Mass scales in string theory

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1 Preliminaries

In critical (ten) dimensions, any consistent superstring theory has two parameters: a mass (or length) scale M_s ($l_s = M_s^{-1}$), and a dimensionless string coupling λ_s given by the vacuum expectation value (VEV) of the dilaton field $e^{<\phi>} = \lambda_s$ [1, 2]:

$$D = 10: \qquad M_s = l_s^{-1} \qquad \lambda_s\,. \tag{1}$$

Upon compactification in $D = 4$ dimensions on a compact manifold of volume V, these parameters determine the four-dimensional (4d) Planck mass (or length) M_p ($l_p = M_p^{-1}$) and the dimensionless gauge coupling g at the string scale. For simplicity, in the following we drop all numerical factors from our formulae, while, when needed, we use the numerical values:

$$D = 4: \qquad M_p \simeq 1.2 \times 10^{19}\ \text{GeV} \qquad g \simeq 1/5\,. \tag{2}$$

Moreover, the weakly coupled condition implies that $\lambda_s << 1$. Our method in the following consists in expressing the 10d parameters (M_s, λ_s) in terms of the 4d ones and the compactification volume, in heterotic ($s = H$), type I ($s = I$) and type II ($s = II$) string theories, and then discuss the conditions on possible large volume or low string scale realizations, keeping the string coupling small.

An important point is that the compactification volume will always be chosen to be bigger than unity in string units, $V > l_s^6$. This can be done by a T-duality transformation which exchanges the role of the Kaluza-Klein (KK) momenta p with the string winding modes w. For instance, in the case of one compact dimension on a circle of radius R, they read:

$$p = \frac{m}{R} \qquad ; \qquad w = \frac{nR}{l_s^2}\,, \tag{3}$$

with integers m, n. T-duality inverts the compactification radius and rescales the string coupling:

$$R \to \frac{l_s^2}{R} \qquad \lambda_s \to \lambda_s \frac{l_s}{R}\,, \tag{4}$$

so that the lower-dimensional coupling $\lambda_s\sqrt{l_s/R}$ remains invariant. When R is smaller than the string scale, the winding modes become very light, while T-duality trades them as KK momenta in terms of the dual radius $\tilde{R} \equiv l_s^2/R$. The enhancement of the string coupling is then due to their multiplicity which diverges in the limit $R \to 0$ (or $\tilde{R} \to \infty$).

2 Heterotic string and motivations for large volume compactifications

In heterotic string, gauge and gravitational interactions appear at the same (tree) level of perturbation theory (spherical world-sheet topology), and the corresponding effective action is [1, 2]:

$$S = \int d^4x \frac{V}{\lambda_H^2}(l_H^{-8}\mathcal{R} + l_H^{-6}F^2)\,, \tag{5}$$

upon compactification in four dimensions. Here, for simplicity, we kept only the gravitational and gauge kinetic terms, in a self-explanatory notation. Identifying their respective coefficients with the 4d parameters $1/l_p^2$ and $1/g^2$, one obtains:

$$M_H = gM_p \qquad\qquad \lambda_H = g\frac{\sqrt{V}}{l_H^3}\,. \tag{6}$$

Using the values (2), one obtains that the heterotic string scale is near the Planck mass, $M_H \simeq 10^{18}$, while the string is weakly coupled when the internal volume is of order of the string scale, $V \sim l_s^6$. However, despite this fact, there are physical motivations which suggest that large volume compactifications, and thus strong coupling, may be relevant in physics [3]. These come from gauge coupling unification and supersymmetry breaking by compactification, which we discuss below.

2.1 Gauge coupling unification

It is a known fact that the three gauge couplings of the Standard Model, when extrapolated at high energies assuming the particle content of its $N = 1$ minimal supersymmetric extension (MSSM), they meet at an energy scale $M_{\rm GUT} \simeq 2 \times 10^{16}$ GeV. At the one-loop level, one has:

$$\frac{1}{g_a^2(\mu)} = \frac{1}{g^2} + \frac{b_a}{4\pi}\ln\frac{M_{\rm GUT}^2}{\mu^2}\,, \tag{7}$$

where μ is the energy scale and a denotes the 3 gauge group factors of the Standard Model $SU(3) \times SU(2) \times U(1)$. The value of $M_{\rm GUT}$ is very near the heterotic string scale, but it differs by roughly two orders of magnitude. If one takes seriously this discrepancy, a possible way to explain it is by introducing large compactification volume.

Consider for instance one large dimension of size R, so that $V \sim Rl_H^5$. Identifying $M_{\rm GUT}$ with the compactification scale R^{-1}, this requires $R \sim 100 l_H$. Alternatively, one can use string threshold corrections which grow linearly with R [4]. Assuming that they can account for the discrepancy, one needs roughly $R/l_H \sim \ln(M_H^2/M_{\rm GUT}^2) \sim 10$. As a result, the string coupling (6) equals $\lambda_H \sim 0.5 - 2$ which enters in the strongly coupled regime.

2.2 Supersymmetry breaking by compactification

In contrast to ordinary supergravity, where supersymmetry breaking can be introduced at an arbitrary scale, through for instance the gravitino, gaugini and other soft masses, in string theory this is not possible (perturbatively). The only way to break supersymmetry at a scale hierarchically smaller than the (heterotic) string scale is by introducing a large compactification radius whose size is set by the breaking scale. This has to be therefore of the order of a few TeV in order to protect the gauge hierarchy. An explicit proof exists for toroidal and fermionic constructions, although the result is believed to apply to all compactifications [5, 6]. This is one of the very few general predictions of perturbative (heterotic) string theory that leads to the spectacular prediction of the possible existence of extra dimensions accessible to future accelerators [3]. The main theoretical problem is though the strong coupling, as mentioned above.

The strong coupling problem can be understood from the effective field theory point of view from the fact that at energies higher than the compactification scale, the KK excitations of gauge bosons and other Standard Model particles will start being produced and contribute to various physical amplitudes. Their multiplicity turns very rapidly the logarithmic evolution of gauge couplings into a power dependence [7], invalidating the perturbative description, as expected in a higher dimensional non-renormalizable gauge theory. A possible way to avoid this problem is to impose conditions which prevent the power corrections to low-energy couplings [3]. For gauge couplings, this implies the vanishing of the corresponding β-functions, which is the case for instance when the KK modes are organized in multiplets of $N = 4$ supersymmetry, containing for every massive spin-1 excitation, 2 Dirac fermions and 6 scalars. Examples of such models are provided by orbifolds with no $N = 2$ sectors with respect to the large compact coordinate(s).

The simplest example of a one-dimensional orbifold is an interval of length πR, or equivalently S^1/Z_2 with Z_2 the coordinate inversion. The Hilbert space is composed of the untwisted sector, obtained by the Z_2-projection of the toroidal states (3), and of the twisted sector which is localized at the two end-points of the interval, fixed under the Z_2 transformations. This sector is chiral and can thus naturally contain quarks and leptons, while gauge fields propagate in the (5d) bulk.

Similar conditions should be imposed to Yukawa's and in principle to higher (non-renormalizable) effective couplings in order to ensure a soft ultraviolet (UV) behavior above the compactification scale. We now know that the problem of strong coupling can be addressed using string S-dualities which invert the string coupling and relate a strongly coupled theory with a weakly coupled one [2]. For instance, as we will discuss below, the strongly coupled heterotic theory with one large dimension is described by a weakly coupled type IIB theory with a tension at intermediate energies $(Rl_H)^{-1/2} \simeq 10^{11}$ GeV [8]. Furthermore, non-abelian gauge interactions emerge from tensionless strings [9] whose effective theory describes a higher-dimensional non-trivial infrared fixed point of the renormalization group [10]. This theory incorporates all conditions to low-energy couplings that guarantee a smooth UV behavior above the compactification scale. In particular, one recovers that KK modes of gauge bosons form $N = 4$ supermultiplets, while matter fields are localized in four dimensions. It is remarkable that the main features of these models were captured already in the context of the heterotic string despite its strong coupling [3].

In the case of two or more large dimensions, the strongly coupled heterotic string is described by a weakly coupled type IIA or type I/I$'$ theory [8]. Moreover, the tension of the dual string becomes of the order or even lower than the compactification scale. In fact, as it will become clear in the following, in the context of any string theory other than the heterotic, the simple relation (6) that fixes the string scale in terms of the Planck mass does not hold and therefore the string tension becomes an arbitrary parameter [11]. It can be anywhere below the Planck scale and as low as a few TeV [12]. The main advantage of having the string tension at the TeV, besides its obvious experimental interest, is that it offers an automatic solution to the problem of gauge hierarchy, alternative to low-energy supersymmetry or technicolor [13, 14, 15].

3 M-theory on S^1/Z_2"$\times$"Calabi-Yau

The strongly coupled $E_8 \times E_8$ heterotic string compactified on a Calabi-Yau manifold (CY) of volume V is described by the 11d M-theory compactified on an interval S^1/Z_2 of length πR_{11} times the same Calabi-Yau [16]. Gravity propagates in the 11d bulk, containing besides the metric and the gravitino a 3-form potential, while gauge interactions are confined on two 10d boundaries (9-branes) localized at the two end-points of the interval and containing one E_8 factor each. The corresponding effective action is

$$S_H = \int d^4x V (\frac{R_{11}}{l_M^9}\mathcal{R} + \frac{1}{l_M^6}F^2) \,. \tag{8}$$

It follows that

$$l_M = (g^2V)^{1/6} \qquad\qquad R_{11} = g^2 \frac{l_M^3}{l_P^2} \,. \tag{9}$$

The validity of the 11d supergravity regime is when $R_{11} > l_M$ and $V > l_M^6$ implying $g < 1$ by virtue of eq.(9). Comparison with the heterotic relations (6) yields:

$$l_M = l_H \lambda_H^{1/3} \qquad\qquad R_{11} = l_H \lambda_H \,, \tag{10}$$

which shows in particular that R_{11} is the string coupling in heterotic units. As a result, at strong coupling $\lambda_H > 1$ the M theory scale and the 11d radius are larger than the heterotic length: $R_{11} > l_M > l_H$.

Imposing the M-theory scale l_M^{-1} to be at 1 TeV, one finds from the relations (9) a value for the radius of the 11th dimension of the size of the solar system, $R_{11} \simeq 10^8$ kms, which is obviously excluded experimentally. On the other hand, imposing a value for $R_{11} \simeq 1$ mm which is the shortest length scale that gravity is tested experimentally, one finds a lower bound for the M-theory scale $l_M^{-1} \gtrsim 10^7$ GeV [17].

While the relations (9) seem to impose no theoretical constraint to l_M, there is however another condition to be imposed beyond the classical approximation [11]. This is because at the next order the factorized space $S^1/Z_2 \times$ CY is not any more solution of the 11d supergravity equations, which require the size of the Calabi-Yau manifold to depend on the 11th coordinate x_{11} along the interval. This can be seen for instance from the supersymmetry transformation of the 3-form potential (with field-strength $G^{(4)}$) which acquires non vanishing contributions from the 10d boundaries:

$$\delta G^{(4)} = l_M^6 \delta(x_{11}) \left(\mathrm{tr} F \wedge F - \frac{1}{2} \mathrm{tr} \mathcal{R} \wedge \mathcal{R} \right) + \left(x_{11} \leftrightarrow \pi R_{11} - x_{11}, F \leftrightarrow F' \right) \,. \tag{11}$$

As a result, the volume of CY varies linearly along the interval, to leading order:

$$V(x_{11}) = V(0) - x_{11} l_M^3 \int_{\mathrm{CY}} \omega \wedge \left(\mathrm{tr} F' \wedge F' - \mathrm{tr} F \wedge F \right) \,, \tag{12}$$

where $\omega \sim V^{1/3}$ is the Kähler form on the six-manifold CY.

It follows that there is an upper bound on R_{11}, otherwise the gauge coupling in one of the two walls blows up when the volume of CY shrinks to zero size. Choosing $V(0) \equiv V$ and imposing $V(\pi R) \geq 0$, eq.(12) yields $R_{11} \lesssim V^{2/3}/l_M^3$ and through the relations (9):

$$l_P \gtrsim g^{5/3} l_M = g^2 V^{1/6} \,. \tag{13}$$

This implies a lower bound for the M-theory scale $l_M^{-1} \gtrsim g^{5/3} M_P$, or equivalently for the unification scale $M_{\mathrm{GUT}} \equiv V^{-1/6} \gtrsim g^2 M_P$. Taking into account the numerical factors, on finds for the lower bound the right order of magnitude $M_{\mathrm{GUT}} \sim 10^{16}$ GeV, providing a solution to the perturbative discrepancy between the unification and heterotic string scales, discussed in section 2.1 [11]. Note that this bound does not hold in the case of symmetric embedding, where one has $\mathrm{tr}F' \wedge F' - \mathrm{tr}F \wedge F = 0$ and thus the correction in eq.(12) vanishes.

4 Type I/I′ string theory and D-branes

In ten dimensions, the strongly coupled $SO(32)$ heterotic string is described by the type I string, or upon T-dualities to type I′ [18, 2].[1] Type I/I′ is a theory of closed and open unoriented strings. Closed strings describe gravity, while gauge interactions are described by open strings whose ends are confined to propagate on D-branes. It follows that the 6 internal compact dimensions are separated into longitudinal (parallel) and transverse to the D-branes. Assuming that the Standard Model is localized on a p-brane with $p \geq 3$, there are $p-3$ longitudinal and $9-p$ transverse compact dimensions. In contrast to the heterotic string, gauge and gravitational interactions appear at different order in perturbation theory and the corresponding effective action reads [1, 2]:

$$S_I = \int d^{10}x \frac{1}{\lambda_I^2 l_I^8} \mathcal{R} + \int d^{p+1}x \frac{1}{\lambda_I l_I^{p-3}} F^2 \,, \tag{14}$$

where the $1/\lambda_I$ factor in the gauge kinetic terms corresponds to the disk diagram.

Upon compactification in four dimensions, the Planck length and gauge couplings are given to leading order by

$$\frac{1}{l_P^2} = \frac{V_{||}V_{\perp}}{\lambda_I^2 l_I^8} \,, \qquad \frac{1}{g^2} = \frac{V_{||}}{\lambda_I l_I^{p-3}} \,, \tag{15}$$

where $V_{||}$ ($V_{\perp}$) denotes the compactification volume longitudinal (transverse) to the p-brane. From the second relation above, it follows that the requirement of weak coupling $\lambda_I < 1$ implies that the size of the longitudinal space must be of order of the string length ($V_{||} \sim l_I^{p-3}$), while the transverse volume $V_{\perp}$ remains unrestricted. One thus has

$$M_P^2 = \frac{1}{g^4 v_{||}} M_I^{2+n} R_{\perp}^n \,, \qquad \lambda_I = g^2 v_{||} \,, \tag{16}$$

to be compared with the heterotic relations (6). Here, $v_{||} \gtrsim 1$ is the longitudinal volume in string units, and we assumed an isotropic transverse space of $n = 9-p$ compact dimensions of radius $R_{\perp}$.

[1] In lower dimensions, type I′ theories can also describe a class of M-theory compactifications.

4.1 Low-scale strings and extra-large transverse dimensions

From the relations (16), it follows that the type I/I′ string scale can be made hierarchically smaller than the Planck mass at the expense of introducing extra large transverse dimensions that interact only gravitationally, while keeping the string coupling weak [14, 19]. The weakness of 4d gravity M_I/M_P is then attributed to the largeness of the transverse space $R_\perp/l_I$. An important property of these models is that gravity becomes strong at the string scale, although the string coupling remains weak. In fact, the first relation of eq.(16) can be understood as a consequence of the $(4+n)$-dimensional Gauss law for gravity, with

$$G_N^{(4+n)} = g^4 l_I^{2+n} v_{||} \tag{17}$$

the Newton's constant in $4+n$ dimensions.

To be more explicit, taking the type I string scale M_I to be at 1 TeV, one finds a size for the transverse dimensions $R_\perp$ varying from 10^8 km, .1 mm (10^{-3} eV), down to .1 fermi (10 MeV) for $n = 1, 2$, or 6 large dimensions, respectively. The case $n = 1$ corresponds to M-theory and is obviously experimentally excluded. On the other hand, all other possibilities are consistent with observations, although barely in the case $n = 2$ [20]. In particular, sub-millimeter transverse directions are compatible with the present constraints from short-distance gravity measurements which tested Newton's law up to the cm [22]. The strongest bounds come from astrophysics and cosmology and concern mainly the case $n = 2$ [20, 21]. In fact, graviton emission during supernovae cooling restricts the 6d Planck scale to be larger than about 50 TeV, implying $M_I \gtrsim 7$ TeV, while the graviton decay contribution to the cosmic diffuse gamma radiation gives even stronger bounds of about 110 TeV and 15 TeV for the two scales, respectively.

If our brane world is supersymmetric, which protects the hierarchy in the usual way, the string scale is an arbitrary parameter and can be at higher energies, in principle up to the Planck scale. However, in the context of type I/I′ theory, the string scale should not be higher than intermediate energies $M_I \lesssim 10^{11}$ GeV, due to the generic existence of other branes with non supersymmetric world volumes [23]. Indeed, in this case, our world would feel the effects of supersymmetry breaking through gravitationally suppressed interactions of order M_I^2/M_P, that should be less than a TeV. In this context, the value $M_I \sim 10^{11}$ GeV could be favored, since it would coincide with the scale of supersymmetry breaking in a hidden sector, without need of non-perturbative effects such as gaugino condensation. Moreover, the gauge hierarchy would be minimized, since one needs to introduce transverse dimensions with size just two orders of magnitude larger than l_I (in the case of $n = 6$) to account for the ratio $M_I/M_P \simeq 10^{-8}$, according to eq.(16). Note also that the weak scale $M_W \sim M_I^2/M_P$ becomes T-dual to the Planck scale.

