FINITE UNIFICATION AT ALL-LOOPS

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Abstract

Finite Unified Theories (FUTs) are very interesting $N = 1$ supersymmetric GUTs in which a complete reduction of couplings has been achieved *at all-loops*, both in the Yukawa and gauge coupling and soft supersymmetry breaking sector. Because of the great reduction of free parameters, these theories have a remarkable predictive power. Reduction of dimensionless couplings in $N = 1$ GUTs is achieved by searching for renormalization group invariant (RGI) relations among them holding beyond the unification scale. Finiteness results from the fact that there exists RGI relations among dimensionless couplings that guarantee the vanishing of the β-functions in certain $N = 1$ GUTs even to all orders. Recent developments in the soft supersymmetry breaking (SSB) sector lead to exact RGI relations also in this sector of the theories. Of particular interest is a RGI sum rule for the soft scalar masses holding to all-orders. The characteristic features of the finite SU(5) models that have been constructed based on the above tools are: a) the old agreement of the top quark mass prediction with the measured value remains unchanged, b) the lightest Higgs boson is predicted to be around 111-130 GeV, c) the s - spectrum starts above several hundreds of GeV.

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1 Introduction

In the last years there has been a large and sustained effort from the theoretical particle physicists to provide with a unified description of all interactions. Theories based on superstrings, non-commutative geometry and quantum groups, although at a different stage of development in each area, have common unification targets and share similar hopes for exhibiting improved renormalization properties in the ultraviolet as compared to ordinary field theories. Moreover, recent progress shows that all above theoretical endeavours could be related and thus they might be understood in a unified manner too. However despite the importance of having frameworks to discuss quantum gravity in a self consistent way, the main goal expected from a unified description of interactions by the particle physics community is to understand the present day large number of free parameters of the Standard Model (SM) in terms of a few fundamental ones. In other words: to achieve *reduction of couplings* at a more fundamental level.

In our recent studies [1]-[7], [14] we have developed a complementary strategy in searching for a more fundamental theory possibly at the Planck scale, whose basic ingredients are GUTs and supersymmetry, but its consequences certainly go beyond the known ones. Our method consists of hunting for renormalization group invariant (RGI) relations holding below the Planck scale, which in turn are preserved down to the GUT scale. This programme, called Gauge–Yukawa unification scheme, applied in the dimensionless couplings of supersymmetric GUTs, such as gauge and Yukawa couplings, had already noticable successes by predicting correctly, among others, the top quark mass in the finite SU(5) GUTs. An impressive aspect of the RGI relations is that one can guarantee their validity to all-orders in perturbation theory by studying the uniqueness of the resulting relations at one-loop, as was proven in the early days of the programme of *reduction of couplings* [15]. Even more remarkable is the fact that it is possible to find RGI relations among couplings that guarantee finiteness to all-orders in perturbation theory [16, 18].

Although supersymmetry seems to be an essential feature for a successful realization of the above programme, its breaking has to be understood too, since it has the ambition to supply the SM with predictions for several of its free parameters. Indeed, the search for RGI relations has been extended to the soft supersymmetry breaking sector (SSB) of these theories [5, 19], which involves parameters of dimension one and two. More recently a very interesting progress has been made [20]-[25] concerning the renormalization properties of the SSB parameters based conceptually and technically on the work of ref. [49]. In ref. [49] the powerful supergraph method [50] for studying supersymmetric theories has been applied to the softly broken ones by using the "spurion" external space-time independent superfields [51]. In the latter method a softly broken supersymmetric gauge theory is considered as a supersymmetric one in which the various parameters such as couplings and masses have been promoted to external superfields that acquire "vacuum expectation values". Based on this method the relations among the soft term renormalization and that of an unbroken supersymmetric theory have been derived. In particular the β-functions of the parameters of the softly broken theory are expressed in terms of partial differential operators involving the dimensionless parameters of the unbroken theory. The key point in the strategy of refs. [23]-[25] in solving the set of coupled differential equations so as to be able to express all parameters in a RGI way, was to transform the partial differential operators involved to total derivative operators. This is indeed possible to be done on the RGI surface which is defined by the solution of the reduction equations.

On the phenomenological side there exist some serious developments too. Previously an appealing "universal" set of soft scalar masses was assumed in the SSB sector of supersymmetric theories, given that apart from economy and simplicity (1) they are part of the constraints that preserve finiteness up to two-loops [12, 13], (2) they are RGI up to two-loops in more general supersymmetric gauge theories, subject to the condition known as $P = 1/3\ Q$ [19] and (3) they appear in the attractive dilaton dominated supersymmetry breaking superstring scenarios [54]. However, further studies have exhibited a number of problems all due to the restrictive nature of the "universality" assumption for the soft scalar masses. For instance (a) in finite unified theories the universality predicts that the lightest supersymmetric particle is a charged particle, namely the superpartner of the τ lepton $\tilde{\tau}$ (b) the MSSM with universal soft scalar masses is inconsistent with the attractive radiative electroweak symmetry breaking [57] and (c) which is the worst of all, the universal soft scalar masses lead to charge and/or colour breaking minima deeper than the standard vacuum [58]. Therefore, there have been attempts to relax this constraint without loosing its attractive features. First an interesting observation was made that in $N = 1$ Gauge–Yukawa unified theories there exists a RGI sum rule for the soft scalar masses at lower orders; at one-loop for the non-finite case [6] and at two-loops for the finite case [7]. The sum rule manages to overcome the above unpleasant phenomenological consequences. Moreover it was proven [25] that the sum rule for the soft scalar massses is RGI to all-orders for both the general as well as for the finite case. Finally the exact β-function for the soft scalar masses in the Novikov-Shifman-Vainstein-Zakharov (NSVZ) scheme [38] for the softly broken supersymmetric QCD has been obtained [25]. Armed with the above tools and results we are in a position to study the spectrum of the full finite SU(5) models in terms of few free parameters with emphasis on the predictions for the masses of the lightest Higgs and LSP and on the constraints imposed by having a large $\tan\beta$.

