$$

where $a(\tau)$ is the conformal factor of the space-like part of the internal membrane metric (last Eq.(54)). Eqs.(55) are of the same form as the equations of motion for a massless point-particle with a world-line "einbein" $e = a^{-1}$ moving in $\mathcal{M}^d$. In other words, the simple solution above describes a membrane living in the extra "internal" dimensions Σ^2 and moving as a whole with the speed of light in "ordinary" space-time $\mathcal{M}^d$.

Let us particularly emphasize the fact that, although the *WILL*-brane is wrapping the extra (compact) dimensions in a topologically non-trivial way (cf. second Eq.(54)), its modes remain *massless* from the projected d-dimensional space-time point of view. This is a new phenomenon from the point of view of Kaluza-Klein theories: here we have particles (membrane modes), which aquire non-zero quantum numbers due to non-trivial winding, while at the same time these particles (modes) remain massless. In contrast, one should recall that in ordinary Kaluza-Klein theory (for a review, see [9]), non-trivial dependence on the extra dimensions is possible for point particles or even standard strings and branes only at a very high energy cost (either by momentum modes or winding modes), which implies a very high mass from the projected d-dimensional space-time point of view.

5.3. *WILL*-Membrane in Spherically-Symmetric Backgrounds

Let us consider general $SO(3)$-symmetric background in $D=4$ embedding space-time:

$$(ds)^2 = -A(z,t)(dt)^2 + B(z,t)(dz)^2 + C(z,t)\left((d\theta)^2 + \sin^2\theta\,(d\phi)^2\right). \tag{56}$$

The usual ansatz:

$$X^0 \equiv t = \tau \,, \quad X^1 \equiv z = z(\tau, \sigma^1, \sigma^2) \,, \quad X^2 \equiv \theta = \sigma^1 \,, \quad X^3 \equiv \phi = \sigma^2 \,, \tag{57}$$
$$\gamma_{ij} = a(\tau) \left((d\sigma^1)^2 + \sin^2(\sigma^1)(d\sigma^2)^2 \right)$$

yields:

(i) equations for $z(\tau, \sigma^1, \sigma^2)$:

$$\frac{\partial z}{\partial \tau} = \pm \sqrt{\frac{A}{B}} \,, \qquad \frac{\partial z}{\partial \sigma^i} = 0 \,; \tag{58}$$

(ii) a restriction on the background itself (comes from the gauge-fixed equations of motion for the dual gauge potential u (39)) :

$$\frac{dC}{d\tau} \equiv \left(\frac{\partial C}{\partial t} \pm \sqrt{\frac{A}{B}} \frac{\partial C}{\partial z} \right) \Big|_{t=\tau,\, z=z(\tau)} = 0 \,; \tag{59}$$

(iii) an equation for the conformal factor $a(\tau)$ of the internal membrane metric:

$$\partial_\tau a + \left(\frac{\frac{\partial}{\partial t}\sqrt{AB} \pm \partial_z A}{\sqrt{AB}} \Big|_{t=\tau,\, z=z(\tau)} \right) a(\tau) - \frac{\frac{\partial}{\partial t} C}{A} \Big|_{t=\tau,\, z=z(\tau)} = 0 \ . \tag{60}$$

Eq.(59) tells that the (squared) sphere radius $R^2 \equiv C(z,t)$ must remain constant along the *WILL*-brane trajectory.
In particular, let us take static spherically-symmetric gravitational background in $D = 4$:

$$(ds)^2 = -A(r)(dt)^2 + B(r)(dr)^2 + r^2 \left[(d\theta)^2 + \sin^2(\theta)\,(d\phi)^2 \right] . \tag{61}$$

Specifically we have:

$$A(r) = B^{-1}(r) = 1 - \frac{2GM}{r} \tag{62}$$

for Schwarzschild black hole,

$$A(r) = B^{-1}(r) = 1 - \frac{2GM}{r} + \frac{Q^2}{r^2} \tag{63}$$

for Reissner-Nordström black hole,

$$A(r) = B^{-1}(r) = 1 - \kappa\, r^2 \tag{64}$$

for (anti-) de Sitter space, *etc.*.