4.2 Relation type I/I′ – heterotic

We will now show that the above type I/I′ models describe particular strongly coupled heterotic vacua with large dimensions [24, 8]. More precisely, we will consider the heterotic string compactified on a 6d manifold with k large dimensions of radius $R \gg l_H$ and $6-k$ string-size dimensions and show that for $k \geq 4$ it has a perturbative type I′ description [8].

In ten dimensions, heterotic and type I theories are related by an S-duality:

$$\lambda_I = \frac{1}{\lambda_H} \qquad\qquad l_I = \lambda_H^{1/2} l_H \,, \tag{18}$$

which can be obtained for instance by comparing eqs.(6) with eqs.(15) in the case of 9-branes ($p = 9$, $V_\perp = 1$, $V_{||} = V$). Using from eq.(6) that $\lambda_H \sim (R/l_H)^{k/2}$, one finds

$$\lambda_I \sim \left(\frac{R}{l_H}\right)^{-k/2} \qquad\qquad l_I \sim \left(\frac{R}{l_H}\right)^{k/4} l_H \,. \tag{19}$$

It follows that the type I scale M_I appears as a non-perturbative threshold in the heterotic string at energies much lower than M_H [17]. For $k < 4$, it appears at intermediate energies $R^{-1} < M_I < M_H$, for $k = 4$, it becomes of the order of the compactification scale $M_I \sim R^{-1}$, while for $k > 4$, it appears at low energies $M_I < R^{-1}$ [24]. Moreover, since $\lambda_I \ll 1$, one would naively think that weakly coupled type I theory could describe the heterotic string with any number $k \geq 1$ of large dimensions. However, this is not true because there are always some dimensions smaller than the type I size ($6-k$ for $k < 4$ and 6 for $k > 4$) and one has to perform T-dualities (4) in order to account for the multiplicity of light winding modes in the closed string sector, as we discussed in section 1.1. Note that open strings have no winding modes along longitudinal dimensions and no KK momenta along transverse directions. The T-dualities have two effects: (i) they transform the corresponding longitudinal directions to transverse ones by exchanging KK momenta with winding modes, and (ii) they increase the string coupling according to eq.(4) and therefore it is not clear that type I′ theory remains weakly coupled.

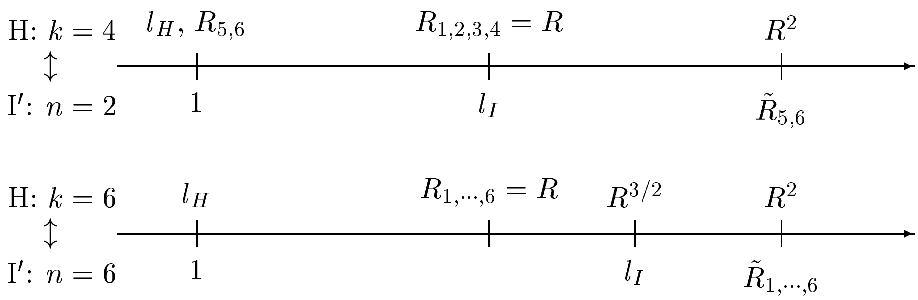
Indeed for $k < 4$, after performing $6-k$ T-dualities on the heterotic size dimensions, with respect to the type I scale, one obtains a type I′ theory with D$(3+k)$-branes but strong coupling:

$$l_H \to \tilde{l}_H = \frac{l_I^2}{l_H} \sim \left(\frac{R}{l_H}\right)^{k/2} l_H \qquad \lambda_I \to \tilde{\lambda}_I = \lambda_I \left(\frac{l_I}{l_H}\right)^{6-k} \sim \left(\frac{R}{l_H}\right)^{k(4-k)/4} \gg 1 \,. \tag{20}$$

For $k \geq 4$, we must perform T-dualities in all six internal directions.[2] As a result, the type I′ theory has D3-branes with $6-k$ transverse dimensions of radius $\tilde{l}_H$ given in eq.(20) and k transverse dimensions of radius $\tilde{R} = l_I^2/R \sim (R/l_H)^{k/2-1}$, while its coupling remains weak (of order unity):

$$\lambda_I \to \tilde{\lambda}_I = \lambda_I \left(\frac{l_I}{l_H}\right)^{6-k} \left(\frac{l_I}{R}\right)^k \sim 1\,. \tag{21}$$

It follows that the type I′ theory with n extra-large transverse dimensions offers a weakly coupled dual description for the heterotic string with $k = 4,5,6$ large dimensions [8]. $k=4$ is described by $n=2$, $k=6$ (for $SO(32)$ gauge group) is described by $n=6$, while for $n=5$ one finds a type I′ model with 5 large transverse dimensions and one extra-large. The case $k=4$ is particularly interesting: the heterotic string with 4 large dimensions, say at a TeV, is described by a perturbative type I′ theory with the string scale at the TeV and 2 transverse dimensions of millimeter size that are T-dual to the 2 heterotic string size coordinates. This is depicted in the following diagram, together with the case $k=6$, where we use heterotic length units $l_H = 1$:



5 Type II theories

Upon compactification to 6 dimensions or lower, the heterotic string admits another dual description in terms of type II (IIA or IIB) string theory [25, 2]. Since in 10 dimensions type II theories have $N=2$ supersymmetry,[3] in contrast to the heterotic string which has $N=1$, the compactification manifolds on the two sides should be different, so that the resulting theories in lower dimensions have the same number of supersymmetries. The first example arises in 6 dimensions, where the $E_8 \times E_8$ heterotic string compactified on the four-torus T^4 is S-dual to type IIA compactified on the

[2] The case $k=4$ can be treated in the same way, since there are 4 dimensions that have type I string size and remain inert under T-duality.

[3] Type IIA (IIB) has two 10d supercharges of opposite (same) chirality.

$K3$ manifold that has $SU(2)$ holonomy and breaks half of the supersymmetries. In lower dimensions, type IIA and type IIB are related by T-duality (or mirror symmetry).

Here, for simplicity, we shall restrict ourselves to 4d compactifications of type II on $K3 \times T^2$, yielding $N = 4$ supersymmetry, or more generally on Calabi-Yau manifolds that are $K3$ fibrations, yielding $N = 2$ supersymmetry. They are obtained by replacing T^2 by a "base" two-sphere over which $K3$ varies, and they are dual to corresponding heterotic compactifications on $K3 \times T^2$. More interesting phenomenological models with $N = 1$ supersymmetry can be obtained by a freely acting orbifold on the two sides, although the most general $N = 1$ compactification would require F-theory on Calabi-Yau fourfolds, which is poorly understood at present [26].

In contrast to heterotic and type I strings, non-abelian gauge symmetries in type II models arise non-perturbatively (even though at arbitrarily weak coupling) in singular compactifications, where the massless gauge bosons are provided by D2-branes in type IIA (D3-branes in IIB) wrapped around non-trivial vanishing 2-cycles (3-cycles). The resulting gauge interactions are localized on $K3$ (similar to a Neveu-Schwarz five-brane), while matter multiplets would arise from further singularities, localized completely on the 6d internal space [27].

5.1 Low-scale IIA strings and tiny coupling

In type IIA non-abelian gauge symmetries arise in six dimensions from D2-branes wrapped around non-trivial vanishing 2-cycles of a singular $K3$.[4] It follows that gauge kinetic terms are independent of the string coupling λ_{IIA} and the corresponding effective action is [2]:

$$S_{IIA} = \int d^{10}x \frac{1}{\lambda_{IIA}^2 l_{IIA}^8} \mathcal{R} + \int d^6 x \frac{1}{l_{IIA}^2} F^2 \,, \tag{24}$$

which should be compared with (8) of heterotic and (14) of type I/I′. As a result, upon compactification in four dimensions, for instance on a two-torus T^2, the gauge couplings are determined by its size V_{T^2}, while the Planck mass is controlled by the 6d string coupling λ_{6IIA}:

$$\frac{1}{g^2} = \frac{V_{T^2}}{l_{IIA}^2} \qquad\qquad \frac{1}{l_P^2} = \frac{V_{T_2}}{\lambda_{6IIA}^2 l_{IIA}^4} = \frac{1}{\lambda_{6IIA}^2} \frac{1}{g^2 l_{IIA}^2} \,. \tag{25}$$

The area of T^2 should therefore be of order l_{IIA}^2, while the string scale is expressed by

$$M_{IIA} = g\lambda_{6IIA} M_P = g\lambda_{IIA} M_P \frac{l_{IIA}^2}{\sqrt{V_{K3}}} \,, \tag{26}$$

[4]Note though that the abelian Cartan subgroup is already in the perturbative spectrum of the Ramond-Ramond sector.

with V_{K3} the volume of $K3$. Thus, in contrast to the type I relation (16) where only the volume of the internal six-manifold appears, we now have the freedom to use both the string coupling and the $K3$ volume to separate the Planck mass from a string scale, say, at 1 TeV [12, 8]. In particular, we can choose a string-size internal manifold, and have an ultra-weak coupling $\lambda_{IIA} = 10^{-14}$ to account for the hierarchy between the electroweak and the Planck scales [8]. As a result, despite the fact that the string scale is so low, gravity remains weak up to the Planck scale and string interactions are suppressed by the tiny string coupling, or equivalently by the 4d Planck mass. Thus, there are no observable effects in particle accelerators, other than the production of KK excitations along the two TeV dimensions of T^2 with gauge interactions. Furthermore, the excitations of gauge multiplets have $N = 4$ supersymmetry, even when $K3 \times T^2$ is replaced by a Calabi-Yau threefold which is a $K3$ fibration, while matter multiplets are localized on the base (replacing the T^2) and have no KK excitations, as the twisted states of heterotic orbifolds.

Above, we discussed the simplest case of type II compactifications with string scale at the TeV and all internal radii having the string size. In principle, one can allow some of the $K3$ (transverse) directions to be large, keeping the string scale low. From eq.(26), it follows that the string coupling λ_{IIA} increases making gravity strong at distances $l_P\sqrt{V_{K3}}/l_{IIA}^2$ larger than the Planck length. In particular, it becomes strong at the string scale (TeV), when λ_{IIA} is of order unity. This corresponds to $V_{K3}/l_{IIA}^4 \sim 10^{28}$, implying a fermi size for the four $K3$ compact dimensions.

5.2 Large dimensions in type IIB

Above we assumed that both directions of T^2 have the string size, so that its volume is of order l_{IIA}^2, as implied by eq.(25). However, one could choose one direction much bigger than the string scale and the other much smaller. For instance, in the case of a rectangular torus of radii r and R, $V_{T^2} = rR \sim l_{IIA}^2$ with $r \gg l_{IIA} \gg R$. This can be treated by performing a T-duality (4) along R to type IIB: $R \to l_{IIA}^2/R$ and $\lambda_{IIA} \to \lambda_{IIB} = \lambda_{IIA} l_{IIA}/R$ with $l_{IIA} = l_{IIB}$. One thus obtains:

$$\frac{1}{g^2} = \frac{r}{R} \qquad\qquad \frac{1}{l_P^2} = \frac{V_{T_2}}{\lambda_{6IIB}^2 l_{IIB}^4} = \frac{R^2}{\lambda_{6IIB}^2}\frac{1}{g^2 l_{IIB}^4}\,. \tag{27}$$

which shows that the gauge couplings are now determined by the ratio of the two radii, or in general by the shape of T^2, while the Planck mass is controlled by its size, as well as by the 6d type IIB string coupling. The string scale can thus be expressed as [8]:

$$M_{IIB}^2 = g\lambda_{6IIB}\frac{M_P}{R}\,. \tag{28}$$

Comparing these relations with eqs.(25) and (26), it is clear that the situation in type IIB is the same as in type IIA, unless the size of T^2 is much larger than the string length, $R \gg l_{IIB}$. Since T^2 is felt by gauge interactions, its size cannot be larger than $\mathcal{O}(\text{TeV}^{-1})$ implying that the type IIB string scale should be much larger than TeV. From eq.(28) and $\lambda_{6IIB} < 1$, one finds $M_{IIB} \lesssim \sqrt{M_P/R}$, so that the largest value for the string tension, when $R \sim 1\text{TeV}^{-1}$, is an intermediate scale $\sim 10^{11}$ GeV when the string coupling is of order unity.

As we will show below, this is precisely the case that describes the heterotic string with one TeV dimension, which we discussed is section 2. It is the only example of longitudinal dimensions larger than the string length in a weakly coupled theory. In the energy range between the KK scale $1/R$ and the type IIB string scale, one has an effective 6d theory without gravity at a non-trivial superconformal fixed point described by a tensionless string [9, 10]. This is because in type IIB gauge symmetries still arise non-perturbatively from vanishing 2-cycles of $K3$, but take the form of tensionless strings in 6 dimensions, given by D3-branes wrapped on the vanishing cycles. Only after further compactification does this theory reduce to a standard gauge theory, whose coupling involves the shape rather than the volume of the two-torus, as described above. Since the type IIB coupling is of order unity, gravity becomes strong at the type IIB string scale and the main experimental signals at TeV energies are similar to those of type IIA models with tiny string coupling.

Similar constructions can be also realized in the context of the heterotic string when the standard model is embedded in non-perturbative gauge group arising from small instantons. In this case, the heterotic string scale can also be lowered in the TeV region [29].

5.3 Relation type II – heterotic

We will now show that the above low-scale type II models describe some strongly coupled heterotic vacua and, in particular, the cases with $k = 1, 2, 3$ large dimensions that have not a perturbative description in terms of type I′ theory [8]. As we described in the beginning of section 5, in 6 dimensions the heterotic $E_8 \times E_8$ superstring compactified on T^4 is S-dual to type IIA compactified on $K3$:

$$\lambda_{6IIA} = \frac{1}{\lambda_{6H}} \qquad\qquad l_{IIA} = \lambda_{6H} l_H \,, \tag{29}$$

which can be obtained, for instance, by comparing eqs.(25) with (6), using $\lambda_{6H} = \lambda_H l_H^2/\sqrt{V_{T^4}}$. However, in contrast to the case of heterotic – type I/I′ duality, the compactification manifolds on the two sides are not the same and a more detailed analysis is needed to study the precise mapping of T^4 to $K3$, besides the general relations (29).

This can be done easily in the context of M-theory compactified on the product space of a line interval of length πR_I with four circles of radii $R_1, \cdots, R_4$ [28, 8]: $S^1/Z_2(R_I) \times S^1(R_1) \times T^3(R_2, R_3, R_4)$. One can then interpret this compactification in various ways by choosing appropriately one of the radii as that of the eleventh dimension. Considering for instance $R_I = R_{11}$, one finds the (strongly coupled) heterotic string compactified on $T^4(R_1, \cdots, R_4)$, while choosing $R_1 = R_{11}$, one finds type IIA compactified on $K3$ of "squashed" shape $S^1/Z_2(R_I) \times T^3(\tilde{R}_2, \tilde{R}_3, \tilde{R}_4)$, where the 3 radii $\tilde{R}_i$ will be determined below. In each of the two cases, one can use the duality relations (10) to obtain

$$R_I = \lambda_H l_H = \lambda_{6H} \frac{V_{T^4}^{1/2}}{l_H} \qquad\qquad R_1 = \lambda_{IIA} l_{IIA} = \lambda_{6IIA} \frac{V_{K3}^{1/2}}{l_{IIA}}, \tag{30}$$

while using eqs.(29) one finds a mapping between the volume of the internal 4-manifold of one theory and a preferred radius of the other, in corresponding string units:

$$\frac{R_I}{l_{IIA}} = \frac{V_{T^4}^{1/2}}{l_H^2} \qquad\qquad \frac{R_1}{l_H} = \frac{V_{K3}^{1/2}}{l_{IIA}}. \tag{31}$$

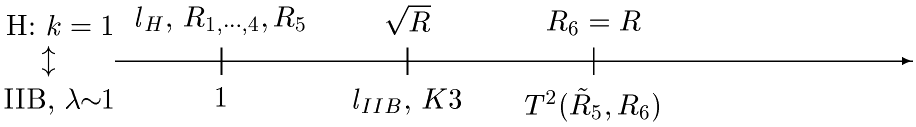
The correspondence among the remaining 3 radii can be found, for instance, by noticing that the S-duality transformations leave invariant the shape of T^3:

$$\frac{R_i}{R_j} = \frac{\tilde{R}_i}{\tilde{R}_j} \qquad\qquad i, j = 2, 3, 4, \tag{32}$$

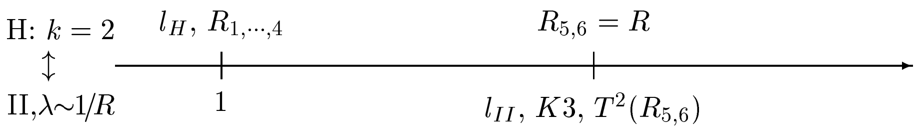
which yields $\tilde{R}_i = l_M^3/(R_j R_k)$ with $i \neq j \neq k \neq i$ and $l_M^3 = \lambda_H l_H^3$. This relation, together with eq.(31), gives the precise mapping between T^4 and $K3$, which completes the S-duality transformations (29). We recall that on the type II side, the four $K3$ directions corresponding to R_I and $\tilde{R}_i$ are transverse to the 5-brane where gauge interactions are localized.