2 The Reduction of Couplings Method

Let us outline briefly the idea of reduction of couplings. Any RGI relation among couplings (which does not depend on the renormalization scale μ explicitly) can be expressed, in the implicit form $\Phi(g_1, \cdots, g_A) = \text{const.}$, which has to satisfy the partial differential equation (PDE)

$$\mu \frac{d\Phi}{d\mu} = \vec{\nabla} \cdot \vec{\beta} = \sum_{a=1}^{A} \beta_a \frac{\partial \Phi}{\partial g_a} = 0 , \qquad (1)$$

where β_a is the β-function of g_a. This PDE is equivalent to a set of ordinary differential equations, the so-called reduction equations (REs) [15],

$$\beta_g \frac{dg_a}{dg} = \beta_a , \; a = 1, \cdots, A , \qquad (2)$$

where g and β_g are the primary coupling and its β-function, and the counting on a does not include g. Since maximally $(A-1)$ independent RGI "constraints" in the A-dimensional space of couplings can be imposed by the Φ_a's, one could in principle express all the couplings in terms of a single coupling g. The strongest requirement is to demand power series solutions to the REs,

$$g_a = \sum_{n=0} \rho_a^{(n)} g^{2n+1} , \qquad (3)$$

which formally preserve perturbative renormalizability. Remarkably, the uniqueness of such power series solutions can be decided already at the one-loop level [15]. To illustrate this, let us assume that the β-functions have the form

$$\begin{aligned} \beta_a &= \frac{1}{16\pi^2}[\sum_{b,c,d\neq g} \beta_a^{(1)\,bcd} g_b g_c g_d + \sum_{b\neq g} \beta_a^{(1)\,b} g_b g^2] + \cdots , \\ \beta_g &= \frac{1}{16\pi^2} \beta_g^{(1)} g^3 + \cdots , \end{aligned} \qquad (4)$$

where $\cdots$ stands for higher order terms, and $\beta_a^{(1)\,bcd}$'s are symmetric in b, c, d. We then assume that the $\rho_a^{(n)}$'s with $n \le r$ have been uniquely determined. To obtain $\rho_a^{(r+1)}$'s, we insert the power series (3) into the REs (2) and collect terms of $O(g^{2r+3})$ and find

$$\sum_{d \ne g} M(r)_a^d \, \rho_d^{(r+1)} = \text{lower order quantities} \ , \tag{5}$$

where the r.h.s. is known by assumption, and

$$\begin{aligned} M(r)_a^d &= 3 \sum_{b,c \ne g} \beta_a^{(1)\,bcd} \, \rho_b^{(1)} \, \rho_c^{(1)} + \beta_a^{(1)\,d} - (2r+1) \, \beta_g^{(1)} \, \delta_a^d \ , & (6) \\ 0 &= \sum_{b,c,d \ne g} \beta_a^{(1)\,bcd} \, \rho_b^{(1)} \, \rho_c^{(1)} \, \rho_d^{(1)} + \sum_{d \ne g} \beta_a^{(1)\,d} \, \rho_d^{(1)} - \beta_g^{(1)} \, \rho_a^{(1)} \ . & (7) \end{aligned}$$

Therefore, the $\rho_a^{(n)}$'s for all $n > 1$ for a given set of $\rho_a^{(1)}$'s can be uniquely determined if $\det M(n)_a^d \ne 0$ for all $n \ge 0$.

Our experience examining specific examples has taught us that the various couplings in supersymmetric theories have easily the same asymptotic behaviour. Therefore searching for a power series solution of the form (3) to the REs (2) is justified and moreover, one can rely that keeping only the first terms a good approximation is obtained in realistic applications.

An Instructive Example

Now that we have established the tools, let us illustrate an interesting example. Consider a gauge theory based on the gauge group $SU(N)$. The matter content is as follows: $\phi^i(\mathbf{N})$ and $\hat{\phi}_i(\overline{\mathbf{N}})$ are complex scalars, $\psi^i(\mathbf{N})$ and $\hat{\psi}_i(\overline{\mathbf{N}})$ are left-handed Weyl spinor, and $\lambda^a (a = 1, \ldots, N^2 - 1)$ is a right-handed Weyl spinor in the adjoint representation of $SU(N)$.

The Lagrangian is:

$$\begin{aligned} \mathcal{L} &= -\frac{1}{4} F_{\mu\nu}^a F^{a\,\mu\nu} + i\sqrt{2} [\ g_Y \overline{\psi} \lambda^a T^a \phi - \hat{g}_Y \overline{\hat{\psi}} \lambda^a T^a \hat{\phi} + \text{h.c.}\ ] - \\ & \quad V(\phi, \overline{\phi}) \ , & (8) \\ V(\phi, \overline{\phi}) &= \frac{1}{4} \lambda_1 (\phi^i \phi_i^*)^2 + \frac{1}{4} \lambda_2 (\hat{\phi}_i \hat{\phi}^{*\,i})^2 + \lambda_3 (\phi^i \phi_i^*)(\hat{\phi}_j \hat{\phi}^{*\,j}) + \\ & \quad \lambda_4 (\phi^i \phi_j^*)(\hat{\phi}_i \hat{\phi}^{*\,j}) \ , & (9) \end{aligned}$$

which is the most general renormalizable form of dimension four, consistent with the $SU(N) \times SU(N)$ global symmetry.

