Constructing super-p-brane action

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Abstract

An algorithm of constructing of supersymmetric p-brane actions is presented. It is based on the linear and nonlinear realisation approaches to supersymmetry. Having been proved to be valid when all the supersymmetries are spontaneously broken, the relationship between these approaches is applied when the partial spontaneous supersymmetry along with translation symmetry breaking occurs.

1 Introduction

In this short report we would like to present a method of constructing supersymmetric p-brane action. This talk does not pretend to embrace all aspects of the issue so we omit some details such as those concerning supersymmetry theory concepts. All necessary information for the unfamiliars with supersymmetry may be found, for example, in [1, 2, 3].

A super-p-brane in D-dimensional space–time spontaneously breaks $D-p-1$ translation generators corresponding to the transverse directions of the super-p-brane and one half of supersymmetries. As we know when any symmetry breaks spontaneously a massless Goldstone fermion arises. In addition, when the partial spontaneous breaking of supersymmetry occurs (as it is in the case of super-p-brane) the most adequate way to describe this situation is a nonlinear realisation method [4].

Nonlinear realisation gives model independent description of partial breaking. This means that nonlinear realisation approach gives only general features of theories with spontaneously broken symmetries irrespectively of specific mechanism of the symmetry breaking. In other words, this approach describes only the long-wave limit excitations of theory with

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spontaneously broken symmetries. Having been applied to supersymmetries it gives the possibility to get the action of the super-p-brane in static gauge.

As well as in the case of internal symmetries [5], there is a relationship between the linear and nonlinear approaches when dealing with supersymmetry [6]. We have applied ideas of the paper [6] when there are broken bosonic generators along with supersymmetric ones. Although the validity of this procedure has not been proved rigorously, it seems to be correct.

To demonstrate how the proposed method works we consider two examples – quite simple one and a little bit complicated one – following the algorithm.

1. *Superalgebra*
   Firstly, we should fix the algebra corresponding to the pattern of partial spontaneous supersymmetry breaking we consider. We will not touch such an interesting question as spontaneous breaking of Lorentz symmetry and therefore saying superalgebra we will mean translation and supertranslation sector of super-Poincare algebra.

2. *Realisation*
   Secondly, we have to have a representation on which the supersymmetry algebra can be realised. It is important that this representation involve a superfield which can be identified with the Lagrangian of super-p-brane.

3. *Finite transformations*
   Thirdly, we should construct some quantities, called supersymmetry finite transformations, by successive application of the supersymmetry field transformations and replacing the supersymmetry parameters with corresponding Goldstone superfields of nonlinear realisation.

4. *Constraints*
   Then we should impose covariant constraints putting the constructed finite supersymmetry transformations to be equal to zero.

5. *Solution*
   Finally, we should solve the constraints. It gives us an expression for the Lagrangian of super-p-brane.

The algorithm contains no explanations. They will be given when considering the two examples – $N = 1$ $D = 4$ supermembrane and $N = 1$ $D = 6$ super-3-brane. Let us stress that the section 2 completely follows the paper [7], while the section 3 is original.

2 N=1 D=4 supermebrane

2.1 Superalgebra

The supermembrane (super-2-brane) in $D=4$ spontaneously breaks half of the $N=1$ $D=4$ supersymmetries and $D-p-1=4-2-1=1$ translation. $N=1$ $D=4$ supersymmetry algebra can be thought of as $N=2$ $d=3$ supersymmetry with one central charge (that one can imagine as a translation generator in spontaneously broken direction). And the superalgebra is

$$\{Q_\alpha, Q_\beta\} = P_{\alpha\beta}, \quad \{S_\alpha, S_\beta\} = P_{\alpha\beta}, \quad \{Q_\alpha, S_\beta\} = \varepsilon_{\alpha\beta} Z. \tag{1}$$

All commutators of the superalgebra are equal to zero. Here Q_α, S_α – the supersymmetry generators, $P_{\alpha\beta} = i\sigma^m_{\alpha\beta}\partial_m$ – the momentum operators, Z – the central charge operator, $m = 0, 1, 2$, $\alpha, \beta = 1, 2$.