Using the above results, one can now study the possible perturbative type II descriptions of 4d heterotic compactifications on $T^4(R_1, \cdots, R_4) \times T^2(R_5, R_6)$ with a certain number k of large dimensions of common size R and string coupling $\lambda_H \sim (R/l_H)^{k/2} \gg 1$. From eq.(29), the type II string tension appears as a non-perturbative threshold at energies of the order of the T^2 compactification scale, $l_{II} \sim \sqrt{R_5 R_6}$. Following the steps we used in the context of heterotic – type I duality, after T-dualizing the radii which are smaller than the string size, one can easily show that the T^2 directions must be among the k large dimensions in order to obtain a perturbative type II description.

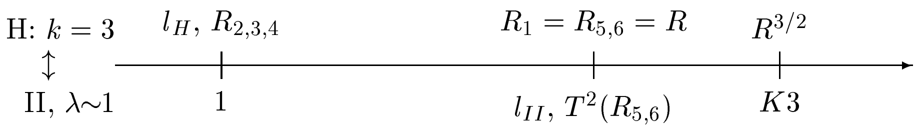
It follows that for $k = 1$ with, say, $R_6 \sim R \gg l_H$, the type II threshold appears at an intermediate scale $l_{II} \sim \sqrt{R l_H}$, together with all 4 directions of $K3$, while the second, heterotic size, direction of T^2 is T-dual (with respect to l_{II}) to R: $\tilde{R}_5 \equiv l_{II}^2/l_H \sim R$. Thus, one finds a type IIB description with two large longitudinal dimensions along the T^2 and string coupling of order unity, which is the example discussed in sections 2.2 and 5.2.



For $k \geq 2$, the type II scale becomes of the order of the compactification scale, $l_{II} \sim R$. For $k = 2$, all directions of $K3 \times T^2$ have the type II size, while the type II string coupling is infinitesimally small, $\lambda_{II} \sim l_H/R$, which is the example discussed in section 5.1.



For $k = 3$, $l_{II} \sim R_{5,6} \sim R$, while the four (transverse) directions of $K3$ are extra large: $R_I \sim \tilde{R}_i \sim R^{3/2}/l_H$.



For $k = 4$, the type II dual theory provides a perturbative description alternative to the type I$'$ with $n = 2$ extra large transverse dimensions. For $k = 5$, there is no perturbative type II description, while for $k = 6$, the heterotic $E_8 \times E_8$ theory is described by a weakly coupled type IIA with all scales of order R apart one $K3$ direction (R_I) which is extra large. This is equivalent to type I$'$ with $n = 1$ extra large transverse dimension. Note that this case was not found from heterotic $SO(32)$ – type I duality since the heterotic $SO(32)$ string is equivalent to $E_8 \times E_8$ only up to T-duality, which cannot be performed when $k = 6$ and there are no leftover dimensions of heterotic size.

6 Theoretical implications

We will now focus on some theoretical implications of the low scale string scenario. Unless explicitly stated otherwise, we will restrict ourselves to the context of type I strings.

6.1 U.V./ I.R. correspondence

In addition to the open strings describing the gauge degrees of freedom, consistency of string theory requires the presence of closed strings associated with gravitons and different kind of moduli fields m_a.

There are two types of extended objects: D-branes and orientifolds. The former are hypersurfaces on which open strings end while the latter are hypersurfaces located at fixed points when acting simultaneously with a Z_2 parity on the transverse space and world-sheet coordinates.

Closed strings can be emitted by D-branes and orientifolds, the lowest order diagrams being described by a cylindric topology. In this way D-branes and orientifolds appear as to lowest order classical point-like sources in the transverse space. For weak type-I string coupling this can be described by a lagrangian of the form

$$\int d^n x_\perp \left[\frac{1}{g_s^2}(\partial_{x_\perp} m_a)^2 + \frac{1}{g_s}\sum_s f_s(m_a)\delta(x_\perp - x_{\perp s})\right], \tag{36}$$

where $x_{\perp s}$ is the location of the source s (D-branes and orientifolds) while $f_s(m_a)$ encodes the coupling of this source to the moduli m_a. As a result while m_a have constant values in the four-dimensional space, their expectation values will generically vary as a function of the transverse coordinates $x_\perp$ of the n directions with size $\sim R_\perp$ large compared to the string length l_s.

Solving the classical equation of motion for m_a in (36) leads to contributions to the parameters (couplings) on the brane of the low energy effective action given by a sum of Green's functions of the form [15]:

$$\frac{1}{V_\perp}\sum_{|p_\perp|<M_s}\frac{1}{p_\perp^2}\,F(\vec{p}_\perp), \tag{37}$$

where $V_\perp = {R_\perp}^{d_\perp}$ is the volume of the transverse space, $\vec{p}_\perp = (m_1/R_\perp \cdots m_{d_\perp}/R_\perp)$ is the transverse momentum exchanged by the massless closed string, $F(\vec{p}_\perp)$ are the Fourier-transformed to momentum space of derivatives of $f_s(m_a)$. An explicit expression can be given in the simple case of toroidal compactification with vanishing antisymmetric tensor, where the global tadpole cancellation fixes the number of D-branes to be 32:

$$F(\vec{p}_\perp) \sim \left(32\prod_{i=1}^{d_\perp}\frac{1+(-)^{m_i}}{2} - 2\sum_{a=1}^{16}\cos(\vec{p}_\perp \vec{x}_a)\right), \tag{38}$$

where $\vec{p}_\perp = (m_1/R_\perp \cdots m_{d_\perp}/R_\perp)$, the orientifolds are located at the corners of the cell $[0, \pi R_\perp]^{d_\perp}$ and are responsible for the first term in (38), and $\pm\vec{x}_a$ are the transverse positions of the 32 D-branes (corresponding to Wilson lines of the T-dual picture) responsible of the second term.

In a compact space where flux lines can not escape to infinity, the Gauss-law implies that the total charge, thus global tadpoles, should vanish $F(0) = 0$ while local tadpoles may not vanish $F(\vec{p}_\perp) \neq 0$ for $\vec{p} \neq 0$. In that case, obtained for generic positions of the D-branes, the tadpole contribution (37) leads to the following behavior in the large radius limit for the moduli m_a:

$$m_a(x_{\perp s}) \sim \begin{cases} O(R_\perp M_s) & \text{for} \quad d_\perp = 1 \\ O(\ln R_\perp M_s) & \text{for} \quad d_\perp = 2 \\ O(1) & \text{for} \quad d_\perp > 2 \end{cases} , \tag{39}$$

which is dictated by the large-distance behavior of the two-point Green function in the $d_\perp$-dimensional transverse space.

There are some important implications of these results:

- The tree-level exchange diagram of a closed string can also be seen as one-loop exchange of open strings. While from the former point of view, a long cylinder represents an infrared limit where one computes the effect of exchanging light closed strings at long distances, in the second point of view the same diagram is conformally mapped to an annulus describing the one-loop running in the ultraviolet limit of very heavy open strings stretching between the two boundaries of the cylinder. Thus, from the brane gauge theory point of view, there are ultraviolet effects that are not cut-off by the string scale M_s but instead by the winding mode scale $R_\perp M_s^2$.

- In the case of one large dimension $d_\perp = 1$, the corrections are linear in $R_\perp$. Such correction appears for instance for the dilaton field which sits in front of gauge kinetic terms, that drive the theory rapidly to a strong coupling singularity and, thus, forbid the size of the transverse space to become much larger than the string length. It is possible to avoid such large corrections if the tadpoles cancel locally. This happens when D-branes are equally distributed at the two fixed points of the orientifold.

- The case $d_\perp = 2$ is particularly attractive because it allows the effective couplings of the brane theory to depend logarithmically on the size of the transverse space, or equivalently on M_P, exactly as in the case of softly broken supersymmetry at M_s. Both higher derivative and higher string loop corrections to the bulk supergravity lagrangian are expected to be small for slowly (logarithmically) varying moduli. The *classical* equations of motion of the effective 2d supergravity in the transverse space are analogous to the renormalization group equations used to resum large corrections to the effective field theory parameters with appropriate boundary conditions.

It turns out that low-scale type II theories with infinitesimal string coupling share many common properties with type I$'$ when $d_\perp = 2$ [8]. In fact, the limit of vanishing coupling does not exist due to subtleties related to the singular character of the compactification manifold and to the non perturbative origin of gauge symmetries. In general, there are corrections depending logarithmically on the string coupling, similarly to the case of type I$'$ strings with 2 transverse dimensions.

6.2 Unification

One of the main success of low-energy supersymmetry is that the three gauge couplings of the Standard Model, when extrapolated at high energies assuming the particle content of its $N = 1$ minimal supersymmetric extension (MSSM), meet at an energy scale $M_{\rm GUT} \simeq 2 \times 10^{16}$ GeV. This running is described at the the one-loop level by:

$$\frac{1}{g_a^2(\mu)} = \frac{1}{g^2} + \frac{b_a}{4\pi} \ln \frac{M_{\rm GUT}^2}{\mu^2}\,, \tag{40}$$

where μ is the energy scale and a denotes the 3 gauge group factors of the Standard Model $SU(3) \times SU(2) \times U(1)$. Note that even in the absence of any GUT group, if one requires keeping unification of all gauge couplings then the string relations we discussed in section 4 suggest that the gauge theories arise from the same kind of branes.

Decreasing the string scale below energies of order M_{GUT} is expected to cut-off the running of the couplings before they meet and thus spoils the unification. Is there a way to reconcile the apparent unification with a low string scale?

One possibility is to use power-law running that may accelerate unification in an energy region where the theory becomes higher dimensional [30]. Within the effective field theory, the summation over the KK modes above the compactification scale and below some energy scale $E \gg R^{-1}$ yields:

$$\frac{1}{g_a^2(E)} = \frac{1}{g_a^2(R^{-1})} - \frac{b_a^{SM}}{2\pi} \ln(ER) - \frac{b_a^{KK}}{2\pi} \left\{2\,(ER - 1) - \ln(ER)\right\}\,, \tag{41}$$

where we considered one extra (longitudinal) dimension. The first logarithmic term corresponds to the usual 4d running controlled by the Standard Model beta-functions b_a^{SM}, while the next term is the contribution of the KK tower dominated by the power-like dependence (ER) associated to the effective multiplicity of KK modes and controlled by the corresponding beta-functions b_a^{KK}.

Supersymmetric theories in higher dimensions have at least$N = 2$ extended supersymmetry thus the KK excitations form supermultiplets of

$N=2$. There are two kinds of such supermultiplets, the vector multiplets containing spin-1 field, a Dirac fermion and 2 real scalars in the adjoint representation and hypermultiplets containing an $N=1$ chiral multiplet and its mirror. As the gauge degrees of freedom are to be identified with bulk fields, their KK excitations will be part of $N=2$ vector multiplets. The higgs and matter fields, quarks and leptons, can on the other hand be chosen to be either localized without KK excitations or instead identified with bulk states with KK excitations forming $N=2$ hypermultiplets representations. Analysis of unification with the corresponding coefficients b_a^{KK} has been performed in [31].

There are two remarks to be made on this approach: (i) the result is very sensitive (power-like) to the initial conditions and thus to string threshold corrections, in contrast to the usual unification based on logarithmic evolution, (ii) only the case of one extra-dimension appears to lead to power-like corrections in type I models.

In fact the one-loop corrected gauge couplings in $N=1$ orientifolds are given by the following expression [34]:

$$\frac{1}{g_a^2(\mu)} = \frac{1}{g^2} + s_a m + \frac{b_a}{4\pi} \ln \frac{M_I^2}{\mu^2} - \sum_{i=1}^{3} \frac{b_{a,i}^{N=2}}{4\pi} \left\{ \ln T_i + f(U_i) \right\} , \qquad (42)$$

where the first two terms in the r.h.s. correspond to the tree-level (disk) contribution and the remaining ones are the one-loop (genus-1) corrections. Here, we assumed that all gauge group factors correspond to the same type of D-branes, so that gauge couplings are the same to lowest order (given by g). m denotes a combination of the twisted moduli, whose VEVs blow-up the orbifold singularities and allow the transition to smooth (Calabi-Yau) manifolds. However, in all known examples, these VEVs are fixed to $m=0$ from the vanishing of the D-terms of anomalous $U(1)$'s.

As expected, the one-loop corrections contain an infrared divergence, regulated by the low-energy scale μ, that produces the usual 4d running controlled by the $N=1$ beta-functions b_a. The last sum displays the string threshold corrections that receive contributions only from $N=2$ sectors, controlled by the corresponding $N=2$ beta-functions $b_{a,i}^{N=2}$. They depend on the geometric moduli T_i and U_i, parameterizing the size and complex structure of the three internal compactification planes. In the simplest case of a rectangular torus of radii R_1 and R_2, $T = R_1 R_2 / l_s^2$ and $U = R_1/R_2$. The function $f(U) = \ln\left(\mathrm{Re}U |\eta(iU)|^4\right)$ with η the Dedekind-eta function; for large U, $f(U)$ grows linearly with U. Thus, from expression (42), it follows that when $R_1 \sim R_2$, there are logarithmic corrections (as explained for transverse directions to the brane in the previous subsection) $\sim \ln(R_1/l_s)$, while when $R_1 > R_2$, the corrections grow linearly as R_1/R_2. Note that in both cases, the corrections are proportional to the $N=2$

β-functions and there no power law corrections in the case of more than one large compact dimensions.

Obviously, unification based on logarithmic evolution requires the two (transverse) radii to be much larger than the string length, while power-low unification can happen either when there is one longitudinal dimension a bit larger than the string scale ($R_1/R_2 \sim R_{\|}/l_s$ keeping $g_s < 1$), or when one transverse direction is bigger than the rest of the bulk.

The most advantageous possibility is to obtain large logarithmic thresholds depending on two large dimensions transverse to the brane ($d_\perp = 2$). One hopes that such logarithmic corrections may restore the "old" unification picture with a GUT scale given by the winding scale, which for millimeter-size dimensions has the correct order of magnitude [32, 15, 33]. In this way, the running due to a large desert in energies is replaced by an effective running due to a "large desert" in transverse distances from our world-brane. However, the logarithmic contributions are model dependent [34] and at present there is no compelling explicit realization of this idea.

6.3 Supersymmetry breaking and scales hierarchy

When decreasing the string scale, the question of hierarchy of scales i.e. of why the Planck mass is much bigger than the weak scale, is translated into the question of why there are transverse dimensions much larger than the string scale, or why the string coupling is very small. For instance for a string scale in the TeV range, From eq.(16) in type I/I′ strings, the required hierarchy $R_\perp/l_I$ varies from 10^{15} to 10^5, when the number of extra dimensions in the bulk varies from $n = 2$ to $n = 6$, respectively, while in type II strings with no large dimensions, the required value of the coupling λ_{II} is 10^{-14}.

There are two issues that one needs to address:

- We have seen in section 6.1 that although the string scale is very low, there might be large quantum corrections that arise, depending on the size of the large dimensions transverse to the brane. This is as if the UV cutoff of the effective field theory on the brane is not the string scale but the winding scale $R_\perp M_I^2$, dual to the large transverse dimensions and which can be much larger than the string scale. In particular such correction could spoil the nullification of gauge hierarchy that remain the main theoretical motivation of TeV scale strings.

- Another important issue is to understand the dynamical question on the origin of the hierarchy.

TeV scale strings offer a solution to the technical (at least) aspect of gauge hierarchy without the need of supersymmetry, provided there is no effective propagation of bulk fields in a single transverse dimension, or else closed string tadpoles should cancel locally. The case of $d_\perp = 2$ leads to a logarithmic dependence of the effective potential on $R_\perp/l_s$ which allows the possible radiative generation of the hierarchy between $R_\perp$ and l_s as for no-scale models. Moreover, it is interesting to notice that the ultraviolet behavior of the theory is very similar with the one with soft supersymmetry breaking at $M_s \sim TeV$. It is then natural to ask the question whether there is any motivation leftover for supersymmetry or not. This bring us to the problems of the stability of the new hierarchy and of the cosmological constant [14].

In fact, in a non-supersymmetric string theory, the bulk energy density behaves generically as $\Lambda_{\text{bulk}} \sim M_s^{4+n}$, where n is the number of transverse dimensions much larger than the string length. In the type I/I$'$ context, this induces a cosmological constant on our world-brane which is enhanced by the volume of the transverse space $V_\perp \sim R_\perp^n$. When expressed in terms of the 4d parameters using the type I/I$'$ mass-relation (16), it is translated to a quadratically dependent contribution on the Planck mass:

$$\Lambda_{\text{brane}} \sim M_I^{4+n} R_\perp^n \sim M_I^2 M_P^2 \,, \tag{43}$$

where we used $s = I$. This contribution is in fact the analogue of the quadratic divergent term $\text{Str}\mathcal{M}^2$ in softly broken supersymmetric theories, with M_I playing the role of the supersymmetry breaking scale.

The brane energy density (43) is far above the (low) string scale M_I and in general destabilizes the hierarchy that one tries to enforce. One way out is to resort to special models with broken supersymmetry and vanishing or exponentially small cosmological constant [35]. Alternatively, one could conceive a different scenario, with supersymmetry broken primordially on our world-brane maximally, i.e. at the string scale which is of order of a few TeV. In this case the brane cosmological constant would be, by construction, $\mathcal{O}(M_I^4)$, while the bulk would only be affected by gravitationally suppressed radiative corrections and thus would be almost supersymmetric [14, 36]. In particular, one would expect the gravitino and other soft masses in the bulk to be extremely small $O(M_I^2/M_P)$. In this case, the cosmological constant induced in the bulk would be

$$\Lambda_{\text{bulk}} \sim M_I^4/R_\perp^n \sim M_I^{6+n}/M_P^2 \,, \tag{44}$$

i.e. of order (10 MeV)^6 for $n = 2$ and $M_I \simeq 1$ TeV. The scenario of brane supersymmetry breaking is also required in models with a string scale at intermediate energies $\sim 10^{11}$ GeV (or lower), discussed in section 4.1. It can occur for instance on a brane distant from our world and is then mediated to us by gravitational (or gauge) interactions.