We find the following β-functions of the couplings of the theory

$$
\begin{aligned}
\beta^{(1)}(g) &= (1-3N)g^3 \ , \\
\beta^{(1)}(g_Y) &= g_Y[\,-3(N+\frac{N^2-1}{2N})\,g^2+(\frac{1}{2}+\frac{3}{2}\frac{N^2-1}{N})\,g_Y^2+\frac{1}{2}\hat{g}_Y^2\ ]\ , \\
\beta^{(1)}(\hat{g}_Y) &= \hat{g}_Y[\,-3(N+\frac{N^2-1}{2N})\,g^2+(\frac{1}{2}+\frac{3}{2}\frac{N^2-1}{N})\,\hat{g}_Y^2+\frac{1}{2}g_Y^2\ ]\ , \\
\beta^{(1)}(\lambda_1) &= (4+N)\lambda_1^2+4\lambda_4^2+4N\lambda_3^2+8\lambda_3\lambda_4+3(N+1-\frac{4}{N}+\frac{2}{N^2})g^4 \\
&\quad -8(N-\frac{2}{N}+\frac{1}{N^2})g_Y^4+\frac{N^2-1}{N}\lambda_1(4g_Y^2-6g^2)\ , \qquad (10) \\
\beta^{(1)}(\lambda_2) &= (4+N)\lambda_2^2+4\lambda_4^2+4N\lambda_3^2+8\lambda_3\lambda_4+3(N+1-\frac{4}{N}+\frac{2}{N^2})g^4 \\
&\quad -8(N-\frac{2}{N}+\frac{1}{N^2})\hat{g}_Y^4+\frac{N^2-1}{N}\lambda_2(4\hat{g}_Y^2-6g^2)\ , \qquad (11) \\
\beta^{(1)}(\lambda_3) &= 4\lambda_3^2+2\lambda_4^2+(1+N)\lambda_3(\lambda_1+\lambda_2)+\lambda_4(\lambda_1+\lambda_2) \\
&\quad +\frac{3}{2}(1+\frac{2}{N^2})g^4-4(1+\frac{1}{N^2})g_Y^2\hat{g}_Y^2 \\
&\quad +\frac{N^2-1}{N}\lambda_3(2g_Y^2+2\hat{g}_Y^2-6g^2)\ , \qquad (12) \\
\beta^{(1)}(\lambda_4) &= 2N\lambda_4^2+8\lambda_3\lambda_4+\lambda_4(\lambda_1+\lambda_2)+\frac{3}{2}(N-\frac{4}{N})g^4 \\
&\quad +8\frac{1}{N}g_Y^2\hat{g}_Y^2+\frac{N^2-1}{N}\lambda_4(2g_Y^2+2\hat{g}_Y^2-6g^2)\ . \qquad (13)
\end{aligned}
$$

Searching for a power series solution (3) to the REs (2), we find in lowest order the following one:

$$
g_Y=\hat{g}_Y=g\ ,\ \lambda_1=\lambda_2=\frac{N-1}{N}g^2\ ,\ \lambda_3=\frac{1}{2N}g^2\ ,\ \lambda_4=-\frac{1}{2}g^2\ , \qquad (14)
$$

which corresponds to an $N=1$ supersymmetric gauge theory. Besides this interesting symmetric solution, the reduction equations have also non-symmetric solutions [30]. It has been checked that in both kinds of solutions the coefficients in the power series can be uniquely determined according to the above described theorem [15]. However both kind of solutions have been examined for several but specific values of N.

3 Reduction of Couplings and Finiteness in $N = 1$ SUSY Gauge Theories

Consider a chiral, anomaly free, $N = 1$ globally supersymmetric gauge theory based on a group G with gauge coupling constant g. The superpotential of the theory is given by

$$W = \frac{1}{2}\, m_{ij}\, \phi_i\, \phi_j + \frac{1}{6}\, C_{ijk}\, \phi_i\, \phi_j\, \phi_k \ , \tag{15}$$

where m_{ij} and C_{ijk} are gauge invariant tensors and the matter field ϕ_i transforms according to the irreducible representation R_i of the gauge group G. The renormalization constants associated with the superpotential (15), assuming that supersymmetry is preserved, are

$$\phi_i^0 = (Z_i^j)^{(1/2)}\, \phi_j \ , \tag{16}$$

$$m_{ij}^0 = Z_{ij}^{i'j'}\, m_{i'j'} \ , \tag{17}$$

$$C_{ijk}^0 = Z_{ijk}^{i'j'k'}\, C_{i'j'k'} \ . \tag{18}$$

The $N = 1$ non-renormalization theorem [10] ensures that there are no mass and cubic-interaction-term infinities and therefore

$$\begin{aligned} Z_{ijk}^{i'j'k'}\, Z_{i'}^{1/2\,i''}\, Z_{j'}^{1/2\,j''}\, Z_{k'}^{1/2\,k''} &= \delta_{(i}^{i''}\, \delta_j^{j''}\, \delta_{k)}^{k''} \ , \\ Z_{ij}^{i'j'}\, Z_{i'}^{1/2\,i''}\, Z_{j'}^{1/2\,j''} &= \delta_{(i}^{i''}\, \delta_{j)}^{j''} \ . \end{aligned} \tag{19}$$

As a result the only surviving possible infinities are the wave-function renormalization constants Z_i^j, i.e., one infinity for each field. The one-loop β-function of the gauge coupling g is given by [11]

$$\beta_g^{(1)} = \frac{dg}{dt} = \frac{g^3}{16\pi^2}\, [\sum_i l(R_i) - 3\, C_2(G)\,] \ , \tag{20}$$

where $l(R_i)$ is the Dynkin index of R_i and $C_2(G)$ is the quadratic Casimir of the adjoint representation of the gauge group G. The β-functions of C_{ijk}, by virtue of the non-renormalization theorem, are related to the anomalous dimension matrix γ_{ij} of the matter fields ϕ_i as:

$$\beta_{ijk} = \frac{dC_{ijk}}{dt} = C_{ijl}\, \gamma_k^l + C_{ikl}\, \gamma_j^l + C_{jkl}\, \gamma_i^l \ . \tag{21}$$

At one-loop level γ_{ij} is [11]

$$\gamma_{ij}^{(1)} = \frac{1}{32\pi^2}\, [\, C^{ikl}\, C_{jkl} - 2\, g^2\, C_2(R_i)\delta_{ij}\,], \tag{22}$$

where $C_2(R_i)$ is the quadratic Casimir of the representation R_i, and $C^{ijk} = C^*_{ijk}$. According to the discussion in Chapter 3, the non-renormalization theorem ensures there are no extra mass and cubic-interaction-term renormalizations, implying that the β-functions of C_{ijk} can be expressed as linear combinations of the anomalous dimensions γ_{ij} of ϕ^i. Therefore, all the one-loop β-functions of the theory vanish if $\beta_g^{(1)}$ and $\gamma_{ij}^{(1)}$, given in Eqs. (20) and (22) respectively, vanish, i.e.

$$\sum_i \ell(R_i) = 3C_2(G)\,, \tag{23}$$

$$C^{ikl}C_{jkl} = 2\delta^i_j g^2 C_2(R_i)\,, \tag{24}$$

A very interesting result is that the conditions (23,24) are necessary and sufficient for finiteness at the two-loop level [11].

In case supersymmetry is broken by soft terms, one-loop finiteness of the soft sector imposes further constraints on it [12]. In addition, the same set of conditions that are sufficient for one-loop finiteness of the soft breaking terms render the soft sector of they theory two-loop finite [13].