In what follows we assume that the generators S_α and Z are spontaneously broken while Q_α – not broken.

2.2 Realisation

The pattern of the partial spontaneous breaking that we consider can be realised on chiral $N=1$ $d=3$ superfields φ and ξ_α transforming under the spontaneously broken symmetries, which are S-supersymmetry and Z-translation, as follows

$$\begin{aligned} &\delta\varphi(x,\theta) = i\eta^\alpha \xi_\alpha, \\ &\delta\xi_\alpha(x,\theta) = \eta_\alpha(1 - \frac{i}{2}D^2\varphi) - \frac{i}{2}\eta^\beta \partial_{\alpha\beta}\varphi. \end{aligned} \tag{2}$$

Here η_α – S-supersymmetry transformation parameters, θ^α – Grassmann coordinates corresponding to the unbroken Q-supersymmetry, $D_\alpha = \partial/\partial\theta^\alpha - i/2\theta^\beta\partial_{\alpha\beta}$ – the spinor covariant derivative of the unbroken Q-supersymmetry, $D^2 \equiv D^\alpha D_\alpha$.

The presence of the field-independent term η_α in the transformations (2) shows that S-supersymmetry is spontaneously broken and ξ_α is a Goldstone fermion of linear realisation. One should mention again that the central charge transformation is broken as well. And this is just what we need for the supermembrane in four dimensions.

One of the fields we have at our disposal can be treated as a chiral Lagrangian density of the supermembrane. Really, as clearly explained in [7] it is the superfield φ that satisfies all requirements imposed on Lagrangian density.

2.3 Finite transformations

The next step of our algorithm is the construction of the finite transformations for the fields φ, ξ_α. What does it mean? We should construct the following objects

$$\begin{aligned}\tilde{\varphi} &\equiv \varphi + \delta\varphi + \frac{1}{2!}\delta^2\varphi + \frac{1}{3!}\delta^3\varphi + \dots, \\ \tilde{\xi}_\alpha &\equiv \xi_\alpha + \delta\xi_\alpha + \frac{1}{2!}\delta^2\xi_\alpha + \frac{1}{3!}\delta^3\xi_\alpha + \dots,\end{aligned} \tag{3}$$

where $\delta^2\varphi = \delta(\delta\varphi)$, $\delta^3\varphi = \delta(\delta^2\varphi)$ and replace in these expressions the transformation parameter η_α with the Goldstone fermion $\psi_\alpha(x,\theta)$ of the nonlinear realisation taken with the sign minus: $\eta_\alpha \to -\psi_\alpha$. It seems reasonable to make some comments on this point. First of all despite the dots in the right hand sides of (3) these sequences terminate due to the Grassmann nature of the supersymmetry parameter η_α. So in the case of the supermembrane each of these expressions contain only three terms. Second, as we have already mentioned, there are two approaches to supersymmetry – linear and nonlinear realisation. In the case of spontaneously broken supersymmetry one has Goldstone fermions of each of these approaches. Nonlinear one is more adequate to the pattern of spontaneously broken supersymmetry. And namely the Goldstone fermion of the nonlinear realisation should replace the supersymmetry parameter ξ_α in the finite transformations. This leads to

$$\begin{aligned}&\tilde{\xi}_\alpha(\eta_\alpha \to -\psi_\alpha) = \xi_\alpha - \psi_\alpha(1 - \frac{i}{2}D^2\varphi) + \frac{i}{2}\psi^\beta\partial_{\alpha\beta}\varphi - \frac{1}{4}\psi^2\partial_{\alpha\beta}\xi^\beta, \\ &\tilde{\varphi}(\eta_\alpha \to -\psi_\alpha) = \varphi - i\psi^\alpha\xi_\alpha + \frac{i}{2}\psi^2(1 - \frac{i}{2}D^2\varphi).\end{aligned} \tag{4}$$

2.4 Constraints

Having introduced the Goldstone fermion ψ_α, we increase the number of degrees of freedom. So we should impose constraints to eliminate these degrees. Moreover, even in the linear realisation approach we have to restrict the superfields to get the Lagrangian of the supermembrane. The work [6] gives the answer to this problem stating what constraints should be imposed.