In the absence of gravity, brane supersymmetry breaking can occur in a non-BPS system of D-branes. The simplest examples are based on orientifold projections of type IIB, in which some of the orientifold 5-planes have opposite charge, requiring an open string sector living on anti-D5 branes in order to cancel the RR (Ramond-Ramond) charge. As a result, supersymmetry is broken on the intersection of D9 and anti-D5 branes that coincides with the world volume of the latter. The simplest construction of this type is a T^4/Z_2 orientifold with a flip of the Ω-projection (world-sheet parity) in the twisted orbifold sector. It turns out that several orientifold models, where tadpole conditions do not admit naive supersymmetric solutions, can be defined by introducing non-supersymmetric open sector containing anti-D-branes. A typical example of this type is the ordinary $Z_2 \times Z_2$ orientifold with discrete torsion.

The resulting models are chiral, anomaly-free, with vanishing RR tadpoles and no tachyons in their spectrum [36]. Supersymmetry is broken at the string scale on a collection of anti-D5 branes while, to lowest order, the closed string bulk and the other branes are supersymmetric. In higher orders, supersymmetry breaking is of course mediated to the remaining sectors, but is suppressed by the size of the transverse space or by the distance from the brane where supersymmetry breaking primarily occurred. The models contain in general uncancelled NS (Neveu-Schwarz) tadpoles reflecting the existence of a tree-level potential for the NS moduli, which is localized on the (non-supersymmetric) world volume of the anti-D5 branes.

As a result, this scenario implies the absence of supersymmetry on our world-brane but its presence in the bulk, a millimeter away! The bulk supergravity is needed to guarantee the stability of gauge hierarchy against large gravitational quantum radiative corrections.

Explicit examples and methods of supersymmetry breaking in type I string theory are described in the Appendix.

6.4 Electroweak symmetry breaking in TeV-scale strings

The existence of non-supersymmetric type I string vacua allows us to address the question of gauge symmetry breaking. From the effective field theory point of view, one expects quadratic divergences in one-loop contribution to the masses of scalar fields. It is then important to address the following questions: (i) which scale plays the role of the Ultraviolet cut-off (ii) could these one-loop corrections be used to to generate radiatively the electroweak symmetry breaking, and explain the mild hierarchy between the weak and a string scale at a few TeVs.

A simple framework to address such issues is non-supersymmetric tachyon-free Z_2 orientifold of type IIB superstring compactified to four dimensions on $T^4/Z_2 \times T^2$ [36]. Cancellation of Ramond-Ramond charges requires the presence of 32 D9 and 32 anti-D5 ($D\bar{5}$) branes. The bulk (closed strings) as well as the D9 branes are $N = 2$ supersymmetric while supersymmetry is broken on the world-volume of the $D\bar{5}$'s. It is possible [37] to compute the effective potential involving the scalars of the $D\bar{5}$ branes, namely in this simple example the adjoints and bifundamentals of the $USp(16) \times USp(16)$ gauge group. The resulting potential has a non-trivial minimum which fixes the VEV of the Wilson line or, equivalently, the distance between the branes in the T-dual picture. Although the obtained VEV is of the order of the string scale, the potential provides a negative squared-mass term when expanded around the origin.

In the limit where the radii of the transverse space are large, $R_\perp \to \infty$ and for arbitrary longitudinal radius $R_{||}$, the result is:

$$\mu^2(R_{||}) = -\varepsilon^2(R_{||})\, g^2\, M_s^2 \tag{45}$$

with

$$\varepsilon^2(R_{||}) = \frac{1}{2\pi^2}\int_0^\infty \frac{dl}{(2\,l)^{5/2}}\frac{\theta_2^4}{4\eta^{12}}\left(il + \frac{1}{2}\right) R_{||}^3 \sum_n n^2 e^{-2\pi n^2 R_{||}^2 l}\,. \tag{46}$$

For the asymptotic value $R_{||} \to 0$ (corresponding upon T-duality to a large transverse dimension of radius $1/R_{||}$), $\varepsilon(0) \simeq 0.14$, and the effective cutoff for the mass term at the origin is M_s, as can be seen from Eq. (45). At large $R_{||}$, $\mu^2(R_{||})$ falls off as $1/R_{||}^2$, which is the effective cutoff in the limit $R_{||} \to \infty$, in agreement with field theory results in the presence of a compactified extra dimension [38]. In fact, in the limit $R_{||} \to \infty$ an analytic approximation to $\varepsilon(R)$ gives:

$$\varepsilon(R_{||}) \simeq \frac{\varepsilon_\infty}{M_s\, R_{||}}\,, \qquad \varepsilon_\infty^2 = \frac{3\,\zeta(5)}{4\,\pi^4} \simeq 0.008\,. \tag{47}$$

While the mass term (45) was computed for the Wilson line it also applies, by gauge invariance, to the charged massless fields which belong to the same representation. By orbifolding the previous example, the Wilson line is projected away from the spectrum and we are left with the charged massless fields with quartic tree-level terms and one-loop negative squared masses. By identifying them with the Higgs field we can achieve radiative electroweak symmetry breaking, and obtain the mild hierarchy between the weak and string scales in terms of a loop factor. More precisely, in the minimal case where there is only one such Higgs doublet h, the scalar potential would be:

$$V = \lambda(h^\dagger h)^2 + \mu^2(h^\dagger h)\,, \tag{48}$$

where λ arises at tree-level and is given by an appropriate truncation of a supersymmetric theory. This property remains valid in any model where the higgs field comes from an open string with both ends fixed on the same type of D-branes (untwisted state). Within the minimal spectrum of the Standard Model, $\lambda = (g_2^2 + g'^2)/8$, with g_2 and g' the $SU(2)$ and $U(1)_Y$ gauge couplings, as in the MSSM. On the other hand, μ^2 is generated at one loop and can be estimated by Eqs. (45) and (46).

The potential (48) has a minimum at $\langle h \rangle = (0, v/\sqrt{2})$, where v is the VEV of the neutral component of the h doublet, fixed by $v^2 = -\mu^2/\lambda$. Using the relation of v with the Z gauge boson mass, $M_Z^2 = (g_2^2 + g'^2)v^2/4$, and the fact that the quartic Higgs interaction is provided by the gauge couplings as in supersymmetric theories, one obtains for the Higgs mass a prediction which is the MSSM value for $\tan\beta \to \infty$ and $m_A \to \infty$:

$$M_h = M_Z \ . \tag{49}$$

Furthermore, one can compute M_h in terms of the string scale M_s, as $M_h^2 = -2\mu^2 = 2\varepsilon^2 g^2 M_s^2$, or equivalently

$$M_s = \frac{M_h}{\sqrt{2}\, g\varepsilon} \tag{50}$$

The determination of the precise value of the string scale suffers from two ambiguities. The first is the value of the gauge coupling g at M_s, which depends on the details of the model. A second ambiguity concerns the numerical coefficient ε which is in general model dependent. Varying R from 0 to 5, that covers the whole range of values for a transverse dimension $1 < 1/R_\perp < \infty$, as well as a reasonable range for a longitudinal dimension $1 < R_{||} \lesssim 5$, one obtains $M_s \simeq 1 - 5$ TeV. In the $R_{||} \gg 1$ (large longitudinal dimension) region our theory is effectively cutoff by $1/R_{||}$ and the Higgs mass is then related to it by,

$$\frac{1}{R_{||}} = \frac{M_h}{\sqrt{2}\, g\, \varepsilon_\infty} \, . \tag{51}$$

Using now the value for ε_∞ in the present model, Eq. (47), we find $1/R_{||} \gtrsim 1$ TeV.

The tree level Higgs mass has been shown to receive important radiative corrections from the top-quark sector. For present experimental values of the top-quark mass, the Higgs mass in Eqs. (49) and (50) is raised to values around 120 GeV [39]. In addition there might be large string threshold corrections in the case of $d_\perp = 2$ large transverse dimensions, due to large logarithms discussed in section 6.1.

7 Scenario for studies of experimental constraints

In order to pursue further, we need to provide the quantum numbers and couplings of the relevant light states. In the scenario we consider:

- Gravitons [5] which describe fluctuations of the metric propagate in the whole 10- or 11-dimensional space.

- In all generality, gauge bosons propagate on a $(3 + d_{||})$-brane, with $d_{||} = 0, ..., 6$. However, as we have seen in the previous sections, a freedom of choice for the values of the string and compactification scales requires that gravity and gauge degrees of freedom live in spaces with different dimensionalities. This means that $d_{||max} = 5$ or 6 for 10- or 11-dimensional theories, respectively. The value of $d_{||}$ represents the number of dimensions felt by KK excitations of gauge bosons.

  To simplify the discussion, we will mainly consider the case $d_{||} = 1$ where some of the gauge fields arise from a 4-brane. Since the couplings of the corresponding gauge groups are reduced by the size of the large dimension $R_{||}M_s$ compared to the others, if $SU(3)$ has KK modes all three group factors must have. Otherwise it is difficult to reconcile the suppression of the strong coupling at the string scale with the observed reverse situation. As a result, there are 5 distinct cases [41] that we denote (l, l, l), (t, l, l), (t, l, t), (t, t, l) and (t, t, t), where the three positions in the brackets correspond to the 3 gauge group factors of the standard model $SU(3)_c \times SU(2)_w \times U(1)_Y$ and those with l feel the extra-dimension, while those with t (transverse) do not.

- The matter fermions, quarks and leptons, are localized on the intersection of a 3-brane with the $(3 + d_{||})$-brane and have no KK excitations along the $d_{||}$ directions. Their coupling to KK modes of gauge bosons are given by:

  $$g_n = \sqrt{2}\delta^{-|\frac{\vec{n}}{R}|^2/M_s^2} g \tag{52}$$

  where $\delta > 1$ is a model dependent number ($\delta = 4$ in the case of Z_2). This is the main assumption in our analysis and limits derived in the next subsection depend on it. In a more general study it could be relaxed by assuming that only part of the fermions are localized. However, if all states are propagating in the bulk, then the KK excitations are stable and a discussion of the cosmology will be necessary in order to explain why they have not been seen as isotopes.

  Let's denote generically the localized states as T while the bulk states with KK momentum n/R by U_n, thus the only trilinear allowed couplings are $g_n TTU_n$ and $gU_nU_mU_{n+m}$ where g_n is given by Eq. (52). Hence because matter fields are localized, their interactions do not

[5] Along with gravitons, string models predict the presence of other very weakly coupled states as gravitini, dilatons, moduli, Ramond-Ramond fields, etc.

preserve the momenta in the extra-dimension and single KK excitations can be produced. This means for example that QCD processes $q\bar{q} \to G^{(n)}$ with q representing quarks and $G^{(n)}$ massive KK excitations of gluons are allowed. In contrast, processes such as $GG \to G^{(n)}$ are forbidden as gauge boson interactions conserve the internal momenta.

8 Extra-dimensions along the world brane: KK excitations of gauge bosons

The experimental signatures of extra-dimensions are of two types [42, 43, 41]:

- Observation of resonances due to KK excitations. This needs a collider energy $\sqrt{s} \gtrsim 1/R_{\|}$ at LHC.
- Virtual exchange of the KK excitations which lead to measurable deviations in cross-sections compared to the standard model prediction.

The necessary data needed to evaluate the size of these contributions are: the coupling constants given in (52), the KK masses, and the associated widths. The latter are given by decay rates into standard model fermions f:

$$\Gamma\left(X_n \to f\bar{f}\right) = g_\alpha^2 \frac{M_{\vec{n}}}{12\pi} C_f (v_f^2 + a_f^2) \tag{53}$$

and, in the case of supersymmetric brane there is an additional contribution from decays into the scalar superpartners

$$\Gamma\left(X_n \to \widetilde{f}_{(R,L)} \widetilde{\bar{f}}_{(R,L)}\right) = g_\alpha^2 \frac{M_{\vec{n}}}{48\pi} C_f (v_f \pm a_f)^2 \,, \tag{54}$$

with $C_f = 1$ (3) for color singlets (triplets) and v_f, a_f stand for the standard model vector and axial couplings. These widths determine the size of corresponding resonance signals and will be important when discussing on-shell production of KK excitations.

In the studies of virtual effects, our strategy for extracting exclusion bounds will depend on the total number of analyzed events. If it is small then we will consider out of reach compactification scales which do not lead to prediction of at least 3 new events. In the case of large number of events, one estimates the deviation from the background fluctuation ($\sim \sqrt{N_T^{\rm SM}(s)}$) by computing the ratio [42, 43, 41]

$$\Delta_T = \left| \frac{N_T(s) - N_T^{\rm SM}(s)}{\sqrt{N_T^{\rm SM}(s)}} \right| \tag{55}$$

where $N_T(s)$ is the total number of events while $N_T^{\rm SM}(s)$ is the corresponding quantity expected from the standard model. These numbers are computed using the formula:

$$N_T = \sigma A \int \mathcal{L} dt \tag{56}$$

where σ is the relevant cross-section, $\int \mathcal{L} dt$ is the integrated luminosity while A is a suppression factor taking into account the corresponding efficiency times acceptance factors.

In the next two subsections we derive limits for the case (l, l, l) where all the gauge factors feel the large extra-dimension. We will return later to the other possibilities.

8.1 Production at hadron colliders

At collider experiments, there are three different channels l^+l^-, $l^\pm \nu$ and dijets where exchange of KK excitations of photon+Z, $W^\pm$ and gluons can produce observable deviations from the standard model expectations.

Let's illustrate in details the first case with exchange of neutral bosons. KK excitations are produced in Drell–Yan processes $pp \to l^+l^-X$ at the LHC, or $p\bar{p} \to l^+l^-X$ at the Tevatron, with $l = e, \mu, \tau$ which originate from the subprocess $q\bar{q} \to l^+l^-X$ of centre–of–mass energy M.

The two colliding partons take a fraction

$$x_a = \frac{M}{\sqrt{s}}\, e^y \quad \text{and} \quad x_b = \frac{M}{\sqrt{s}}\, e^{-y} \tag{57}$$

of the momentum of the initial proton (a) and (anti)proton (b), with a probability described by the quark or antiquark distribution functions-$f_{q,\bar{q}}^{(a)}(x_a, M^2)$ and $f_{q,\bar{q}}^{(b)}(x_b, M^2)$. The total cross-section, due to the production is given by:

$$\sigma = \sum_{q=\text{quarks}} \int_0^{\sqrt{s}} dM \int_{\ln(M/\sqrt{s})}^{\ln(\sqrt{s}/M)} dy\; g_q(y, M) S_q(y, M)\ , \tag{58}$$

where

$$g_q(y, M) = \frac{M}{18\pi} x_a x_b \left[f_q^{(a)}(x_a, M^2) f_{\bar{q}}^{(b)}(x_b, M^2) + f_{\bar{q}}^{(a)}(x_a, M^2) f_q^{(b)}(x_b, M^2)\right], \tag{59}$$

and

$$S_q(y, M) = \sum_{\alpha,\beta\gamma,Z,KK} g_\alpha^2(M) g_\beta^2(M) \frac{(v_e^\alpha v_e^\beta + a_e^\alpha a_e^\beta)(v_l^\alpha v_l^\beta + a_l^\alpha a_l^\beta)}{(s - m_\alpha^2 + i\Gamma_\alpha m_\alpha)(s - m_\beta^2 - i\Gamma_\beta m_\beta)}\ . \tag{60}$$

At the Tevatron, the CDF collaboration has collected an integrated luminosity $\int \mathcal{L}dt = 110$ pb^{-1} during the 1992-95 running period. A lower bound on the size of compactification scale can be extracted from the absence of candidate events at e^+e^- invariant mass above 400 GeV. A similar analysis can be carried over for the case of run-II of the Tevatron with a centre–of–mass energy $\sqrt{s} = 2$ TeV and integrated luminosity $\int \mathcal{L}dt = 2\ fb^{-1}$. The expected number of events at these experiments are plotted in Fig. 1 while the bounds are summarized in Table 1 (the factor A in (56) has be taken to be 50 %) [43, 41].