The one- and two-loop finiteness conditions (23,24) restrict considerably the possible choices of the irreps. R_i for a given group G as well as the Yukawa couplings in the superpotential (15). Note in particular that the finiteness conditions cannot be applied to the supersymmetric standard model (SSM), since the presence of a $U(1)$ gauge group is incompatible with the condition (23), due to $C_2[U(1)] = 0$. This naturally leads to the expectation that finiteness should be attained at the grand unified level only, the SSM being just the corresponding, low-energy, effective theory.

Another important consequence of one- and two-loop finiteness is that supersymmetry (most probably) can only be broken by soft breaking terms. Indeed, due to the unacceptability of gauge singlets, F-type spontaneous symmetry breaking [8] terms are incompatible with finiteness, as well as D-type [9] spontaneous breaking which requires the existence of a $U(1)$ gauge group.

A natural question to ask is what happens at higher loop orders. The answer is contained in a theorem [16] which states the necessary and sufficient conditions to achieve finiteness at all orders. Before we discuss the theorem let us make some introductory remarks. The finiteness conditions impose relations between gauge and Yukawa couplings. To require such relations which render the couplings mutually dependent at a given renormalization point is trivial. What is not trivial is to guarantee that relations leading to a reduction of the couplings hold at any renormalization point.

As we have seen, the necessary, but also sufficient, condition for this to happen is to require that such relations are solutions to the REs (2) and hold at all orders. As we have seen, remarkably the existence of all-order solutions to (2) can be decided at the one-loop level.

3.1 The all-loop finiteness theorem

Let us now turn to the all-order finiteness theorem [16], which states when a $N = 1$ supersymmetric gauge theory can become finite to all orders in the sense of vanishing β-functions, that is of physical scale invariance. It is based on (a) the structure of the supercurrent in $N = 1$ SYM [32, 33, 34], and on (b) the non-renormalization properties of $N = 1$ chiral anomalies [16, 17]. Details on the proof can be found in refs. [16, 31]. Here, following mostly ref. [31] we present a comprehensible sketch of the proof.

Consider a $N = 1$ supersymmetric gauge theory, with simple Lie group G. The content of this theory is given at the classical level by the matter supermultiplets S_i, which contain a scalar field ϕ_i and a Weyl spinor ψ_{ia}, and the vector superfields V_a, which contain a gauge vector field A^a_μ and a gaugino Weyl spinor λ^a_α.

Let us first recall certain facts about the theory:
(1) A massless $N = 1$ supersymmetric theory is invariant under a $U(1)$ chiral transformation R under which the various fields transform as follows

$$A'_\mu = A_\mu, \quad \lambda'_\alpha = \exp(-i\theta)\lambda_\alpha \quad \phi' = \exp(-i\frac{2}{3}\theta)\phi, \quad \psi'_\alpha = \exp(-i\frac{1}{3}\theta)\psi_\alpha, \quad \cdots \tag{25}$$

The corresponding axial Noether current $J^\mu_R(x)$ is

$$J^\mu_R(x) = \bar{\lambda}\gamma^\mu\gamma^5\lambda + \cdots \tag{26}$$

is conserved classically, while in the quantum case is violated by the axial anomaly

$$\partial_\mu J^\mu_R = r(\epsilon^{\mu\nu\sigma\rho}F_{\mu\nu}F_{\sigma\rho} + \cdots). \tag{27}$$

From its known topological origin in ordinary gauge theories [35], one would expect that the axial vector current J^μ_R to satisfy the Adler-Bardeen theorem [36] and receive corrections only at the one-loop level. Indeed it has been shown that the same non-renormalization theorem holds also in supersymmetric theories [17]. Therefore

$$r = \hbar\beta^{(1)}_g. \tag{28}$$

(2) The massless theory we consider is scale invariant at the classical level and, in general, there is a scale anomaly due to radiative corrections. The

scale anomaly appears in the trace of the energy momentum tensor $T_{\mu\nu}$, which is traceless classically. It has the form

$$T^{\mu}_{\mu} \quad = \quad \beta_g F^{\mu\nu} F_{\mu\nu} + \cdots \tag{29}$$

(3) Massless, $N=1$ supersymmetric gauge theories are classically invariant under the supersymmetric extension of the conformal group – the superconformal group. Examining the superconformal algebra, it can be seen that the subset of superconformal transformations consisting of translations, supersymmetry transformations, and axial R-transformations is closed under supersymmetry, i.e. these transformations form a representation of supersymmetry. It follows that the conserved currents corresponding to these transformations make up a supermultiplet represented by an axial vector superfield called supercurrent [32] J,

$$J \equiv \{J'^{\mu}_{R},\ Q^{\mu}_{\alpha},\ T^{\mu}_{\nu}, ...\}, \tag{30}$$

where J'^{μ}_{R} is the current associated to R invariance, Q^{μ}_{α} is the one associated to supersymmetry invariance, and T^{μ}_{ν} the one associated to translational invariance (energy-momentum tensor).

The anomalies of the R-current J'^{μ}_{R}, the trace anomalies of the supersymmetry current, and the energy-momentum tensor, form also a second supermultiplet, called the supertrace anomaly

$$\begin{aligned} S \quad &= \{Re\ S,\ Im\ S,\ S_{\alpha}\} = \\ &\{T^{\mu}_{\mu},\ \partial_{\mu} J'^{\mu}_{R},\ \sigma^{\mu}_{\alpha\dot{\beta}} \bar{Q}^{\dot{\beta}}_{\mu}\ +\ \cdots\} \end{aligned} \tag{31}$$

where T^{μ}_{μ} in eq.(29) and

$$\partial_{\mu} J'^{\mu}_{R} \quad = \quad \beta_g \epsilon^{\mu\nu\sigma\rho} F_{\mu\nu} F_{\sigma\rho} + \cdots \tag{32}$$

$$\sigma^{\mu}_{\alpha\dot{\beta}} \bar{Q}^{\dot{\beta}}_{\mu} \quad = \quad \beta_g \lambda^{\beta} \sigma^{\mu\nu}_{\alpha\beta} F_{\mu\nu} + \cdots \tag{33}$$

(4) It is very important to note that the Noether current defined in (26) is not the same as the current associated to R-invariance that appears in the supermultiplet J in (30), but they coincide in the tree approximation. So starting from a unique classical Noether current $J^{\mu}_{R(class)}$, the Noether current J^{μ}_{R} is defined as the quantum extension of $J^{\mu}_{R(class)}$ which allows for the validity of the non-renormalization theorem. On the other hand J'^{μ}_{R}, is defined to belong to the supercurrent J, together with the energy-momentum tensor. The two requirements cannot be fulfilled by a single current operator at the same time.