In the case of (61) Eqs.(58)–(59) reduce to:

$$\frac{\partial r}{\partial \tau} = \pm A(r) \,, \qquad \frac{\partial r}{\partial \sigma^i} = 0 \,, \qquad \frac{\partial r}{\partial \tau} = 0 \,, \tag{65}$$

yielding:

$$r = r_0 \equiv \text{const} \,, \quad \text{where} \quad A(r_0) = 0 \,. \tag{66}$$

Further, Eq.(60) implies for the intrinsic *WILL*-membrane metric:

$$\|\gamma_{ij}\| = c_0 \, e^{\mp\tau/r_0} \begin{pmatrix} 1 & 0 \\ 0 & \sin^2(\sigma^1) \end{pmatrix} \,, \tag{67}$$

where c_0 is an arbitrary integration constant.

From (66) we conclude that the *WILL*-membrane with spherical topology (and with exponentially blowing-up/deflating radius w.r.t. internal metric, see Eq.(67)) automatically "sits" on (materializes) the event horizon of the pertinent black hole in $D = 4$ embedding space-time. This conforms with the well-known general property of closed light-like hypersurfaces in $D = 4$ (*i.e.*, their section with the hyper-plane t=const being a compact 2-dimensional manifold) which automatically serve as horizons [10]. On the other hand, let us stress that our *WILL*-membrane model (29) provides an explicit *dynamical* realization of event horizons.

6. Coupled Einstein-Maxwell-*WILL*-Membrane System: *WILL*-Membrane as a Source for Gravity and Electromagnetism

We can extend the results from the previous section to the case of the self-consistent Einstein-Maxwell-*WILL*-membrane system, *i.e.*, we will consider the *WILL*-membrane as a dynamical material and electrically charged source for gravity and electromagnetism. The relevant action reads:

$$\begin{aligned} S \;=\; & \int d^4x \sqrt{-G} \left[\frac{R(G)}{16\pi G_N} - \frac{1}{4} \mathcal{F}_{\mu\nu}(\mathcal{A}) \, \mathcal{F}_{\kappa\lambda}(\mathcal{A}) \, G^{\mu\kappa} G^{\nu\lambda}\right] \\ & + S_{\mathrm{WILL-brane}} \,, \end{aligned} \tag{68}$$

where $\mathcal{F}_{\mu\nu}(\mathcal{A}) = \partial_\mu \mathcal{A}_\nu - \partial_\nu \mathcal{A}_\mu$ is the space-time electromagnetic field-strength, and $S_{\mathrm{WILL-brane}}$ indicates the the *WILL*-membrane action coupled to the space-time gauge field $\mathcal{A}_\mu$ – either (42) or its dual (43).

The equations of motion for the *WILL*-membrane subsystem are of the same form as Eqs.(46)–(48). The Einstein-Maxwell equations of motion read:

$$R_{\mu\nu} - \frac{1}{2} G_{\mu\nu} R = 8\,\pi\, G_N \left(T^{(EM)}_{\mu\nu} + T^{(brane)}_{\mu\nu}\right) \,, \tag{69}$$

$$-\partial_\nu \left(\sqrt{-G}\, G^{\mu\kappa} G^{\nu\lambda} \mathcal{F}_{\kappa\lambda}\right) + j^\mu = 0 \,, \tag{70}$$

where:

$$T^{(EM)}_{\mu\nu} \equiv \mathcal{F}_{\mu\kappa}\,\mathcal{F}_{\nu\lambda}\,G^{\kappa\lambda} - G_{\mu\nu}\,\frac{1}{4}\,\mathcal{F}_{\rho\kappa}\,\mathcal{F}_{\sigma\lambda}\,G^{\rho\sigma}G^{\kappa\lambda}\,, \tag{71}$$

$$T^{(brane)}_{\mu\nu} \equiv -G_{\mu\kappa}\,G_{\nu\lambda}\int\! d^3\sigma\,\frac{\delta^{(4)}\Big(x - X(\sigma)\Big)}{\sqrt{-G}}\,\chi\,\sqrt{-\gamma}\gamma^{ab}\partial_a X^\kappa \partial_b X^\lambda\,, \tag{72}$$

$$j^\mu \equiv q\int\! d^3\sigma\,\delta^{(4)}\Big(x - X(\sigma)\Big)\,\varepsilon^{abc}\,F_{bc}\partial_a X^\mu\,. \tag{73}$$