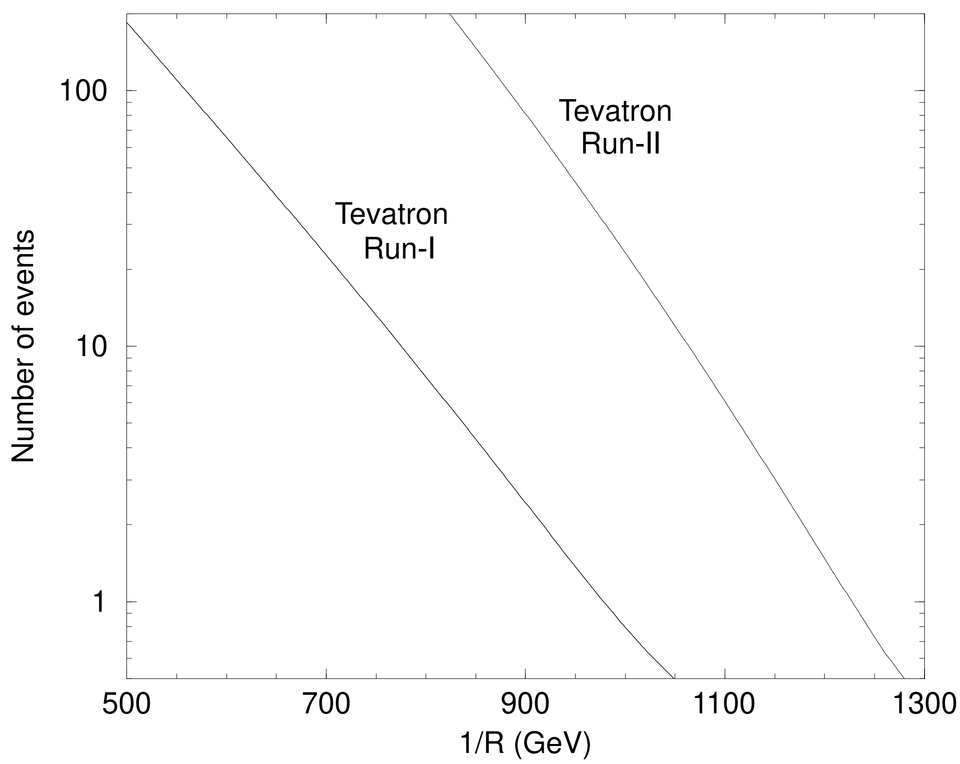


Figure 1: *Number of l^+l^--pair events with centre–of–mass energy above 400 GeV (600 GeV) expected at the Tevatron run-I (run-II) with integrated luminosity $\int \mathcal{L}dt = 110\ pb^{-1}$ ($\int \mathcal{L}dt = 2\ fb^{-1}$) and efficiency times acceptance of $\sim$ 50%, as a function of R^{-1}.*

The most promising for probing TeV-scale extra-dimensions are the LHC future experiments at $\sqrt{s} = 14$ TeV with an integrated luminosity

Table 1: *Limits on* $R_{\|}^{-1}$ *in TeV at present and future colliders. The luminosity is given in* fb^{-1}.

Collider	Luminosity	Gluons	$W^{\pm}$	$\gamma + Z$
Discovery of Resonances				
LHC	100	5	6	6
Observation of Deviation				
LEP 200	4×0.2	-	-	1.9
TevatronI	0.11	-	-	0.9
TevatronII	2	-	-	1.2
TevatronII	20	4	-	1.3
LHC	10	15	8.2	6.7
LHC	100	20	14	12
NLC500	75	-	-	8
NLC1000	200	-	-	13

$\int \mathcal{L} dt = 100\ fb^{-1}$. Fig. 2 shows the expected deviation from the standard model predictions of the total number of events in the l^+l^-, $l^{\pm}\nu$ due to KK excitations $\gamma^{(n)} + Z^{(n)}$ and $W_{\pm}^{(n)}$ respectively. The results were obtained by requiring for the dilepton final state one lepton to be in the central region, $|\eta_l| \leq 1$, the other one having a looser cut $|\eta_{l'}| \leq 2.4$. Moreover the lower bound on the transverse and invariant mass was chosen to be 400 GeV [41, 42, 43].

In the case of (l, l, l) scenario, looking for an excess of dijet events due to KK excitations of gluons could be the most efficient channel to constrain the size of extra-dimensions. Fig. 3 shows the corresponding expected deviation Δ_T as defined in (55). This analysis uses summation over all jets, top excluded, a rapidity cut, $|\eta| \leq 0.5$, on both jets and requirement on the invariant mass to be $M_{jj'} \geq 2$ TeV, which reduces the SM background and gives the optimal ratio $S/\sqrt{B}$ especially for large masses [41].

In addition to these virtual effects, the LHC experiments allow the production on-shell of KK excitations. The discovery limits for these KK excitations are given in Table 1. An interesting observation is the case of excitations $\gamma^{(1)} + Z^{(1)}$ where interferences lead to a "deep" just before the resonance as illustrated in Fig. 4

There are some ways to distinguish the corresponding signals from other possible origin of new physics, such as models with new gauge bosons. In the case of observation of resonances, one expects three resonances in the (l, l, l) case and two in the (t, l, l) and (t, l, t) cases, located practically at the same mass value. This property is not shared by most of other new gauge

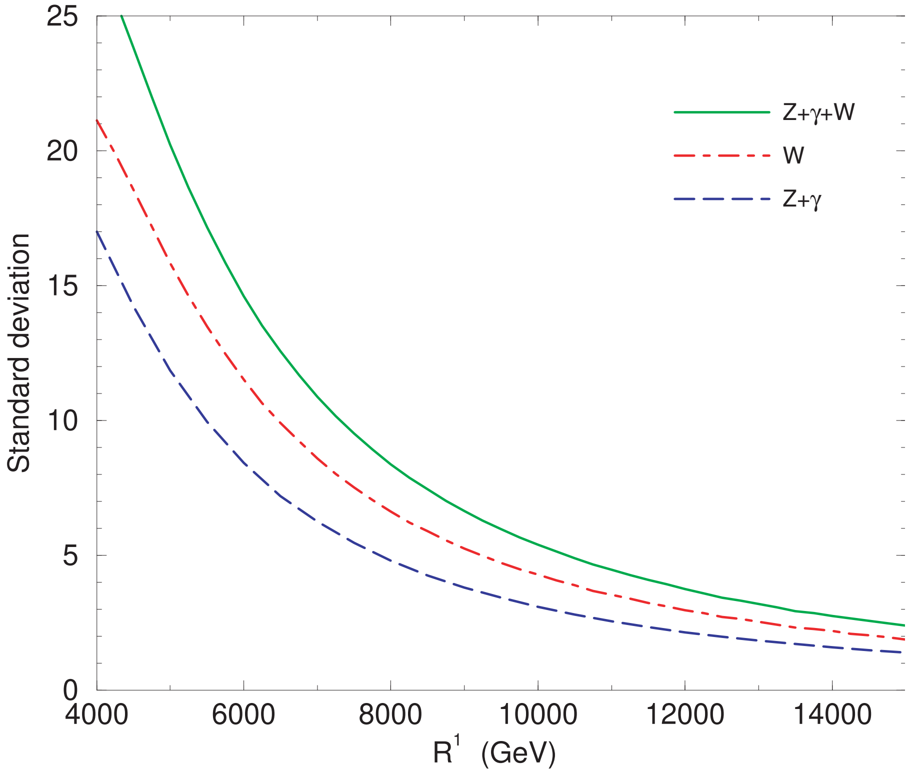


Figure 2: *Number of standard deviation in the number of l^+l^- pairs and $\nu_l l$ pairs produced from the expected standard model value due to the presence of one extra-dimension of radius R.*

boson models. Moreover, the heights and widths of the resonances are directly related to those of standard model gauge bosons in the corresponding channels. In the case of virtual effects, these are not reproduced by a tail of Bright-Wigner shape and a deep is expected just before the resonance of the photon+Z, due to the interference between the two. However, good statistics will be necessary [42].

8.2 High precision data low-energy bounds

Using the lagrangian describing interactions of the standard model states, it is possible to compute all physical observables in term of few input data. Then one can compare the predictions with experimental values.

Following [44, 45] we will use as input parameters, the Fermi constant $G_F = 1.166 \times 10^{-5}$ GeV^{-2}, the fine-structure constant $\alpha = 1/137.036$ (or $\alpha(M_Z) = 1/128.933$) and the mass of the Z gauge-boson $M_Z = 91.1871$

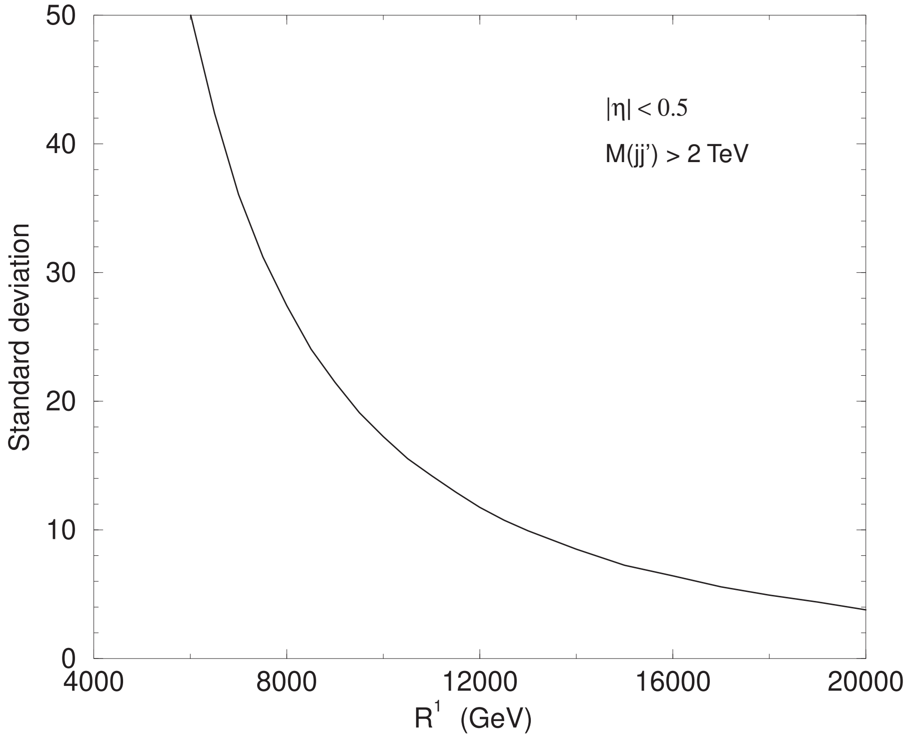


Figure 3: *Number of standard deviation in number of observed dijets from the expected standard model value, due to the presence of a TeV-scale extra-dimension of compactification radius R.*

GeV. The observables given in Table 2 are then computed with the new lagrangian including the contribution of KK excitations. The effects of the latter will be computed as a leading order expansion in the small parameter

$$X = \sum_{n=1}^{\infty} \frac{2}{n^2} \frac{m_Z^2}{M_c^2} = \frac{\pi^2}{3} (m_Z R_{||})^2 \,, \tag{61}$$

as one expects $m_Z \ll 1/R_{||}$.

Performing a χ^2 fit, one finds that if the Higgs is a bulk state like the gauge bosons, $R^{-1} \gtrsim 3.5$ TeV. Inclusion of Q_W measurement, which does not give a good agreement with the standard model itself, raises the bound to $R^{-1} \gtrsim 3.9$ TeV [45]. Different choices for localization of matter states and Higgs lead to slightly different bounds, lying in the 1 to 5 TeV range, and the analysis can be found in [45].

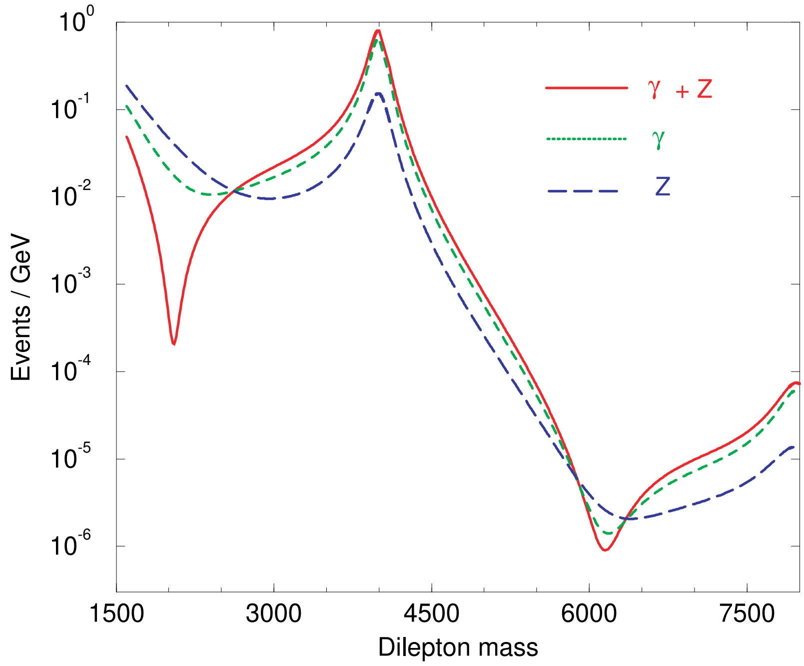


Figure 4: *First resonances in the LHC experiment due to a KK excitation of photon and Z for one extra-dimension at 4 TeV. From highest to lowest: excitation of photon+Z, photon and Z boson.*

8.3 One extra dimension for other cases:

Except for the (l, l, l) scenario, in all other cases there are no excitations of gluons and there no important limits from the dijets channels [41]. The KK excitations $W_{\pm}^{(n)}$, $\gamma^{(n)}$ and $Z^{(n)}$ are present and lead to the same limits in the (t, l, l) case: 6 TeV for discovery and 15 TeV for the exclusion bounds. In the (t, l, t) case, only the $SU(2)$ factor feels the extra-dimension and the limits are set by KK excitations of $W^{\pm}$ and are again 6 TeV for discovery and 14 TeV for the exclusion bounds. In the (t, t, l) channel where only $U(1)_Y$ feels the extra-dimension the limits are weaker, the exclusion bound is in fact around 8 TeV.

In addition to these simple possibilities, brane constructions lead often to cases where part of $U(1)_Y$ is t and part is l, while $SU(3)$ and $SU(2)$ are either t or l. If $SU(3)$ is l then the bounds come from dijets, if instead $SU(3)$ is t and $SU(2)$ is l the limits could come from $W^{\pm}$ while if both are t then it will be difficult to distinguish this case from a generic extra

Table 2: *Set of physical observables. The Standard Model predictions are computed for a Higgs mass $M_H = M_Z$ ($M_H = 300$ GeV) and a top-quark mass $m_t = 173 \pm 4$ GeV.*

Observable	Experimental value	Standard Model prediction
M_W (GeV)	80.394±0.042	80.377±0.023 (−0.036)
$\Gamma_{\ell\ell}$ (MeV)	83.958±0.089	84.00±0.03 (−0.04)
Γ_{had} (GeV)	1.7439±0.0020	1.7433±0.0016 (−0.0005)
A^{ℓ}_{FB}	0.01701±0.00095	0.0162±0.0003 (−0.0004)
Q_W	−72.06±0.46	−73.12±0.06 (+0.01)
$\sum_{i=1}^{3} \lvert V_{1i}\rvert^2$	0.9969±0.0022	1 (unitarity)

$U(1)'$. A good statistics would be needed to see the deviation in the tail of the resonance as being due to effects additional to those of a generic $U(1)'$ resonance.

8.4 More than one extra dimensions

The computation of virtual effects of KK excitations involves summing on effects of a priori infinite number of tree-level diagrams as terms of the form:

$$\sum_{|\vec{n}|} \frac{g^2(|\vec{n}|)}{|\vec{n}|^2} \tag{62}$$

arising from interference between the exchange of the photon and Z-boson and their KK excitations, with $g^2(|\vec{n}|)$ the KK-mode couplings. In the case of one extra-dimension the sum in (62) converges rapidly and for $RM_s \sim \mathcal{O}(10)$ the result is not sensitive to the value of M_s. This allowed us to discuss bounds on only one parameter, the scale of compactification.

In the case of two or more dimensions, Eq. (62) is divergent and needs to be regularized using:

$$g(|\vec{n}|) \sim g\, a_{(|\vec{n}|)}\, e^{\frac{-c|\vec{n}|^2}{2R^2M_s^2}}\,, \tag{63}$$

where c is a constant and $a_{(|\vec{n}|)}$ takes into account the normalization of the gauge kinetic terms, as only the even combination couples to the boundary. For the case of two extra-dimensions [43] $a_{(0,0)} = 1$, $a_{(0,p)} = a_{(q,0)} = \sqrt{2}$ and $a_{(q,p)} = 2$ with (p,q) positive (> 0) integers. The result will depend on both the compactification and string scales. Other features are that cross-sections are bigger and resonances are closer. The former property arises because the degeneracy of states within each mass level increases with the number of extra dimensions while the latter property implies that more resonances could be reached by a given hadronic machine.

9 Extra-dimensions transverse to the brane world: KK excitations of gravitons

The localization of (infinitely massive) branes in the $(D-d)$ dimensions breaks translation invariance along these directions. Thus, the corresponding momenta are not conserved: particles, as gravitons, could be absorbed or emitted from the brane into the $(D-d)$ dimensions. Non observation of the effects of such processes allow us to get bounds on the size of these transverse extra dimensions. In order to simplify the analysis, it is usually assumed that among the $D-d$ dimensions n have very large common radius $R_\perp \gg M_s^{-1}$, while the remaining $D-d-n$ have sizes of the order of the string length.

9.1 Signals from missing energy experiments

During a collision of center of mass energy $\sqrt{s}$, there are $(\sqrt{s}R_\perp)^n$ KK excitations of gravitons with mass $m_{KK\perp} < \sqrt{s} < M_s$, which can be emitted. Each of these states looks from the four-dimensional point of view as a massive, quasi-stable, extremely weakly coupled (s/M_P^2 suppressed) particle that escapes from the detector. The total effect is a missing-energy cross-section roughly of order:

$$\frac{(\sqrt{s}R_\perp)^n}{M_P^2} \sim \frac{1}{s}(\frac{\sqrt{s}}{M_s})^{n+2} \tag{64}$$

For illustration, the simplest process is the gluon annihilation into a graviton which escapes into the extra dimensions. The corresponding cross-section is given by (in the weak coupling limit) [14]:

$$\sigma(E) \sim \frac{E^n}{M_s^{n+2}} \frac{\Gamma\left(1-2E^2/M_s^2\right)^2}{\Gamma\left(1-E^2/M_s^2\right)^4}\,, \tag{65}$$

where E is the center of mass energy and n the number of extra large transverse dimensions. The above expression exhibits 3 kinematic regimes with different behavior. At high energies $E \gg M_s$, it falls off exponentially due to the UV softness of strings. At energies of the order of the string scale, it exhibits a sequence of poles at the position of Regge resonances. Finally, at low energies $E \ll M_s$, it falls off as a power $\sigma(E) \sim E^n/M_s^{n+2}$, dictated by the effective higher dimensional gravity which requires the presence of the $(4+n)$-dimensional Newton's constant $G_N^{(4+n)} \sim l_s^{n+2}$ from eq.(17).