Although the Noether current J^μ_R which obeys (27) and the current J'^μ_R belonging to the supercurrent multiplet J are not the same, there is a relation [16] between quantities associated with them

$$r = \beta_g(1 + x_g) + \beta_{ijk}x^{ijk} - \gamma_A r^A \ , \tag{34}$$

where r was given in eq. (28). The r^A are the non-renormalized coefficients of the anomalies of the Noether currents associated to the chiral invariances of the superpotential, and –like r– are strictly one-loop quantities. The γ_A's are linear combinations of the anomalous dimensions of the matter fields, and x_g, and x^{ijk} are radiative correction quantities. The structure of equality (34) is independent of the renormalization scheme.

One-loop finiteness, i.e. vanishing of the β-functions at one-loop, implies that the Yukawa couplings λ_{ijk} must be functions of the gauge coupling g. To find a similar condition to all orders it is necessary and sufficient for the Yukawa couplings to be a formal power series in g, which is solution of the REs (2).

We can now state the theorem for all-order vanishing β-functions.

Theorem: Consider an $N = 1$ supersymmetric Yang-Mills theory, with simple gauge group. If the following conditions are satisfied

1. There is no gauge anomaly.
2. The gauge β-function vanishes at one-loop

$$\beta_g^{(1)} = 0 = \sum_i l(R_i) - 3\, C_2(G). \tag{35}$$

3. There exist solutions of the form

$$\lambda_{ijk} = \rho_{ijk} g, \qquad \rho_{ijk} \in \mathbb{C} \tag{36}$$

to the conditions of vanishing one-loop matter fields anomalous dimensions

$$\gamma_j^{i\ (1)} = 0 = \frac{1}{32\pi^2}\ [\ C^{ikl}\, C_{jkl} - 2\ g^2\ C_2(R_i)\delta_{ij}]. \tag{37}$$

4. these solutions are isolated and non-degenerate when considered as solutions of vanishing one-loop Yukawa β-functions:

$$\beta_{ijk} = 0. \tag{38}$$

Then, each of the solutions (36) can be uniquely extended to a formal power series in g, and the associated super Yang-Mills models depend on the single coupling constant g with a β function which vanishes at all orders.

It is important to note a few things: The requirement of isolated and non-degenerate solutions guarantees the existence of a formal power series solution to the reduction equations. The vanishing of the gauge β-function at one-loop, $\beta_g^{(1)}$, is equivalent to the vanishing of the R-current anomaly (27). The vanishing of the anomalous dimensions at one-loop implies the vanishing of the Yukawa couplings β-functions at that order. It also implies the vanishing of the chiral anomaly coefficients r^A. This last property is a necessary condition for having β-functions vanishing at all orders.

Proof: Insert β_{ijk} as given by the REs into the relationship (34) between the axial anomalies coefficients and the β-functions. Since these chiral anomalies vanish, we get for β_g an homogeneous equation of the form

$$0 = \beta_g(1 + O(\hbar)). \tag{39}$$

The solution of this equation in the sense of a formal power series in $\hbar$ is $\beta_g = 0$, order by order. Therefore, due to the REs (2), $\beta_{ijk} = 0$ too.

3.2 Other criteria leading to all-loop finiteness

Another method of searching for all loop finite theories was suggested some time ago in refs. [18] and was revived and improved recently in ref. [39]. It is based on the all orders relation among the gauge β-function β_g and the anomalous dimensions of the superfields γ which was first derived using instanton calculus [38] given by

$$\beta_g^{NSVZ} \propto [\sum_i \lambda(R_i) - 3C_2(G) - \sum_i \lambda(R_i)\gamma_i], \tag{40}$$

which is believed also to hold non-perturbatively [46]. In addition the second method is based on the general relation among the β-functions for the Yukawa couplings and anomalous dimensions of the superfields given in eq. (22) in accordance with the non-renormalization theorem [40].

It is worth noting that both β-functions are linear functionals of the anomalous dimensions of the matter superfields. These anomalous dimensions now are complicated functions of the couplings, however the relations among β's and γ's are very simple. The criterion for having a finite theory is that all β-functions vanish simultaneously. This puts n constraints on the n couplings g, C_{ijk}. In the case that these constraints are linearly independent, then one expects that their solutions are isolated points in the space of couplings. (Note that this case looks the same as the condition (4) of the theorem [16] mentioned above. The difference is that the theorem refers to one-loop β-functions.) However if only p constraints are linearly independent, then one is led to an $n-p$ dimensional manifold of fixed points. Thus, if some of the β-functions are linearly dependent, and there is at least one generic fixed point somewhere in the space of couplings, then the theory will have a manifold of fixed points. Of course it is always possible that the constraints have no solutions at all, as for instance when they put contradictory conditions on the anomalous dimensions.

In practice, when searching for all loop finite theories the method of ref. [39] gives a faster answer, since it does not require calculation of the anomalous dimensions even at one-loop. However in general one has to be careful. The first method is certainly better for real calculations in a given theory and gives unambiguous results, but requires more work. Before closing our reference to the second method it is worth mentioning the relation among the β_g^{NSVZ} and the β_g and γ, when calculated in the DRED scheme. The relationship between β_g^{NSVZ} and β_g^{DRED} has been explored [45], with the conclusion that there exists an analytic redefinition of g, $g \longrightarrow g'(g,C)$ which connects them, however the two schemes start to deviate at three loops.

