Anti-de Sitter 3-dimensional gravity with torsion*

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Abstract

Using the canonical formalism, we study the asymptotic symmetries of the topological 3-dimensional gravity with torsion. In the anti-de Sitter sector, the symmetries are realized by two independent Virasoro algebras with classical central charges. In the simple case of the teleparallel vacuum geometry, the central charges are equal to each other and have the same value as in general relativity, while in the general Riemann-Cartan geometry, they become different.

Introduction

Three-dimensional (3d) gravity has been used as a theoretical laboratory to test some of the conceptual problems of both classical and quantum gravity [1, 2]. One can identify several particularly important achievements in the development of these ideas. Brown and Henneaux demonstrated that, under suitable asymptotic conditions, the asymptotic symmetry of 3d gravity has an extremely rich structure, described by two independent canonical Virasoro algebras with classical central charges [3]. Soon after that, Witten found that general relativity with a cosmological constant (GR_Λ) can be formulated as a Chern-Simons gauge theory [4]. The equivalence between gravity and an ordinary gauge theory was crucial for a deeper understanding of quantum gravity. Next, the discovery of the black hole solution by Bañados, Teitelboim and Zanelli, had a powerful impact on 3d gravity [5]. It turned out that the Virasoro algebra of the asymptotic symmetry plays

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a central role in our understanding of the quantum nature of black hole [6–11].

Following a widely spread belief that the dynamics of gravity is to be described by general relativity, investigations of 3d gravity have been carried out mostly in the realm of *Riemannian* geometry. However, since the early 1990s, the possibility of *Riemann-Cartan* geometry has been also explored [12–18]; it is a geometry in which both the *curvature* and the *torsion* are present as independent geometric characteristics of spacetime [19, 20]. In this way, one expects to clarify the influence of geometry on the dynamical content of spacetime.

Dynamics of a theory is determined not only by its action, but also by the asymptotic conditions. The dynamical content of asymptotic conditions is best seen in topological theories, where the non-trivial dynamics is bound to exist only at the boundary. General action for topological 3d gravity in Riemann–Cartan spacetime has been proposed by Mielke and Baekler [12, 13]. This model is our starting point for exploring the structure of 3d gravity with torsion. In particular, we shall investigate

- the existence of the black hole with torsion, and
- the asymptotic structure of 3d gravity with torsion.

We restrict ourselves to the anti-de Sitter (AdS) sector of the theory, with negative effective cosmological constant. For a particular choice of parameters, the Mielke-Baekler action leads to the *teleparallel* (Weizenböck) geometry in vacuum [21, 22, 20], defined by the requirement of vanishing curvature, which we choose as the simplest framework for studying the influence of torsion on the spacetime dynamics.

The paper is organized as follows. In Sect. 2 we introduce Riemann–Cartan spacetime as a general geometric arena for 3d gravity with torsion, and discuss the teleparallel description of gravity in vacuum. In Sect. 3 we construct the teleparallel black hole solution. Then, in Sect. 4, we introduce the concept of asymptotically AdS configuration, and show that the related asymptotic symmetry is the same as in general relativity—the conformal symmetry. In the next section, the gauge structure of the theory is incorporated into the canonical formalism by investigating the Poisson bracket algebra of the asymptotic generators. The asymptotic symmetry is characterized by two independent canonical Virasoro algebras with classical central charges, the values of which are the same as in Riemannian spacetime of general relativity. In Sect. 6 we discuss the general case of Riemann-Cartan geometry, and show that the related classical central charges are different. Finally, Sect. 7 is devoted to concluding remarks.[1]

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Concluding remarks

- 3d gravity with torsion, defined by the Mielke-Baekler action, possesses the teleparallel black hole solution, a generalization of the Riemannian BTZ black hole.

[1]For the complete version of the text, see the preprint gr-qc/0412072.

▪ Assuming the AdS asymptotic conditions, the canonical asymptotic symmetry is realized by two commuting Virasoro algebras with central extensions:
− in GR_Λ and in the teleparallel theory: $c_1 = c_2 = 3\ell/2G$,
− in Riemann-Cartan theory: $c_1 \neq c_2$. The implications of this result for the quantum structure of black hole are to be explored.

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