Explicit computation of these effects leads to the bounds given in Table 3, while Fig. 5 shows the cross-section for graviton emission in the bulk, corresponding to the process $pp \to jet + gravition$ at LHC, together with the Standard Model background [47]. The results require some remarks:

Table 3: *Limits on $R_\perp$ in mm from missing-energy processes.*

Experiment	$R_\perp(n=2)$	$R_\perp(n=4)$	$R_\perp(n=6)$
Collider bounds			
LEP 2	4.8×10^{-1}	1.9×10^{-8}	6.8×10^{-11}
Tevatron	5.5×10^{-1}	1.4×10^{-8}	4.1×10^{-11}
LHC	4.5×10^{-3}	5.6×10^{-10}	2.7×10^{-12}
NLC	1.2×10^{-2}	1.2×10^{-9}	6.5×10^{-12}
Present non-collider bounds			
SN1987A	3×10^{-4}	1×10^{-8}	6×10^{-10}
COMPTEL	5×10^{-5}	-	-

- The amplitude for emission of each of the KK gravitons is taken to be well approximated by the tree-level coupling of the massless graviton as derived from General Relativity. Eq. 52 suggests that this is likely to be a good approximation for $R_\perp M_s \gg 1$.

- The cross-section depends on the size $R_\perp$ of the transverse dimensions and allows to derive bounds on this *physical* scale. As it can be seen from Eq. (52), transforming these bounds to limits on M_s there is an ambiguity on different factors involved, such as the string coupling. This is sometimes absorbed in the so called "fundamental quantum gravity scale $M_{(4+n)}$". Generically $M_{(4+n)}$ is bigger than M_s, and in some cases, as in type II strings or in heterotic strings with small instantons, it can be many orders of magnitude higher than M_s. It corresponds to the scale where the perturbative expansion of string theory seems to break down [48].

- There is a particular energy and angular distribution of the produced gravitons that arises from the distribution in mass of KK states. It might be a smoking gun for the extra-dimensional nature of such observable signal.

- For given value of M_s, the cross-section for graviton emission decreases with the number of large transverse dimensions. The effects are more likely to be observed for the lowest values of M_s and n.

- Finally, while the obtained bounds for $R_\perp^{-1}$ are smaller than those that could be checked in table-top experiments probing macroscopic gravity at small distances (see next subsection), one should keep in mind that larger radii are allowed if one relaxes the assumption of

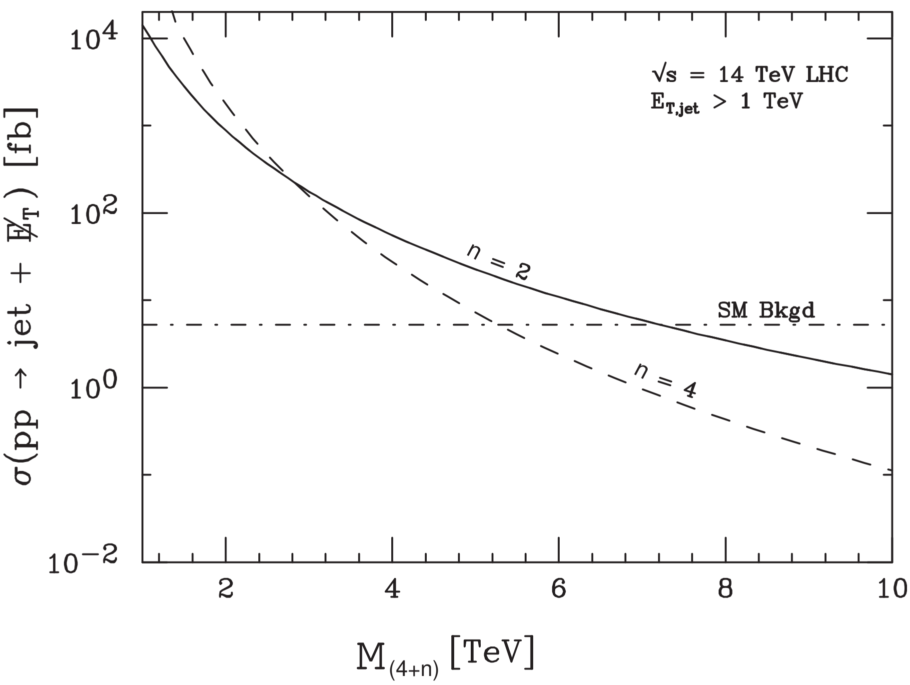


Figure 5: *Missing energy due to graviton emission in the LHC experiment, as a function of the quantum gravity scale $M_{(4+n)}$ for n large transverse dimensions, together with the Standard model baclground.*

isotropy, by taking for instance two large dimensions with different radii.

In Table 3, we have also included astrophysical and cosmological bounds. Astrophysical bounds [20] arise from the requirement that the radiation of gravitons should not carry on too much of the gravitational binding energy released during core collapse of supernovae. In fact, the measurements of Kamiokande and IMB for SN1987A suggest that the main channel is neutrino fluxes.

The best cosmological bound [21] is obtained from requiring that decay of bulk gravitons to photons do not generate a spike in the energy spectrum of the photon background measured by the COMPTEL instrument. Bulk gravitons are expected to be produced just before nucleosynthesis due to thermal radiation from the brane. The limits assume that the temperature was at most 1 MeV as nucleosynthesis begins, and become stronger if the temperature is increased.

9.2 Gravity modification and sub-millimeter forces

Besides the spectacular experimental predictions in particle accelerators, string theories with large volume compactifications and/or low string scale predict also possible modifications of gravitation in the sub-millimeter range, which can be tested in "table-top" experiments that measure gravity at short distances. There are three categories of such predictions:
(i) Deviations from the Newton's law $1/r^2$ behavior to $1/r^{2+n}$, for n extra large transverse dimensions, which can be observable for $n = 2$ dimensions of (sub)-millimeter size. This case is particularly attractive on theoretical grounds because of the logarithmic sensitivity of Standard Model couplings on the size of transverse space [15], which allows to determine the desired hierarchy [49], but also for phenomenological reasons since the effects in particle colliders are maximally enhanced [47]. Notice also the coincidence of this scale with the possible value of the cosmological constant in the universe that recent observations seem to support.
(ii) New scalar forces in the sub-millimeter range, motivated by the problem of supersymmetry breaking, and mediated by light scalar fields φ with masses [50, 46, 14, 36]:

$$m_\varphi \simeq \frac{m_{susy}^2}{M_P} \simeq 10^{-4} - 10^{-6}\ \mathrm{eV}\,, \tag{66}$$

for a supersymmetry breaking scale $m_{susy} \simeq 1 - 10$ TeV. These correspond to Compton wavelengths in the range of 1 mm to 10 μm. m_{susy} can be either the KK scale $1/R$ if supersymmetry is broken by compactification [46, 50], or the string scale if it is broken "maximally" on our world-brane [14, 36]. A model independent and universal attractive scalar force is mediated by the radius modulus (in Planck units)

$$\varphi \equiv \ln R\,, \tag{67}$$

with R the radius of the longitudinal ($\|$) or transverse ($\perp$) dimension(s), respectively. In the former case, the result (66) follows from the behavior of the vacuum energy density $\Lambda \sim 1/R_\|^4$ for large $R_\|$ (up to logarithmic corrections). In the latter case, supersymmetry is broken primarily on the brane only, and thus its transmission to the bulk is gravitationally suppressed, leading to masses (66). Note that in the case of two-dimensional bulk, there may be an enhancement factor of the radion mass by $\ln R_\perp M_s \simeq 30$ which decreases its wavelength by roughly an order of magnitude [49].

The coupling of the radius modulus (67) to matter relative to gravity can be easily computed and is given by:

$$\sqrt{\alpha_\varphi} = \frac{1}{m}\frac{\partial m}{\partial \varphi} \quad ; \quad \alpha_\varphi = \begin{cases} \frac{\partial \ln \Lambda_{\rm QCD}}{\partial \ln R} \simeq \frac{1}{3} & \text{for } R_\| \\ \\ \frac{2n}{n+2} = 1 - 1.5 & \text{for } R_\perp \end{cases}\,, \tag{68}$$

where m denotes a generic physical mass. In the upper case of a longitudinal radius, the coupling arises dominantly through the radius dependence of the QCD gauge coupling [46], while in the lower case of transverse radius, it can be deduced from the rescaling of the metric which changes the string to the Einstein frame and depends on the dimensionality of the bulk n (varying from $\alpha = 1$ for $n = 2$ to $\alpha = 1.5$ for $n = 6$) [49]. Moreover, in the case of $n = 2$, there may be again model dependent logarithmic corrections of the order of $(g_s/4\pi) \ln RM_s \simeq \mathcal{O}(1)$. Such a force can be tested in microgravity experiments and should be contrasted with the change of Newton's law due the presence of extra dimensions that is observable only for $n = 2$ [22]. In principle there can be other light moduli which couple with even larger strengths. For example the dilaton, whose VEV determines the (logarithm of the) string coupling constant, if it does not acquire large mass from some dynamical supersymmetric mechanism, can lead to a force of strength 2000 times bigger than gravity [51].
(iii) Non universal repulsive forces much stronger than gravity, mediated by possible abelian gauge fields in the bulk [20, 52]. Such gauge fields may acquire tiny masses of the order of M_s^2/M_P, as in (66), due to brane localized anomalies [52]. Although the corresponding gauge coupling is infinitesimally small, $g_A \sim M_s/M_P \simeq 10^{-16}$, it is still bigger that the gravitational coupling $\sim E/M_P$ for typical energies E of the order of the proton mass, and the strength of the new force would be $10^6 - 10^8$ stronger than gravity. This an interesting region which will be soon explored in micro-gravity experiments (see Fig. 6). Note that in this case the supernova constraints impose that there should be at least four large extra dimensions in the bulk [20].

In Fig. 6 we depict the actual information from previous, present and upcoming experiments [49]. The vertical axis is the strength, α, of the force relative to gravity; the horizontal axis is the Compton wavelength, λ, of the exchanged particle. The solid lines indicate the present limits from the experiments indicated. The excluded regions lie above these solid lines. Measuring gravitational strength forces at such short distances is quite challenging. The most important background is the Van der Walls force which becomes equal to the gravitational force between two atoms when they are about 100 microns apart. Since the Van der Walls force falls off as the 7th power of the distance, it rapidly becomes negligible compared to gravity at distances exceeding 100 μm. The dashed thick lines give the expected sensitivity of the present and upcoming experiments, which will improve the actual limits by roughly two orders of magnitude, while the horizontal dashed lines correspond to the theoretical predictions for the graviton in the case of two large extra dimensions and for the radion in the case of transverse radius.

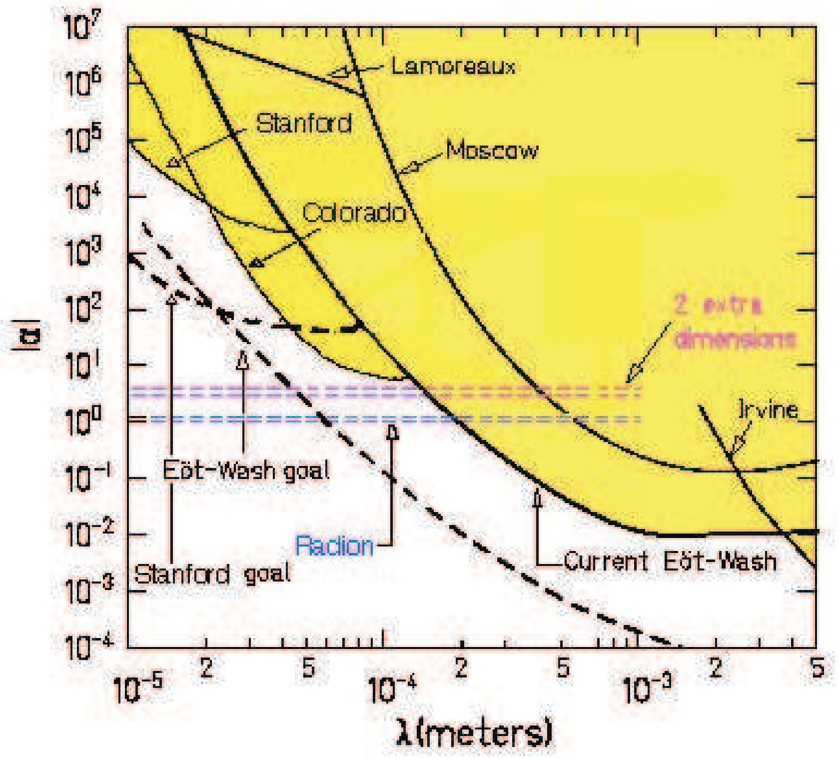


Figure 6: Limits on non-Newtonian forces at short distances, compared to new forces mediated by the graviton in the case of two large extra dimensions, and by the radion.

10 Dimension-eight operators and limits on the string scale

At low energies, the interaction of light (string) states is described by an effective field theory. Non-renormalizable dimension-six operators are due to the exchange of KK excitations of gauge bosons between localized states. If these are absent, then there are deviations to the standard model expectations from dimension-eight operators. There are two generic sources for such operators: exchange of virtual KK excitations of bulk fields (gravitons,...) and form factors due to the extended nature of strings.

The exchange of virtual KK excitations of bulk gravitons is described in the effective field theory by an amplitude involving the sum $\frac{1}{M_P^2}\sum_n \frac{1}{s-\frac{\vec{n}^2}{R_\perp^2}}$. For $n > 1$, this sum diverges and one cannot compute it in field theory but only in a fundamental (string) theory. In analogy with the case of exchange of gauge bosons, one expects the string scale to act as a cut-off with the result:

$$Ag_s^2 \frac{T_{\mu\nu}T^{\mu\nu} - \frac{1}{1+d_\perp} T_\mu^\mu T_\nu^\nu}{M_s^4} \, . \tag{69}$$

The approximation $A = \log \frac{M_s^2}{s}$ for $d_\perp = 2$ and $A = \frac{2}{d_\perp - 2}$ for $d_\perp > 2$ is usually used for quantitative discussions. There are some reasons which might invalidate this approximation in general. In fact, the result is very much model dependent: in type I string models it reflects the ultraviolet behavior of open string one-loop diagrams which are model (compactification) dependent.

In order to understand better this issue, it is important to remind that in type I string models, gravitons and other bulk particles correspond to excitations of closed strings. Their tree-level exchange is described by a cylinder joining the initial $|Bin>$ and final $|Bout>$ closed strings lying on the brane. This cylinder can be be seen on the other hand as an open string with one of its end-points describing the closed (loop) string $|Bin>$, while the other end draws $|Bout>$. In other words, the cylinder can be seen as an annulus which is a one-loop diagram of open strings with boundaries $|Bin>$ and $|Bout>$. Note that usually the theory requires the presence of other weakly interacting closed strings besides gravitons.

More important is that when the gauge degrees of freedom arise from Dirichlet branes, it is expected that the dominant source of dimension-eight operators is not the exchange of KK states but instead the effects of massive open string oscillators [41, 53, 54]. These give rise to contributions to tree-level scatterings that behave as $g_s s/M_s^4$. Thus, they are enhanced by a string-loop factor g_s^{-1} compared to the field theory estimate based on KK graviton exchanges. Although the precise value of g_s requires a detail analysis of threshold corrections, a rough estimate can be obtained by taking $g_s \simeq \alpha \sim 1/25$, implying an enhancement by one order of magnitude, and in any case a loop-factor as consequence of perturbation theory.

What is the simplest thing one could do in practice?. There are some processes for which there is only one allowed dimension-eight operator; an example is $f\bar{f} \to \gamma\gamma$. The coefficient of this operator can then be computed in terms of g_s and M_s. As a result, in the only framework where computation of such operators is possible to carry out, one cannot rely on the effects of exchange of KK graviton excitations in order to derive bounds on extra-dimensions or the string scale. Instead, one can use the dimension-eight operator arising from stringy form-factors.

Under the assumption that the electrons arise as open strings on a $D3$-brane, and not as living on the intersections of different kind of branes, an estimate at the lowest order approximation of string form factor in type I was used in [53]. For instance for $e^+e^- \to \gamma\gamma$ one has:

$$\frac{d\sigma}{d\cos\theta} = \left.\frac{d\sigma}{d\cos\theta}\right|_{SM} \cdot \left[1 + \frac{\pi^2}{12}\frac{ut}{M_S^4} + \cdots\right] \tag{70}$$

while for Bhabha scattering, it was suggested that

$$\frac{d\sigma}{d\cos\theta}(e^-e^+ \to e^-e^+) = \left.\frac{d\sigma}{d\cos\theta}\right|_{SM} \cdot \left|\frac{\Gamma(1-\frac{s}{M_s^2})\Gamma(1-\frac{t}{M_s^2})}{\Gamma(1-\frac{s}{M_s^2}-\frac{t}{M_s^2})}\right|^2 , \tag{71}$$

where s and t are the Mandelstam kinematic variables. Using these estimates, present LEP data lead to limits on the string scale $M_s \gtrsim 1$ TeV. This translates into a stronger bound on the size of transverse dimension than those obtained from missing energy experiments in the cases $d_\perp > 2$.