3.3 The supertrace anomaly in the presence of gravity

Having established how to construct an all-loop finite gauge theory, it is interesting to examine further the trace anomaly, T_μ^μ of the energy momentum tensor of such a theory in the presence of gravity. In that case one studies the contribution of graviton excitations to the trace anomaly. The latter is given by

$$T_\mu^\mu = (c/16\pi^2) W_{\mu\nu\rho\sigma}^2 - (a/16\pi^2) \tilde{R}_{\mu\nu\rho\sigma} \tilde{R}^{\mu\nu\rho\sigma}, \tag{41}$$

where $W_{\mu\nu\rho\sigma}$ is the Weyl tensor and $\tilde{R}^{\mu\nu\rho\sigma}$ is the dual of the curvature tensor, the second term being the Euler density. In a supersymmetric gauge theory with $N_V = dim\ G$ gauge multiplets and N_χ chiral multiplets the constants are [41]

$$c = 1/24(3N_V + N_\chi), \quad a = 1/48(9N_V + N_\chi) \tag{42}$$

Similarly the supersymmetric partner of the trace anomaly, the divergence of the $J_R^{'\mu}$ is [64]

$$\partial_\mu \sqrt{g} J_R^{'\mu} = (c-a)/24\pi^2 R\tilde{R}, \tag{43}$$

where R and $\tilde{R}$ are the curvature tensor and its dual. In general a gravitational background breaks supersymmetry. In fact the only gravitational vacua compatible with supersymmetry are the flat Minkowski space-time M_{1+n} with the standard Poincaré supersymmetry and the anti-de Sitter one AdS_{2+n} with AdS supersymmetry [37].

Therefore knowing how to construct all-loop finite theories on M_4, we can examine all the potentially all-loop finite models and check to which extent the condition for cancellation of the graviton excitation contributions to the T_μ^μ and to the divergence of the $J_R^{'\mu}$, i.e.

$$c = a \tag{44}$$

can be satisfied. The latter is obeyed by models with $3N_V = N_\chi$.[1] Indeed we have checked all the $N = 1$, chiral, all-loop finite models of ref. [43], which potentially can become all-loop finite and all the potentially interesting $N = 2$, one- and therefore all-loop finite models based on $SU(5)$, $SO(10)$, E_6, E_7, E_8. We found that the only $N = 1$, chiral model of ref. [43] which satisfies the condition (44) is an SO(10) gauge theory with chiral multiplets in the 2 **10** + 1 **54** + 1 **45** + **1** representations. We did not find any of the potentially interesting $N = 2$ finite theories satisfying the condition (44), while on the contrary all $N = 4$ gauge theories satisfy automatically the condition.

The general problem of studying superconformal anomalies of field theories on curved superspace has already been considered in ref. [55] for the massless Wess-Zumino model as well as for gauge theories [56]. Here we have limited ourselves in the case of considering finite gauge theories defined on M_4 including graviton excitations to the supertrace anomaly at one-loop. We found that apart from the phenomenologically uninteresting models, one $N = 1$ based on SO(10) and all $N = 4$, the supertrace anomaly receives gravitational contributions. These contributions cannot be cancelled in non-finite theories.

[1] In particular the isometries of the AdS_{2+n} space are described by the group $SO(2, n)$. The latter is actually the conformal group of the boundary Minkowski space M_{1+n}. According to recent ideas, the supergravity theory in the bulk of AdS is dual to a superconformal theory on the boundary [42].

4 Reduction of dimensionful parameters

The reduction of couplings was originally formulated for massless theories on the basis of the Callan-Symanzik equation [15]. The extension to theories with massive parameters is not straightforward if one wants to keep the generality and the rigor on the same level as for the massless case; one has to fulfill a set of requirements coming from the renormalization group equations, the Callan-Symanzik equations, etc. along with the normalization conditions imposed on irreducible Green's functions [52]. There has been progress in this direction [53]. Here, to simplify the situation, we would like to assume that a mass-independent renormalization scheme has been employed so that all the RG functions have only trivial dependencies of dimensional parameters.

To be general, we consider a renormalizable theory which contain a set of $(N+1)$ dimension-zero couplings, $\{\hat{g}_0, \hat{g}_1, \ldots, \hat{g}_N\}$, a set of L parameters with dimension one, $\{\hat{h}_1, \ldots, \hat{h}_L\}$, and a set of M parameters with dimension two, $\{\hat{m}_1^2, \ldots, \hat{m}_M^2\}$. The renormalized irreducible vertex function satisfies the RG equation

$$\begin{aligned} 0 &= \mathcal{D}\Gamma[\,\mathbf{\Phi}'s; \hat{g}_0, \hat{g}_1, \ldots, \hat{g}_N; \hat{h}_1, \ldots, \hat{h}_L; \hat{m}_1^2, \ldots, \hat{m}_M^2; \mu\,] \;, \qquad (45) \\ \mathcal{D} &= \mu\frac{\partial}{\partial\mu} + \sum_{i=0}^{N} \beta_i \frac{\partial}{\partial \hat{g}_i} + \sum_{a=1}^{L} \gamma_a^h \frac{\partial}{\partial \hat{h}_a} + \sum_{\alpha=1}^{M} \gamma_\alpha^{m^2} \frac{\partial}{\partial \hat{m}_\alpha^2} \\ &\quad + \sum_J \Phi_I {\gamma^{\phi I}}_J \frac{\delta}{\delta \Phi_J} \;. \end{aligned}$$

Since we assume a mass-independent renormalization scheme, the γ's have the form

$$\begin{aligned} \gamma_a^h &= \sum_{b=1}^{L} \gamma_a^{h,b}(g_0, \ldots, g_N)\hat{h}_b \;, \\ \gamma_\alpha^{m^2} &= \sum_{\beta=1}^{M} \gamma_\alpha^{m^2,\beta}(g_0, \ldots, g_N)\hat{m}_\beta^2 + \sum_{a,b=1}^{L} \gamma_\alpha^{m^2,ab}(g_0, \ldots, g_N)\hat{h}_a \hat{h}_b \;, \qquad (46) \end{aligned}$$

where $\gamma_a^{h,b}, \gamma_\alpha^{m^2,\beta}$ and $\gamma_a^{m^2,ab}$ are power series of the dimension-zero couplings g's in perturbation theory.