The main point is that $\tilde{\xi}_\alpha(\eta \to -\psi)$ and $\tilde{\varphi}(\eta \to -\psi)$ transform homogeneously so one can impose three covariant constraints

$$\tilde{\xi}_\alpha(\eta \to -\psi) = \tilde{\varphi}(\eta \to -\psi) = 0,$$

that provide us with the same number of degrees of freedom we have in the linear realisation and with the correct expression for the Lagrangian φ.

2.5 Solution to the constraints

So we have the following set of the equations

$$\begin{cases} \xi_\alpha - \psi_\alpha(1 - \frac{i}{2}D^2\varphi) + \frac{i}{2}\psi^\beta\partial_{\alpha\beta} - \frac{1}{4}\psi^2\partial_{\alpha\beta}\xi^\beta = 0, \\ \varphi - i\psi^\alpha\xi_\alpha + \frac{i}{2}\psi^2(1 - \frac{i}{2}D^2\varphi) = 0. \end{cases} \tag{5}$$

The solution to this system gives us the relations between the Goldstone fermions ξ_α and ψ_α of the linear and nonlinear realisations

$$\psi_\alpha = \frac{\xi_\alpha}{1 - \frac{i}{2}D^2\varphi}$$

and the following expression for the Lagrangian in terms of the Goldstone fermions ξ_α of the linear realisation

$$\varphi = \frac{i\xi^\alpha\xi_\alpha}{1 + \sqrt{1 + D^2(\xi^\beta\xi_\beta)}}.$$

So we get the following static gauge action of the $N = 1$ $D = 4$ supermembrane

$$S = \int d^3x d\theta \frac{i\xi^\alpha\xi_\alpha}{1 + \sqrt{1 + D^2(\xi^\beta\xi_\beta)}}. \tag{6}$$

3 N=1 D=6 super-3-brane

The following example is something more sophisticated but the machinery is the same. The main point is what supermultiplet containing Lagrangian density one should take. So again we follow our algorithm.

3.1 Superalgebra

The super-3-brane in $D = 6$ spontaneously breaks one half of the supersymmetries and $D - p - 1 = 6 - 3 - 1 = 2$ translations. From $D = 4$ point of view $N = 1$ $D = 6$ supersymmetry is equivalent to $N = 2$ $D = 4$ supersymmetry with two real central charges (or one complex central charge). The corresponding superalgebra is

$$\begin{array}{ll} \{Q_\alpha, \bar{Q}_{\dot{\alpha}}\} = 2P_{\alpha\dot{\alpha}}, & \{S_\alpha, \bar{S}_{\dot{\alpha}}\} = 2P_{\alpha\dot{\alpha}}, \\ \{Q_\alpha, S_\beta\} = 2\varepsilon_{\alpha\beta}Z, & \{\bar{Q}_{\dot{\alpha}}, \bar{S}_{\dot{\beta}}\} = 2\varepsilon_{\dot{\alpha}\dot{\beta}}\bar{Z}. \end{array} \tag{7}$$

Like in the previous section, Q_α, S_α – supersymmetry generators, Z – the complex central charge generator, $P_{\alpha\dot\alpha} = -i\partial_{\alpha\dot\alpha} = -i\sigma^m_{\alpha\dot\alpha}\partial_m$ – the momentum generator, proportional to the vector derivatives, $\alpha, \dot\alpha = 1, 2$, $m = 0, 1, 2, 3$.

In what follows we will suppose S_α and Z to be spontaneously broken while the others not broken.