On the other hand, when matter fields live on brane intersections, the presence of dimension-six operators increase the lower bound on the string scale to 2-3 TeV, independently on the number of large extra dimensions [54].

11 D-brane Standard Model

As we discussed in section 6.2, one of the main questions with such a low string scale is to understand the observed values of the low energy gauge couplings. One possibility is to have the three gauge group factors of the Standard Model arising from different collections of coinciding branes. This is unattractive since the three gauge couplings correspond in this case to different arbitrary parameters of the model. A second possibility is to maintain unification by imposing all the Standard Model gauge bosons to arise from the same collection of D-branes. The large difference in the actual values of gauge couplings could then be explained either by introducing power-law running from a few TeV to the weak scale [30, 31], or by an effective logarithmic evolution in the transverse space in the special case of two large dimensions [32, 33, 34]. However, no satisfactory model built along these lines has so far been presented.

Here, we will discuss a third possibility [55], which is alternative to unification but nevertheless maintains the prediction of the weak angle at low energies. Specifically, we consider the strong and electroweak interactions to arise from two different collections of coinciding branes, leading to two different gauge couplings, [19]. Assuming that the low energy spectrum of the (non-supersymmetric) Standard Model can be derived by a type I/I′ string vacuum, the normalization of the hypercharge is determined in terms of the two gauge couplings and leads naturally to the right value of $\sin^2\theta_W$ for a string scale of the order of a few TeV. The electroweak gauge symmetry is broken by the vacuum expectation values of two Higgs doublets, which are both necessary in the present context to give masses to all quarks and leptons.

Another issue of this class of models with TeV string scale is to understand proton stability. In the model presented here, this is achieved by the conservation of the baryon number which turns out to be a perturbatively exact global symmetry, remnant of an anomalous $U(1)$ gauge symmetry broken by the Green-Schwarz mechanism. Specifically, the anomaly is canceled by shifting a corresponding axion field that gives mass to the $U(1)$ gauge boson. Moreover, the two extra $U(1)$ gauge groups are anomalous and the associated gauge bosons become massive with masses of the order of the string scale. Their couplings to the standard model fields up to dimension five are fixed by charges and anomalies.

11.1 Hypercharge embedding and the weak angle

The gauge group closest to the $SU(3) \times SU(2) \times U(1)$ of the Standard Model one can hope to derive from type I/I$'$ string theory in the above context is $U(3) \times U(2) \times U(1)$. The first factor arises from three coincident D-branes ("color" branes). An open string with one end on them is a triplet under $SU(3)$ and carries the same $U(1)$ charge for all three components. Thus, the $U(1)$ factor of $U(3)$ has to be identified with *gauged* baryon number. Similarly, $U(2)$ arises from two coincident "weak" D-branes and the corresponding abelian factor is identified with *gauged* weak-doublet number. A priori, one might expect that $U(3) \times U(2)$ would be the minimal choice. However it turns out that one cannot give masses to both up and down quarks in that case. Therefore, at least one additional $U(1)$ factor corresponding to an extra D-brane ("$U(1)$" brane) is necessary in order to accommodate the Standard Model. In principle this $U(1)$ brane can be chosen to be independent of the other two collections with its own gauge coupling. To improve the predictability of the model, here we choose to put it on top of either the color or the weak D-branes. In either case, the model has two independent gauge couplings g_3 and g_2 corresponding, respectively, to the gauge groups $U(3)$ and $U(2)$. The $U(1)$ gauge coupling g_1 is equal to either g_3 or g_2.

Let us denote by Q_3, Q_2 and Q_1 the three $U(1)$ charges of $U(3)\times U(2)\times U(1)$, in a self explanatory notation. Under $SU(3)\times SU(2)\times U(1)_3\times U(1)_2\times U(1)_1$, the members of a family of quarks and leptons have the following quantum numbers:

$$\begin{array}{ll} Q & (\mathbf{3},\mathbf{2};1,w,0)_{1/6} \\ u^c & (\bar{\mathbf{3}},\mathbf{1};-1,0,x)_{-2/3} \\ d^c & (\bar{\mathbf{3}},\mathbf{1};-1,0,y)_{1/3} \\ L & (\mathbf{1},\mathbf{2};0,1,z)_{-1/2} \end{array} \qquad (72)$$

$$l^c \quad (\mathbf{1},\mathbf{1};0,0,1)_1$$

Here, we normalize all $U(N)$ generators according to $\mathrm{Tr}T^aT^b = \delta^{ab}/2$, and measure the corresponding $U(1)_N$ charges with respect to the coupling $g_N/\sqrt{2N}$, with g_N the $SU(N)$ coupling constant. Thus, the fundamental representation of $SU(N)$ has $U(1)_N$ charge unity. The values of the $U(1)$ charges x, y, z, w will be fixed below so that they lead to the right hypercharges, shown for completeness as subscripts.

The quark doublet Q corresponds necessarily to a massless excitation of an open string with its two ends on the two different collections of branes. The Q_2 charge w can be either $+1$ or -1 depending on whether Q transforms as a $\mathbf{2}$ or a $\bar{\mathbf{2}}$ under $U(2)$. The antiquark u^c corresponds to fluctuations of an open string with one end on the color branes and the other on the $U(1)$ brane for $x = \pm 1$, or on other branes in the bulk for $x = 0$. Ditto for d^c. Similarly, the lepton doublet L arises from an open string with one end on the weak branes and the other on the $U(1)$ brane for $z = \pm 1$, or in the bulk for $z = 0$. Finally, l^c corresponds necessarily to an open string with one end on the $U(1)$ brane and the other in the bulk. We defined its $Q_1 = 1$.

The weak hypercharge Y is a linear combination of the three $U(1)$'s:[6]

$$Y = c_1 Q_1 + c_2 Q_2 + c_3 Q_3 \,. \tag{73}$$

$c_1 = 1$ is fixed by the charges of l^c in eq. (72), while for the remaining two coefficients and the unknown charges x, y, z, w, we obtain four possibilities:

$$\begin{aligned}
c_2 = -\frac{1}{2}\,,\; c_3 = -\frac{1}{3}\,; &\quad x = -1\,,\; y = 0\,,\; z = 0\,,\; w = -1 \\
c_2 = \frac{1}{2}\,,\; c_3 = -\frac{1}{3}\,; &\quad x = -1\,,\; y = 0\,,\; z = -1\,,\; w = 1 \\
c_2 = -\frac{1}{2}\,,\; c_3 = \frac{2}{3}\,; &\quad x = 0\,,\; y = 1\,,\; z = 0\,,\; w = 1 \\
c_2 = \frac{1}{2}\,,\; c_3 = \frac{2}{3}\,; &\quad x = 0\,,\; y = 1\,,\; z = -1\,,\; w = -1
\end{aligned} \tag{74}$$

Orientifold models realizing the $c_3 = -1/3$ embedding in the supersymmetric case with intermediate string scale $M_s \sim 10^{11}$ GeV have been described in [56].

To compute the weak angle $\sin^2\theta_W$, we use from eq. (73) that the

[6] A study of hypercharge embedding in gauge groups obtained from M-branes was considered in Ref. [57]. In the context of Type I ground states such embeddings were considered in [56].

hypercharge coupling g_Y is given by [7]:

$$\frac{1}{g_Y^2} = \frac{2}{g_1^2} + \frac{4c_2^2}{g_2^2} + \frac{6c_3^2}{g_3^2}\,, \tag{75}$$

with $g_1 = g_2$ or $g_1 = g_3$ at the string scale. On the other hand, with the generator normalizations employed above, the weak $SU(2)$ gauge coupling is g_2. Thus,

$$\sin^2\theta_W \equiv \frac{g_Y^2}{g_2^2 + g_Y^2} = \frac{1}{1 + 4c_2^2 + 2g_2^2/g_1^2 + 6c_3^2 g_2^2/g_3^2}\,, \tag{76}$$

which for $g_1 = g_2$ reduces to:

$$\sin^2\theta_W(M_s) = \frac{1}{4 + 6c_3^2 g_2^2(M_s)/g_3^2(M_s)}\,, \tag{77}$$

while for $g_1 = g_3$ it becomes:

$$\sin^2\theta_W(M_s) = \frac{1}{2 + 2(1+3c_3^2)g_2^2(M_s)/g_3^2(M_s)}\,. \tag{78}$$

We now show that the above predictions agree with the experimental value for $\sin^2\theta_W$ for a string scale in the region of a few TeV. For this comparison, we use the evolution of gauge couplings from the weak scale M_Z as determined by the one-loop beta-functions of the Standard Model with three families of quarks and leptons and one Higgs doublet,

$$\frac{1}{\alpha_i(M_s)} = \frac{1}{\alpha_i(M_Z)} - \frac{b_i}{2\pi}\ln\frac{M_s}{M_Z}\ ; \quad i = 3, 2, Y \tag{79}$$

where $\alpha_i = g_i^2/4\pi$ and $b_3 = -7$, $b_2 = -19/6$, $b_Y = 41/6$. We also use the measured values of the couplings at the Z pole $\alpha_3(M_Z) = 0.118 \pm 0.003$, $\alpha_2(M_Z) = 0.0338$, $\alpha_Y(M_Z) = 0.01014$ (with the errors in $\alpha_{2,Y}$ less than 1%).

In order to compare the theoretical relations for the two cases (77) and (78) with the experimental value of $\sin^2\theta_W = g_Y^2/(g_2^2 + g_Y^2)$ at M_s, we plot in Fig. 1 the corresponding curves as functions of M_s.

The solid line is the experimental curve. The dashed line is the plot of the function (77) for $c_3 = -1/3$ while the dotted-dashed line corresponds to the function (78) for $c_3 = 2/3$. Thus, the second case, where the $U(1)$ brane is on top of the color branes, is compatible with low energy data for $M_s \sim 6 - 8$ TeV and $g_s \simeq 0.9$. This selects the last two possibilities of

[7] The gauge couplings $g_{2,3}$ are determined at the tree-level by the string coupling and other moduli, like radii of longitudinal dimensions. In higher orders, they also receive string threshold corrections.

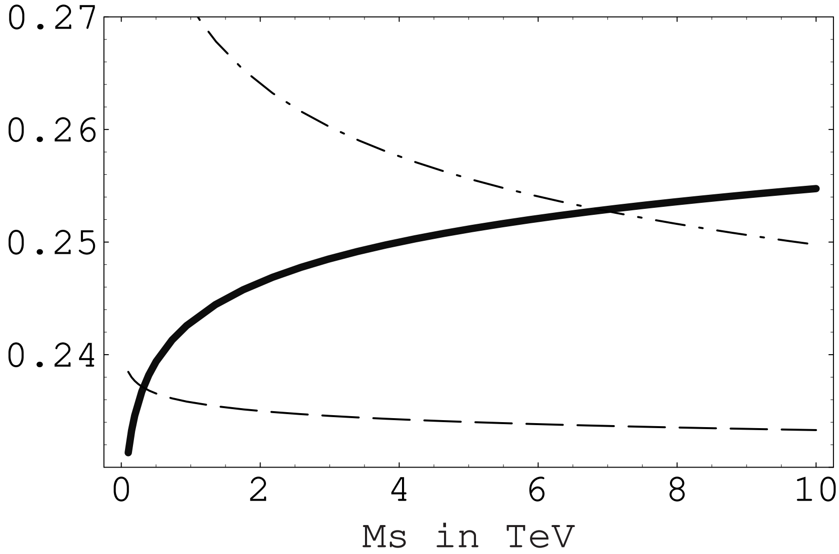


Figure 7: The experimental value of $\sin^2\theta_W$ (thick curve), together with the theoretical predictions (77) with $c_3 = -1/3$ (dashed line) and (78) with $c_3 = 2/3$ (dotted-dashed), are plotted as functions of the string scale M_s.

charge assignments in Eq. (74). Indeed, the curve corresponding to $g_1 = g_3$ and $c_3 = -1/3$ intersects the experimental curve for $\sin^2\theta_W$ at a scale M_s of the order of a few thousand TeV. Since this value appears to be too high to protect the hierarchy, it is less interesting and is not shown in Fig. 1. The other case, where the $U(1)$ brane is on top of the weak branes, is not interesting either. For $c_3 = 2/3$, the corresponding curve does not intersect the experimental one at all and is not shown in the Fig. 1, while the case of $c_3 = -1/3$ leads to M_s of a few hundred GeV and is most likely excluded experimentally. In the sequel we shall restrict ourselves to the last two possibilities of Eq. (74).

From the general solution (74) and the requirement that the Higgs doublet has hypercharge 1/2, one finds the following possible assignments for it, in the notation of Eq. (72):

$$c_2 = -\frac{1}{2} \ : \qquad H \ \ (\mathbf{1},\mathbf{2};0,1,1)_{1/2} \qquad H' \ \ (\mathbf{1},\mathbf{2};0,-1,0)_{1/2} \tag{80}$$

$$c_2 = \frac{1}{2} \ : \qquad \tilde{H} \ \ (\mathbf{1},\mathbf{2};0,-1,1)_{1/2} \qquad \tilde{H}' \ \ (\mathbf{1},\mathbf{2};0,1,0)_{1/2} \tag{81}$$

It is straightforward to check that the allowed (trilinear) Yukawa terms are:

$$c_2 = -\frac{1}{2} \ : \qquad H'Qu^c \ , \quad H^\dagger L l^c \ , \quad H^\dagger Q d^c \tag{82}$$

$$c_2 = \frac{1}{2} \quad : \qquad \tilde{H}' Q u^c \ , \quad \tilde{H}'^{\dagger} L l^c \ , \quad \tilde{H}^{\dagger} Q d^c \tag{83}$$

Thus, two Higgs doublets are in each case necessary and sufficient to give masses to all quarks and leptons. Let us point out that the presence of the second Higgs doublet changes very little the curves of Fig. 1 and consequently our previous conclusions about M_s and $\sin^2\theta_W$.

A few important comments are now in order:

(i) The spectrum we assumed in Eq. (72) does not contain right-handed neutrinos on the branes. They could in principle arise from open strings in the bulk. Their interactions with the particles on the branes would then be suppressed by the large volume of the transverse space [58]. More specifically, conservation of the three U(1) charges allow for the following Yukawa couplings involving the right-handed neutrino ν_R:

$$c_2 = -\frac{1}{2} \ : \quad H' \ L \ \nu_L \qquad ; \qquad c_2 = \frac{1}{2} \ : \quad \tilde{H} \ L \ \nu_R \tag{84}$$

These couplings lead to Dirac type neutrino masses between ν_L from L and the zero mode of ν_R, which is naturally suppressed by the volume of the bulk.

(ii) Implicit in the above was our assumption of three generations (72) of quarks and lepton in the light spectrum. They can arise, for example, from an orbifold action along the lines of the model described in Ref. [56].

(iii) From Eq. (78) and Fig. 1, we find the ratio of the $SU(2)$ and $SU(3)$ gauge couplings at the string scale to be $\alpha_2/\alpha_3 \sim 0.4$. This ratio can be arranged by an appropriate choice of the relevant moduli. For instance, one may choose the color and U(1) branes to be D3 branes while the weak branes to be D7 branes. Then, the ratio of couplings above can be explained by choosing the volume of the four compact dimensions of the seven branes to be $V_4 = 2.5$ in string units. This being larger than one is consistent with the picture above. Moreover it predicts an interesting spectrum of KK states for the Standard model, different from the naive choices that have appeared hitherto: The only Standard Model particles that have KK descendants are the W bosons as well as the hypercharge gauge boson. However since the hypercharge is a linear combination of the three U(1)'s the massive U(1) gauge bosons couple not to hypercharge but to doublet number.

Another possibility would be to move slightly off the orientifold point which may be necessary also for other reasons (see discussion next subsection).

(iv) As we have seen that lepton singlet and the u-quark are generated by strings that must end up in another brane. This brane must also be coincident with the rest, in order for the fermions to be light. This means that

these particles will feel the interactions mediated from the gauge bosons and/or scalars associated with its fluctuations.
(v) Finally, it should be stressed that there are some alternative assignments that may work and these are discussed further in [55].

11.2 The fate of $U(1)$'s and proton stability

The model under discussion has three $U(1)$ gauge interactions corresponding to the generators Q_1, Q_2, Q_3. From the previous analysis, the hypercharge was shown to be either one of the two linear combinations:

$$Y = Q_1 \mp \frac{1}{2}Q_2 + \frac{2}{3}Q_3\,. \tag{85}$$

It is easy to see that the remaining two $U(1)$ combinations orthogonal to Y are anomalous. In particular there are mixed anomalies with the $SU(2)$ and $SU(3)$ gauge groups of the Standard Model.

These anomalies are canceled by two axions coming from the closed string sector, via the standard Green-Schwarz mechanism [59]. The mixed anomalies with the non-anomalous hypercharge are also canceled by dimension five Chern-Simmons type of interactions [55]. The presence of such interactions has so far escaped attention in the context of string theory.

An important property of the above Green-Schwarz anomaly cancellation mechanism is that the two $U(1)$ gauge bosons A and A' acquire masses leaving behind the corresponding global symmetries [59]. This is in contrast to what would had happened in the case of an ordinary Higgs mechanism. These global symmetries remain exact to all orders in type I string perturbation theory around the orientifold vacuum. This follows from the topological nature of Chan-Paton charges in all string amplitudes. On the other hand, one expects non-perturbative violation of global symmetries and consequently exponentially small in the string coupling, as long as the vacuum stays at the orientifold point. Once we move sufficiently far away from it, we expect the violation to become of order unity. So, as long as we stay at the orientifold point, all three charges Q_1, Q_2, Q_3 are conserved and since Q_3 is the baryon number, proton stability is guaranteed.