As in the massless case, we then look for conditions under which the reduction of parameters,

$$\hat{g}_i = \hat{g}_i(g) \;, \; (i = 1, \ldots, N) \;, \qquad (47)$$

$$\hat{h}_a = \sum_{b=1}^{P} f_a^b(g) h_b \ , \ (a = P+1, \ldots, L) \ , \tag{48}$$

$$\hat{m}^2_\alpha = \sum_{\beta=1}^{Q} e_\alpha^\beta(g) m^2_\beta + \sum_{a,b=1}^{P} k_\alpha^{ab}(g) h_a h_b \ , \ (\alpha = Q+1, \ldots, M) \ , \tag{49}$$

is consistent with the RG equation (1), where we assume that $g \equiv g_0$, $h_a \equiv \hat{h}_a \ \ (1 \le a \le P)$ and $m^2_\alpha \equiv \hat{m}^2_\alpha \ \ (1 \le \alpha \le Q)$ are independent parameters of the reduced theory. We find that the following set of equations has to be satisfied:

$$\beta_g \frac{\partial \hat{g}_i}{\partial g} = \beta_i \ , \ (i = 1, \ldots, N) \ , \tag{50}$$

$$\beta_g \frac{\partial \hat{h}_a}{\partial g} + \sum_{b=1}^{P} \gamma_b^h \frac{\partial \hat{h}_a}{\partial h_b} = \gamma_a^h \ , \ (a = P+1, \ldots, L) \ , \tag{51}$$

$$\beta_g \frac{\partial \hat{m}^2_\alpha}{\partial g} + \sum_{a=1}^{P} \gamma_a^h \frac{\partial \hat{m}^2_\alpha}{\partial h_a} + \sum_{\beta=1}^{Q} \gamma_\beta^{m^2} \frac{\partial \hat{m}^2_\alpha}{\partial m^2_\beta} = \gamma_\alpha^{m^2} \ , \ (\alpha = Q+1, \ldots, M) \tag{52}$$

Using eq.(45) for γ's, one finds that eqs.(50-52) reduce to

$$\beta_g \frac{d f_a^b}{dg} + \sum_{c=1}^{P} f_a^c [\gamma_c^{h,b} + \sum_{d=P+1}^{L} \gamma_c^{h,d} f_d^b] - \gamma_a^{h,b} - \sum_{d=P+1}^{L} \gamma_a^{h,d} f_d^b = 0 \ , \tag{53}$$

$$(a = P+1, \ldots, L; b = 1, \ldots, P) \ ,$$

$$\beta_g \frac{d e_\alpha^\beta}{dg} + \sum_{\gamma=1}^{Q} e_\alpha^\gamma [\gamma_\gamma^{m^2,\beta} + \sum_{\delta=Q+1}^{M} \gamma_\gamma^{m^2,\delta} e_\delta^\beta] - \gamma_\alpha^{m^2,\beta}$$

$$- \sum_{\delta=Q+1}^{M} \gamma_\alpha^{m^2,\delta} e_\delta^\beta = 0 \ , \tag{54}$$

$$(\alpha = Q+1, \ldots, M; \beta = 1, \ldots, Q) \ ,$$

$$\beta_g \frac{d k_\alpha^{ab}}{dg} + 2 \sum_{c=1}^{P} (\gamma_c^{h,a} + \sum_{d=P+1}^{L} \gamma_c^{h,d} f_d^a) k_\alpha^{cb} + \sum_{\beta=1}^{Q} e_\alpha^\beta [\gamma_\beta^{m^2,ab}$$

$$+ \sum_{c,d=P+1}^{L} \gamma_\beta^{m^2,cd} f_c^a f_d^b$$

$$+2 \sum_{c=P+1}^{L} \gamma_\beta^{m^2,cb} f_c^a + \sum_{\delta=Q+1}^{M} \gamma_\beta^{m^2,\delta} k_\delta^{ab}] - [\gamma_\alpha^{m^2,ab} + \sum_{c,d=P+1}^{L} \gamma_\alpha^{m^2,cd} f_c^a f_d^b$$

$$+2 \sum_{c=P+1}^{L} \gamma_\alpha^{m^2,cb} f_c^a + \sum_{\delta=Q+1}^{M} \gamma_\alpha^{m^2,\delta} k_\delta^{ab}] = 0 \ , \tag{55}$$

$$(\alpha = Q+1, \ldots, M; a, b = 1, \ldots, P) \ .$$

If these equations are satisfied, the irreducible vertex function of the reduced theory

$$\begin{aligned}
& \Gamma_R[\ \mathbf{\Phi}'s; g; h_1, \ldots, h_P; m_1^2, \ldots, \hat{m}_Q^2; \mu\ ] \\
\equiv\ & \Gamma[\ \mathbf{\Phi}'s; g, \hat{g}_1(g), \ldots, \hat{g}_N(g); h_1, \ldots, h_P, \hat{h}_{P+1}(g,h), \ldots, \hat{h}_L(g,h); \\
& m_1^2, \ldots, \hat{m}_Q^2, \hat{m}_{Q+1}^2(g,h,m^2), \ldots, \hat{m}_M^2(g,h,m^2); \mu\ ] \qquad (56)
\end{aligned}$$

has the same renormalization group flow as the original one.

The requirement for the reduced theory to be perturbative renormalizable means that the functions $\hat{g}_i$, f_a^b, e_α^β and k_α^{ab}, defined in eqs. (47-49), should have a power series expansion in the primary coupling g:

$$\begin{aligned}
\hat{g}_i &= g\sum_{n=0}^{\infty} \rho_i^{(n)} g^n \ , \ f_a^b = g\sum_{n=0}^{\infty} \eta_a^{b\ (n)} g^n \ , \\
e_\alpha^\beta &= \sum_{n=0}^{\infty} \xi_\alpha^{\beta\ (n)} g^n \ , \ k_\alpha^{ab} = \sum_{n=0}^{\infty} \chi_\alpha^{ab\ (n)} g^n \ . \qquad (57)
\end{aligned}$$

To obtain the expansion coefficients, we insert the power series ansatz above into eqs. (50,53–55) and require that the equations are satisfied at each order in g. Note that the existence of a unique power series solution is a non-trivial matter: It depends on the theory as well as on the choice of the set of independent parameters.

Sum rule for Soft Breaking Terms and All-loop Results

The method of reducing the dimensionless couplings has been extended[5], as we have just seen above, to the soft supersymmetry breaking (SSB) dimensionful parameters of $N=1$ supersymmetric theories. In addition it was found [6] that RGI SSB scalar masses in Gauge-Yukawa unified models satisfy a universal sum rule. Here we will describe first how the use of the available two-loop RG functions and the requirement of finiteness of the SSB parameters up to this order leads to the soft scalar-mass sum rule [7].