3.2 Realisation

It is known [8] that the superalgebra (7) can be realised on the $N = 2$ vector Goldstone supermultiplet containing $N = 1$ superfields $\varphi(x, \theta, \bar\theta)$ and $W_\alpha(x, \theta, \bar\theta)$

$$\begin{aligned} &\delta\varphi = f + \eta^\alpha W_\alpha, \qquad f \equiv c - 2i\eta^\alpha\theta_\alpha, \\ &\delta W_\alpha = -\frac{1}{2}\eta_\alpha \bar D^2\bar\varphi - 2i\bar\eta^{\dot\alpha}\partial_{\alpha\dot\alpha}\varphi, \end{aligned} \tag{8}$$

where φ, W_α are chiral $\bar D_{\dot\alpha}\varphi = 0$, $\bar D_{\dot\alpha}W_\alpha = 0$ and W_α satisfies reality condition

$$D^\alpha W_\alpha + \bar D_{\dot\alpha}\bar W^{\dot\alpha} = 0. \tag{9}$$

In the transformations (8) c – complex parameters corresponding to the Z-transformations, η^α – Grassmann parameter of the S-supersymmetry, θ^α – Grassmann coordinates of the Q-supersymmetry, D_α – covariant spinor derivative of Q-supersymmetry. Let us note that (8) is the transformation of the vector supermultiplet components under the spontaneously broken symmetries only.

There is no superfield that can be treated as the Lagrangian density. However, solving the real condition equation (9) we get at our disposal a real $N = 1$ superfield $\mathcal{L}(x, \theta, \bar\theta)$ which turns out to be the Lagrangian density

$$W_\alpha = -\frac{i}{4}\bar D^2 D_\alpha \mathcal{L}.$$

To provide the realisation of the broken part of the $N = 2$ $D = 4$ supersymmetry on the superfield $\mathcal{L}$ one has to add some additional $N = 1$ chiral superfields F, Y_α and Z_α transforming as follows

$$\begin{aligned} &\delta\mathcal{L} = f\bar\varphi + \bar f\varphi + \eta^\alpha Z_\alpha + \bar\eta_{\dot\alpha}\bar Z^{\dot\alpha}, \\ &\delta F = f\varphi + \eta^\alpha Y_\alpha, \\ &\delta Y_\alpha = fW_\alpha - \frac{1}{2}\eta_\alpha \bar D^2\mathcal{L} - 2i\bar\eta^{\dot\alpha}\partial_{\alpha\dot\alpha}F, \\ &\delta Z_\alpha = \bar f W_\alpha - \frac{1}{2}\eta_\alpha \bar D^2\bar F - \bar\eta^{\dot\alpha}\bar D_{\dot\alpha}D_\alpha\mathcal{L} \end{aligned} \tag{10}$$

and restricted by the condition

$$D^{\alpha}Y_{\alpha}+\bar{D}_{\dot{\alpha}}\bar{Z}^{\dot{\alpha}}=0$$

analogous to the reality condition (9). The set of such superfields φ, W_{α}, $\mathcal{L}$, F, Y_{α}, Z_{α} will be called extended vector supermultiplet.

The transformations (10) uniquely arise when requiring the closure of the supersymmetry transformations on each of the extended vector supermultiplet component. Let us mention that these transformations can be obtained from some infinite dimensional linear realisation of the $N=2$, $D=4$ complex charge extended supersymmetry [9]. This realisation is infinitely reducible and the above written transformations are just its part.

3.3 Finite transformations

The next step is the construction of the finite transformations. As one may note the transformations (8), (10) essentially differ from those for the supermembrane by the presence commuting parameter f containing ordinary c-number corresponding to the central charge transformations. This is somewhat unusual situation that has never been before. We have supposed that this even parameter f should be replaced by a Goldstone boson $Q(x,\theta,\bar{\theta})$ of the nonlinear realisation whereas the odd one η_{α} – by the Goldstone fermion $\psi_{\alpha}(x,\theta,\bar{\theta})$ of the nonlinear realisation. So now we have quite symmetric situation having both Goldstone superfields of the nonlinear realisation in consideration.