To break the electroweak symmetry, the Higgs doublets in Eq. (80) or (81) should acquire non-zero VEV's. Since the model is non-supersymmetric, this may be achieved radiatively as we discussed in subsection 6.4 [37]. From Eqs. (82) and (83), to generate masses for all quarks and leptons, it is necessary for both Higgses to get non-zero VEV's. The baryon number conservation remains intact because both Higgses have vanishing Q_3. However, the linear combination which does not contain Q_3, will be broken spontaneously, as follows from their quantum numbers in Eqs. (80) and (81). This leads to an unwanted massless Goldstone boson of the Peccei-Quinn type. The way out is to break this global symmetry explicitly, by moving away from the orientifold point along the direction of the associated modulus so that baryon number remains conserved. Instanton effects in that case will generate the appropriate symmetry breaking couplings in the potential.

In conclusion, we presented a particular embedding of the Standard Model in a non-supersymmetric D-brane configuration of type I/I$'$ string theory. The strong and electroweak couplings are not unified because strong and weak interactions live on different branes. Nevertheless, $\sin^2\theta_W$ is naturally predicted to have the right value for a string scale of the order of a few TeV. The model contains two Higgs doublets needed to give masses to all quarks and leptons, and preserves baryon number as a (perturbatively) exact global symmetry. The model satisfies the main phenomenological requirements for a viable low energy theory and its explicit derivation from string theory deserves further study.

12 Appendix: Supersymmetry breaking in type I strings

12.1 Scherk-Schwarz deformations in type-I strings

Scherk-Schwarz deformations can be introduced in type I strings following [6], but present a few interesting novelties, that may be conveniently exhibited referring to a couple of simple 9D models [60]. To this end, we begin by recalling that, for the type IIB string, (the fermionic part of) the partition function can be written in the compact form

$$\mathcal{T} = |V_8 - S_8|^2 \ , \tag{86}$$

resorting to the level-one SO(8) characters

$$\begin{aligned} O_8 &= \frac{\vartheta_3^4 + \vartheta_4^4}{2\eta^4} \quad , \quad V_8 = \frac{\vartheta_3^4 - \vartheta_4^4}{2\eta^4} \quad , \\ S_8 &= \frac{\vartheta_2^4 - \vartheta_1^4}{2\eta^4} \quad , \quad C_8 = \frac{\vartheta_2^4 + \vartheta_1^4}{2\eta^4} \quad , \end{aligned} \tag{87}$$

where the ϑ are Jacobi theta functions and η is the Dedekind function. In the usual toroidal reduction, where bosons and fermions have the momentum modes

$$p_L = \frac{m}{R} + \frac{nR}{\alpha'} \qquad p_R = \frac{m}{R} - \frac{nR}{\alpha'} \ , \tag{88}$$

the 9D partition function is

$$\mathcal{T} = |V_8 - S_8|^2 \, Z_{mn} \ , \tag{89}$$

where

$$Z_{mn} \equiv \sum_{m,n} \frac{q^{\alpha' p_L^2/4}\ \bar{q}^{\alpha' p_R^2/4}}{\eta\bar{\eta}} \ . \tag{90}$$

A simple modification results in a Scherk-Schwarz breaking of space-time supersymmetry. There are actually two inequivalent choices, described by

$$\begin{aligned} \mathcal{T}_1 &= Z_{m,2n}(V_8\bar{V}_8 + S_8\bar{S}_8) + Z_{m,2n+1}(O_8\bar{O}_8 + C_8\bar{C}_8) \\ &- Z_{m+1/2,2n}(V_8\bar{S}_8 + S_8\bar{V}_8) - Z_{m+1/2,2n+1}(O_8\bar{C}_8 + C_8\bar{O}_8) \end{aligned} \tag{91}$$

and

$$\begin{aligned} \mathcal{T}_2 &= Z_{2m,n}(V_8\bar{V}_8 + S_8\bar{S}_8) + Z_{2m+1,n}(O_8\bar{O}_8 + C_8\bar{C}_8) \\ &- Z_{2m,n+1/2}(V_8\bar{S}_8 + S_8\bar{V}_8) - Z_{2m+1,n+1/2}(O_8\bar{C}_8 + C_8\bar{O}_8) \ , \end{aligned} \tag{92}$$

that may be associated to shifts of the momenta or of the windings of the usual (S_8) fermionic modes relatively to the usual (V_8) bosonic ones. In both cases modular invariance introduces additional sectors, that disappear from the spectrum as the deformation is removed, but the two choices are inequivalent, since T-duality along the circle interchanges type-IIB and type-IIA strings. Both deformed models have tachyon instabilities at the scale of supersymmetry breaking for the low-lying modes, $O(1/R)$ for the momentum deformation of eq. (90) and $O(R/\alpha')$ for the winding deformation of eq. (91).

The open descendants [61] are essentially determined by the choice of Klein-bottle projection $\mathcal{K}$ [62], while the other amplitudes $\mathcal{A}$ and $\mathcal{M}$ reflect the propagation of closed-string modes between boundaries and crosscaps. In displaying the amplitudes of [36], we implicitly confine our attention to internal radii such that (closed-string) tachyon instabilities are absent, and choose Chan-Paton assignments that remove them from the open sectors as well. We also impose some (inessential) NS-NS tadpoles, in order to bring the resulting expressions to their simplest forms.

Starting from the model of eq. (90), corresponding to *momentum shifts*, the additional amplitudes are

$$\begin{aligned} \mathcal{K}_1 &= \frac{1}{2}\,(V_8 - S_8)\ Z_m \ , \\ \mathcal{A}_1 &= \frac{n_1^2 + n_2^2}{2}(V_8 Z_m - S_8 Z_{m+1/2}) + n_1 n_2 (V_8 Z_{m+1/2} - S_8 Z_m) \ , \\ \mathcal{M}_1 &= -\frac{n_1 + n_2}{2}(\hat{V}_8 Z_m - \hat{S}_8 Z_{m+1/2}) \ , \end{aligned} \tag{93}$$

while the tadpole conditions require that $n_1 + n_2 = 32$. Supersymmetry, broken in the whole range $R > \sqrt{\alpha'}$, is recovered asymptotically in the de-compactification limit. This type-I vacuum, first described in [63] and interesting in its own right, describes the type-I string at finite temperature (with Wilson lines), but includes a rather conventional open spectrum, where bosonic and fermionic modes present the usual $O(1/R)$ Scherk-Schwarz splittings of field-theory models.

On the other hand, starting from the model of eq. (91), corresponding to *winding shifts*, the additional amplitudes are [36]

$$\begin{aligned}
\mathcal{K}_2 &= \frac{1}{2}\,(V_8 - S_8)\;Z_{2m} + \frac{1}{2}\,(O_8 - C_8)\;Z_{2m+1}\;, \\
\mathcal{A}_2 &= \Big(\frac{n_1^2+n_2^2+n_3^2+n_4^2}{2}(V_8 - S_8) + (n_1 n_3 + n_2 n_4)(O_8 - C_8)\Big) Z_m \\
&+ \Big((n_1 n_2 + n_3 n_4)(V_8 - S_8) + (n_1 n_4 + n_2 n_3)(O_8 - C_8)\Big) Z_{m+1/2}\;, \\
\mathcal{M}_2 &= -\frac{n_1+n_2+n_3+n_4}{2}\,\hat{V}_8\,Z_m + \frac{n_1-n_2-n_3+n_4}{2}\,\hat{S}_8\,(-1)^m Z_m\;,
\end{aligned} \tag{94}$$

while the tadpole conditions now require that $n_1 + n_2 = n_3 + n_4 = 16$. Supersymmetry is recovered in the limit of vanishing radius R, where the whole tower of winding modes present in the vacuum-channel amplitudes collapses into additional tadpole conditions that eliminate n_2 and n_3. This is precisely the phenomenon of [18], spelled out very clearly by these partition functions. The resulting open sector, described by

$$\begin{aligned}
\mathcal{A}_2 &= \frac{n_1^2+n_4^2}{2}(V_8 - S_8) Z_m + n_1 n_4 (O_8 - C_8) Z_{m+1/2}\;, \\
\mathcal{M}_2 &= -\frac{n_1+n_4}{2}\,\hat{V}_8\,Z_m + \frac{n_1+n_4}{2}\,\hat{S}_8\,(-1)^m Z_m\;,
\end{aligned} \tag{95}$$

has the suggestive gauge group $SO(16)\times SO(16)$, and is rather peculiar. In the limit of small breaking R, aside from the ultra-massive (O, C) sector, it contains a conventional (V, S) sector where supersymmetry, *exact* for the massless modes, is effectively broken *at the string scale* for the massive ones by the unpairing of the corresponding Chan-Paton representations. This is the phenomenon of "brane supersymmetry" [60], here present only for the massless modes. One can then connect, via a sequence of duality transformations, the $SO(16)\times SO(16)$ gauge group to the two Horava-Witten walls [16] of M-theory, with the end result that this peculiar breaking can be associated to an 11D Scherk-Schwarz deformation. We are thus facing a simple perturbative description of a phenomenon whose origin is non-perturbative on the heterotic side. Several generalizations have been discussed, in six and four dimensions, both with partial and with total breaking of supersymmetry [60, 35, 64].

After suitable T-dualities, these results can be put in a very suggestive form: while the conventional Scherk-Schwarz breaking of $\mathcal{T}_1$ results from shifts *parallel* to a brane, the M-theory breaking of $\mathcal{T}_2$ results from shifts *orthogonal* to a brane, and is ineffective on its massless modes.

12.2 Brane supersymmetry breaking

The last phenomenon that we would like to review, "brane supersymmetry breaking" [36], solves an old problem in the construction of open-string models where, in a number of interesting cases, the tadpole conditions have apparently no consistent solution [65]. The simplest example is provided by the six-dimensional $T^4/Z2$ reduction where, as in [62], the Klein-bottle projection is reverted for all twisted states. The resulting projected closed spectrum, described by

$$\begin{aligned} \mathcal{T} &= \frac{1}{2}|Q_o + Q_v|^2\Lambda + \frac{1}{2}|Q_o - Q_v|^2\left|\frac{2\eta}{\theta_2}\right|^4 \\ &+ \frac{1}{2}|Q_s + Q_c|^2\left|\frac{2\eta}{\theta_4}\right|^4 + \frac{1}{2}|Q_sQ_c|^2\left|\frac{2\eta}{\theta_3}\right|^4 , \\ \mathcal{K} &= \frac{1}{4}\left\{(Q_o + Q_v)(P + W) - 2 \times 16(Q_s + Q_c)\right\} , \end{aligned} \tag{96}$$

contains 17 tensor multiplets and 4 hypermultiplets. Turning on a (quantized) NS-NS B_{ab} would lead to similar models with lower numbers tensor multiplets, that may be analyzed in a similar fashion [66]. In writing eq. (96), where Λ is the whole Narain lattice sum, while P and W are its restrictions to only momenta or only windings, we have resorted to the supersymmetric combinations of SO(4) characters

$$\begin{aligned} Q_o &= V_4O_4 - C_4C_4 \quad , \quad Q_v = O_4V_4 - S_4S_4 , \\ Q_s &= O_4C_4 - S_4O_4 \quad , \quad Q_c = V_4S_4 - C_4V_4 . \end{aligned} \tag{97}$$

The reversal of the Klein-bottle projection for the twisted states changes the relative sign of the crosscap contributions for N and D strings or, equivalently, the relative charge of the O5 planes relative to the O9 ones. This is clearly spelled out by the terms at the origin of the lattices,

$$\tilde{\mathcal{K}}_0 = \frac{2^5}{4}\left\{Q_o\left(\sqrt{v} \pm \frac{1}{\sqrt{v}}\right)^2 + Q_v\left(\sqrt{v} \mp \frac{1}{\sqrt{v}}\right)^2\right\} , \tag{98}$$

where the upper signs refer to the standard choice, while the lower ones refer to the reverted Klein bottle of eq. (96). In the latter case one is forced to cancel a *negative* background O5 charge, and this can be achieved introducing antibranes in the vacuum configuration. The corresponding open sector [36]

$$\mathcal{A} = \frac{1}{4}\left\{(Q_o + Q_v)(N^2P + D^2W) + 2ND(Q'_s + Q'_c)\left(\frac{\eta}{\theta_4}\right)^2 \right. \tag{99}$$

$$
\begin{aligned}
&+ \quad (R_N^2 + R_D^2)(Q_o - Q_v)\left(\frac{2\eta}{\theta_2}\right)^2 + 2R_N R_D(-O_4S_4 - C_4O_4 + V_4C_4 + S_4V_4)\left(\frac{\eta}{\theta_3}\right)^2\Bigg\} \\
\mathcal{M} &= -\frac{1}{4}\Big\{NP(\hat{O}_4\hat{V}_4 + \hat{V}_4\hat{O}_4 - \hat{S}_4\hat{S}_4 - \hat{C}_4\hat{C}_4) - DW(\hat{O}_4\hat{V}_4 + \hat{V}_4\hat{O}_4 + \hat{S}_4\hat{S}_4 + \hat{C}_4\hat{C}_4) \\
&- N(\hat{O}_4\hat{V}_4 - \hat{V}_4\hat{O}_4 - \hat{S}_4\hat{S}_4 + \hat{C}_4\hat{C}_4)\left(\frac{2\hat{\eta}}{\hat{\theta}_2}\right)^2 + D(\hat{O}_4\hat{V}_4 - \hat{V}_4\hat{O}_4 + \hat{S}_4\hat{S}_4 - \hat{C}_4\hat{C}_4)\left(\frac{2\hat{\eta}}{\hat{\theta}_2}\right)^2\Bigg\}
\end{aligned}
$$

results from a combination of D9 branes and D5 antibranes. Supersymmetry is broken on the D$\bar{5}$ branes, and indeed the amplitudes involve the new characters Q_s' and Q_c', corresponding to a chirally flipped supercharge, that may be obtained from eq. (97) upon the interchange of S_4 and C_4, as well as other non-supersymmetric combinations. The tadpole conditions determine the gauge group $[SO(16) \times SO(16)]_9 \times [USp(16) \times USp(16)]_5$, and the 99 spectrum is supersymmetric, with (1,0) vector multiplets for the $SO(16) \times SO(16)$ gauge group and a corresponding hypermultiplet in the representations $(\mathbf{16}, \mathbf{16}, \mathbf{1}, \mathbf{1})$. On the other hand, the $\bar{5}\bar{5}$ DD spectrum is non supersymmetric, and contains, aside from the $[USp(16) \times USp(16)]$ gauge vectors, quartets of scalars in $(\mathbf{1}, \mathbf{1}, \mathbf{16}, \mathbf{16})$, right-handed Weyl fermions in $(\mathbf{1}, \mathbf{1}, \mathbf{120}, \mathbf{1})$, $(\mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{120})$ and left-handed Weyl fermions in $(\mathbf{1}, \mathbf{1}, \mathbf{16}, \mathbf{16})$. Finally, the ND sector, also non supersymmetric, comprises doublets of scalars in $(\mathbf{16}, \mathbf{1}, \mathbf{1}, \mathbf{16})$ and in $(\mathbf{1}, \mathbf{16}, \mathbf{16}, \mathbf{1})$, and additional (symplectic) Majorana-Weyl fermions in $(\mathbf{16}, \mathbf{1}, \mathbf{16}, \mathbf{1})$ and $(\mathbf{1}, \mathbf{16}, \mathbf{1}, \mathbf{16})$. These fields are a peculiar feature of six-dimensional space time, where the fundamental Weyl fermion, a spinor of $SU^*(4)$, is psudoreal, and can thus be subjected to a Majorana condition if this is supplemented by the conjugation in a pseudoreal representation. All irreducible gauge and gravitational anomalies cancel, while the residual anomaly polynomial requires a generalized Green-Schwarz mechanism [59] with couplings more general than those found in supersymmetric models.

It should be appreciated that the resulting non-BPS configuration of branes and anti-branes have no tachyonic excitations, while the branes themselves experience no mutual forces. Brane configurations of this type have received some attention lately [67], and form the basis of earlier constructions of non-supersymmetric type I vacua [68] and of their tachyon-free reductions [69]. As a result, the contributions to the vacuum energy, localized on the antibranes, come solely from the Möbius contribution amplitude. The resulting potential, determined by uncancelled NS-NS tadpoles, is

$$
V_{\text{eff}} = c\frac{e^{-\phi_6}}{\sqrt{v}} = ce^{-\phi_{10}} = \frac{c}{g_{\text{YM}}^2}\,, \tag{100}
$$

where ϕ_{10} is the 10D dilaton, that determines the Yang-Mills coupling g_{YM} on the D5 branes, and c is some *positive* numerical constant. This potential (100) is clearly localized on the D5 branes, while the D5 brane contribution to the vacuum energy is positive, consistently with the interpretation of this mechanism as global supersymmetry breaking. One would also expect that, in the limit of vanishing D5 coupling, supersymmetry be recovered, at least from the D9 viewpoint. While not true in six dimensions, due to the peculiar chirality flip that we have described, this expectation is actually realized after compactification to four dimensions, with suitable subgroups of the antibrane gauge group realized as internal symmetries.

Several generalizations of this model have been discussed in [36]. These include the possibility of allowing the simultaneous presence of branes and antibranes of the same type, still in tachyon-free combinations, that extend the construction of [70]. This more general setting has the amusing feature of leading to the effective stabilization of some geometric moduli, while some of the resulting models, related to the Z_3 orientifold of [71], have interesting three-family spectra, of some potential interest for phenomenology.

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