We find the following self-consistent spherically symmetric stationary solution for the coupled Einstein-Maxwell-*WILL*-membrane system (68). For the Einstein subsystem we have a solution:

$$(ds)^2 = -A(r)(dt)^2 + A^{-1}(r)\,(dr)^2 + r^2\,[\,(d\theta)^2 + \sin^2(\theta)\,(d\phi)^2\,]\,, \tag{74}$$

consisting of two different black holes with a *common* event horizon:

- Schwarzschild black hole inside the horizon:

$$A(r) \equiv A_-(r) = 1 - \frac{2GM_1}{r}\;,\quad \text{for}\;\; r < r_0 \equiv r_{\rm horizon} = 2GM_1\,. \tag{75}$$

- Reissner-Norström black hole outside the horizon:

$$A(r) \equiv A_+(r) = 1 - \frac{2GM_2}{r} + \frac{GQ^2}{r^2}\;,\quad \text{for}\;\; r > r_0 \equiv r_{\rm horizon}\,, \tag{76}$$

  where $Q^2 = 8\,\pi\,q^2 r^4_{\rm horizon} \equiv 128\,\pi\,q^2\,G^4\,M_1^4$;

For the Maxwell subsystem we have $\mathcal{A}_1 = \ldots = \mathcal{A}_{D-1} = 0$ everywhere and:

- Coulomb field outside horizon:

$$\mathcal{A}_0 = \frac{\sqrt{2}\,q\,r^2_{\rm horizon}}{r}\;,\quad \text{for}\;\; r \geq r_0 \equiv r_{\rm horizon}\,. \tag{77}$$

- No electric field inside horizon:

$$\mathcal{A}_0 = \sqrt{2}\,q\,r_{\rm horizon} = \text{const}\;,\quad \text{for}\;\; r \leq r_0 \equiv r_{\rm horizon}\,. \tag{78}$$

Using the same (synchronous) gauge choice (34) and ansatz for the dual "gauge potential" (35), as well as taking into account (77)–(78), the *WILL*-membrane equations of motion (46)–(48) acquire the form (recall $(\partial_a X\,\partial_b X) \equiv \partial_a X^\mu\,\partial_b X^\nu\,G_{\mu\nu}$):

$$(\partial_0 X\,\partial_0 X) = 0\,, \qquad (\partial_0 X\,\partial_i X) = 0\,, \tag{79}$$

$$(\partial_i X\,\partial_j X) - \frac{1}{2}\,\gamma_{ij}\,\gamma^{kl}\,(\partial_k X \partial_l X) = 0\,, \tag{80}$$

(these constraints are the same as in the absence of coupling to space-time gauge field (37)–(38));

$$\partial_0 \left(\sqrt{\gamma_{(2)}}\, \gamma^{kl} \left(\partial_k X\, \partial_l X \right) \right) = 0 \,, \tag{81}$$

(once again the same equation as in the absence of coupling to space-time gauge field (39));

$$\widetilde{\Box}^{(3)} X^\mu + \left(-\partial_0 X^\nu\, \partial_0 X^\lambda + \gamma^{kl}\, \partial_k X^\nu\, \partial_l X^\lambda \right) \Gamma^\mu_{\nu\lambda} \\ - q\, \frac{\gamma^{kl} \left(\partial_k X\, \partial_l X \right)}{\sqrt{2}\, \chi}\, \partial_0 X^1\, \partial_r \mathcal{A}_0\, G^{1\mu} = 0 \,, \tag{82}$$

Here $\chi \equiv T_0 - \sqrt{2}\, q\, \mathcal{A}_0 \equiv T_0 - \sqrt{2}\, q\, \frac{Q}{r}$ is the variable brane tension coming from Eqs.(35), (44); $X^0 \equiv t, X^1 \equiv r, X^2 \equiv \theta, X^3 \equiv \phi$; and

$$\widetilde{\Box}^{(3)} \equiv -\frac{1}{\chi \sqrt{\gamma^{(2)}}}\, \partial_0 \left(\chi \sqrt{\gamma^{(2)}}\, \partial_0 \right) + \frac{1}{\chi \sqrt{\gamma^{(2)}}}\, \partial_i \left(\chi \sqrt{\gamma^{(2)}}\, \gamma^{ij}\, \partial_j \right) \,. \tag{83}$$