Each of the finite transformation series terminates having only 6 terms

$$\tilde{\varphi}=\varphi+\delta\varphi+\frac{1}{2!}\delta^{2}\varphi+\frac{1}{3!}\delta^{3}\varphi+\frac{1}{4!}\delta^{4}\varphi+\frac{1}{5!}\delta^{5}\varphi$$

(and analogous expressions for the other superfields). Replacing the parameters with the Goldstone superfields of the nonlinear realisation

$$f\to-Q,\qquad \eta^{\alpha}\to-\psi^{\alpha}$$

we get the finite transformations. Due to the cumbersome form of these expressions we write them down here only up to quadratic terms

$$
\begin{aligned}
&\tilde{\varphi} = \varphi - Q - \psi^\alpha W_\alpha + \dots, \\
&\tilde{W_\alpha} = W_\alpha + \frac{1}{2}\psi_\alpha \bar{D}^2\bar{\varphi} - 2i\bar{\psi}^{\dot{\alpha}}\partial_{\alpha\dot{\alpha}}\varphi + \dots, \\
&\widetilde{D_\alpha\varphi} = D_\alpha\varphi + 2i\psi_\alpha - \psi^\mu D_\alpha W_\mu + \dots, \\
&\tilde{F} = F + \frac{1}{2}Q^2 - \psi^\alpha Y_\alpha - Q\varphi + \dots, \\
&\tilde{Y_\alpha} = Y_\alpha - QW_\alpha + \frac{1}{2}\psi_\alpha\bar{D}^2\bar{\varphi} + \frac{1}{2}\psi_\alpha\bar{D}^2\mathcal{L} + 2i\bar{\psi}^{\dot{\alpha}}\partial_{\alpha\dot{\alpha}}F + \dots, \\
&\tilde{\mathcal{L}} = \mathcal{L} - Q\bar{\varphi} - \bar{Q}\varphi + Q\bar{Q} - \psi^\alpha Z_\alpha - \bar{\psi}_{\dot{\alpha}}\bar{Z}^{\dot{\alpha}} + \dots, \\
&\tilde{Z_\alpha} = Z_\alpha - \bar{Q}W_\alpha + \frac{1}{2}\psi_\alpha\bar{D}^2\bar{F} + \bar{\psi}^{\dot{\alpha}}\bar{D}_{\dot{\alpha}}D_\alpha\mathcal{L} + \dots .
\end{aligned}
\tag{11}
$$

3.4 Constraints and their solutions

The next step is to put the finite transformations (11) to be equal to zero

$$\tilde{\varphi} = \tilde{W_\alpha} = \widetilde{D_\alpha\varphi} = \tilde{F} = \tilde{Y_\alpha} = \tilde{\mathcal{L}} = \tilde{Z_\alpha} = 0$$

and to solve these constraints. It can be solved by iterations. By the moment we have succeeded to find the solution to these constraints up to the 4^{th} order of the superfields φ. The expression for the Lagrangian of the super-3-brane has the form

$$\mathcal{L} = \varphi\bar{\varphi} + \frac{1}{6}(D\varphi)^2(\bar{D}\bar{\varphi})^2 + o(\varphi^6) \tag{12}$$

while the Goldstone fermions of the linear and nonlinear realisations are connected as follows

$$\psi_\alpha = \frac{i}{2}D_\alpha\varphi + o(\varphi^3).$$

The expression (12) is consistent with the exact Lagrangian density of super-3-brane in $D = 6$ derived in [8] using vector–tensor multiplet duality. Let us note that the same exact expression has been also obtained algorithmically in our recent work [9] starting from the above mentioned infinite-dimensional linear representation of the $N = 2$ $D = 4$ supersymmetry.

4 Conclusion

To summarise we would like to say that proposed method is a generalisation of the known one. Unlike the known method the generalisation is also valid for the transformations containing translation parameter of central charge transformations. It allows us to get effective static gauge actions of objects corresponding to the pattern when part of supersymmetries and translation symmetries are broken down. Unfortunately, this method has not been considered whether it could be applied if the Lorentz part of the super-Poincare group is taken into account. Moreover, a very interesting question of κ-symmetry has yet to be gone over.

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