A self-consistent solution to Eqs.(79)–(82) reads:

$$X^0 \equiv t = \tau \,, \qquad \theta = \sigma^1 \,, \qquad \phi = \sigma^2 \,, \tag{84}$$

$$r(\tau, \sigma^1, \sigma^2) = r_{\rm horizon} = {\rm const} \,, \qquad A_\pm(r_{\rm horizon}) = 0 \,, \tag{85}$$

i.e., the *WILL*-membrane automatically positions itself on the common event horizon of the pertinent black holes. Furthermore, inserting (84)–(85) in the expression (72) for the *WILL*-membrane energy-momentum tensor $T^{(brane)}_{\mu\nu}$, the Einstein equations (69) entail the following important matching conditions for the space-time metric components along the *WILL*-membrane surface:

$$\frac{\partial}{\partial r} A_+ \Big|_{r=r_{\rm horizon}} - \frac{\partial}{\partial r} A_- \Big|_{r=r_{\rm horizon}} = -16\pi G \chi \,. \tag{86}$$

Condition (86) in turn yields relations between the parameters of the black holes and the *WILL*-membrane (q being its surface charge density) :

$$M_2 = M_1 + 32\, \pi\, q^2\, G^3\, M_1^3 \tag{87}$$

and for the brane tension χ:

$$\chi \equiv T_0 - 2\, q^2\, r_{\rm horizon} = q^2\, G M_1 \,, \quad \text{i.e.} \quad T_0 = 5\, q^2\, G\, M_1 \tag{88}$$

The matching condition (86) corresponds to the so called statically soldering conditions in the theory of light-like thin shell dynamics in general relativity [11]. Unlike the latter model, where the membranes are introduced *ad hoc*, the present *WILL*-brane models provide a systematic dynamical description of light-like branes (as sources for both gravity and electromagnetism) from first principles starting with concise Weyl-conformally invariant actions (42), (68).

7. Conclusions and Outlook

In the present work we have discussed a novel class of Weyl-invariant p-brane theories whose dynamics significantly differs from ordinary Nambu-Goto p-brane dynamics. The princial features of our construction are as follows:

- Employing alternative non-Riemannian integration measure (volume-form) (16) on the p-brane world-volume independent of the intrinsic Riemannian metric.
- Acceptable dynamics in the novel class of brane models (Eqs.(15),(42)) *naturally* requires the introduction of additional world-volume gauge fields.
- By employing square-root Yang-Mills actions for the pertinent world-volume gauge fields one achieves manifest *Weyl-conformal symmetry* in the new class of p-brane theories *for any* p.
- The brane tension is *not* a constant dimensionfull scale given *ad hoc*, but rather it appears as a *composite* world-volume scalar field (Eqs. (24), (29), (44)) transforming non-trivially under Weyl-conformal transformations.
- The novel class of Weyl-invariant p-brane theories describes intrinsically *light-like* p-branes for any even p (*WILL*-branes).
- When put in a gravitational black hole background, the *WILL*-membrane ($p = 2$) automatically sits on ("materializes") the event horizon.
- When moving in background product-spaces ("Kaluza-Klein" context) the *WILL*-membrane describes *massless* modes, even though the membrane is wrapping the extra dimensions and therefore aquiring non-trivial Kaluza-Klein charges.
- The coupled Einstein-Maxwell- *WILL*-membrane system (68) possesses self-consistent solution where the *WILL*-membrane serves as a material and electrically charged source for gravity and electromagnetism, and it automatically "sits" on (materializes) the common event horizon for a Schwarzschild (in the interior) and Reissner-Nordström (in the exterior) black holes. Thus our model (68) provides an explicit dynamical realization of the so called "membrane paradigm" in the physics of black holes [12].
- The *WILL*-branes could be good representations for the string-like objects introduced by 't Hooft in ref.[13] to describe gravitational interactions associated with black hole formation and evaporation, since as shown above the *WILL*-branes locate themselves automatically in the horizons and, therefore, they could represent degrees of freedom associated particularly with horizons.

The novel class of Weyl-conformal invariant p-branes discussed above suggests various physically interesting directions for further study such as: quantization (Weyl-conformal anomaly and critical dimensions); supersymmetric generalization; possible relevance for the open string dynamics (similar to the role played by Dirichlet- (Dp-)branes); *WILL*-brane dynamics in more complicated gravitational black hole backgrounds (*e.g.*, Kerr-Newman).

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