Consider the superpotential given by (15) along with the Lagrangian for SSB terms

$$\begin{aligned}
-\mathcal{L}_{\rm SB} &= \frac{1}{6}\, h^{ijk}\, \phi_i\phi_j\phi_k + \frac{1}{2}\, b^{ij}\, \phi_i\phi_j + \frac{1}{2}\, (m^2)^j_i\, \phi^{*\,i}\phi_j \\
&\quad + \frac{1}{2}\, M\, \lambda\lambda + \text{H.c.}, \qquad (58)
\end{aligned}$$

where the ϕ_i are the scalar parts of the chiral superfields Φ_i , λ are the gauginos and M their unified mass. Since we would like to consider only finite theories here, we assume that the gauge group is a simple group and the one-loop β-function of the gauge coupling g vanishes. We also assume that the reduction equations admit power series solutions of the form

$$C^{ijk} \;=\; g\sum_{n=0}\rho^{ijk}_{(n)}g^{2n}\ . \tag{59}$$

According to the finiteness theorem of ref. [16], the theory is then finite to all orders in perturbation theory, if, among others, the one-loop anomalous dimensions $\gamma_i^{j(1)}$ vanish. The one- and two-loop finiteness for h^{ijk} can be achieved by

$$h^{ijk} \;=\; -MC^{ijk}+\ldots = -M\rho^{ijk}_{(0)}\,g+O(g^5)\ . \tag{60}$$

Now, to obtain the two-loop sum rule for soft scalar masses, we assume that the lowest order coefficients $\rho^{ijk}_{(0)}$ and also $(m^2)^i_j$ satisfy the diagonality relations

$$\rho_{ipq(0)}\rho^{jpq}_{(0)} \;\propto\; \delta_i^j \text{ for all } p \text{ and } q \;\text{ and }\; (m^2)^i_j = m_j^2\delta^i_j\ , \tag{61}$$

respectively. Then we find the following soft scalar-mass sum rule

$$(\ m_i^2+m_j^2+m_k^2\ )/MM^{\dagger} \;=\; 1+\frac{g^2}{16\pi^2}\,\Delta^{(1)}+O(g^4) \tag{62}$$

for i, j, k with $\rho^{ijk}_{(0)}\neq 0$, where $\Delta^{(1)}$ is the two-loop correction

$$\Delta^{(1)} \;=\; -2\sum_l[(m_l^2/MM^{\dagger})-(1/3)]\ T(R_l), \tag{63}$$

which vanishes for the universal choice in accordance with the previous findings of ref. [13].

If we know higher-loop β-functions explicitly, we can follow the same procedure and find higher-loop RGI relations among SSB terms. However, the β-functions of the soft scalar masses are explicitly known only up to two loops. In order to obtain higher-loop results, we need something else instead of knowledge of explicit β-functions, e.g. some relations among β-functions.

The recent progress made using the spurion technique [50], [51] leads to the following all-loop relations among SSB β-functions, [20]-[24]

$$\beta_M \;=\; 2\mathcal{O}\left(\frac{\beta_g}{g}\right)\ , \tag{64}$$

$$
\begin{aligned}
\beta_h^{ijk} &= \gamma^i{}_l h^{ljk} + \gamma^j{}_l h^{ilk} + \gamma^k{}_l h^{ijl} \\
&\quad -2\gamma_1^i{}_l C^{ljk} - 2\gamma_1^j{}_l C^{ilk} - 2\gamma_1^k{}_l C^{ijl} \ , \qquad (65) \\
(\beta_{m^2})^i{}_j &= \left[\Delta + X\frac{\partial}{\partial g}\right]\gamma^i{}_j \ , \qquad (66) \\
\mathcal{O} &= \left(Mg^2\frac{\partial}{\partial g^2} - h^{lmn}\frac{\partial}{\partial C^{lmn}}\right) \ , \qquad (67) \\
\Delta &= 2\mathcal{O}\mathcal{O}^* + 2|M|^2 g^2\frac{\partial}{\partial g^2} + \tilde{C}_{lmn}\frac{\partial}{\partial C_{lmn}} + \tilde{C}^{lmn}\frac{\partial}{\partial C^{lmn}} \ , \qquad (68)
\end{aligned}
$$

where $(\gamma_1)^i{}_j = \mathcal{O}\gamma^i{}_j$, $C_{lmn} = (C^{lmn})^*$, and

$$
\tilde{C}^{ijk} = (m^2)^i{}_l C^{ljk} + (m^2)^j{}_l C^{ilk} + (m^2)^k{}_l C^{ijl} \ . \qquad (69)
$$

It was also found [24] that the relation

$$
h^{ijk} = -M(C^{ijk})' \equiv -M\frac{dC^{ijk}(g)}{d\ln g} \ , \qquad (70)
$$

among couplings is all-loop RGI. Furthermore, using the all-loop gauge β-function of Novikov *et al.* [38] given by

$$
\beta_g^{\mathrm{NSVZ}} = \frac{g^3}{16\pi^2}\left[\frac{\sum_l T(R_l)(1-\gamma_l/2) - 3C(G)}{1 - g^2 C(G)/8\pi^2}\right] \ , \qquad (71)
$$

it was found the all-loop RGI sum rule [25],

$$
\begin{aligned}
m_i^2 + m_j^2 + m_k^2 &= |M|^2\left\{ \frac{1}{1 - g^2 C(G)/(8\pi^2)}\frac{d\ln C^{ijk}}{d\ln g} + \frac{1}{2}\frac{d^2\ln C^{ijk}}{d(\ln g)^2} \right\} \\
&\quad + \sum_l \frac{m_l^2 T(R_l)}{C(G) - 8\pi^2/g^2}\frac{d\ln C^{ijk}}{d\ln g} \ . \qquad (72)
\end{aligned}
$$

In addition the exact-β-function for m^2 in the NSVZ scheme has been obtained [25] for the first time and is given by

$$
\begin{aligned}
\beta_{m_i^2}^{\mathrm{NSVZ}} &= \left[|M|^2\left\{ \frac{1}{1 - g^2 C(G)/(8\pi^2)}\frac{d}{d\ln g} + \frac{1}{2}\frac{d^2}{d(\ln g)^2} \right\} \right. \\
&\quad \left. + \sum_l \frac{m_l^2 T(R_l)}{C(G) - 8\pi^2/g^2}\frac{d}{d\ln g} \right] \gamma_i^{\mathrm{NSVZ}} \ . \qquad (73)
\end{aligned}
$$

Surprisingly enough, the all-loop result (72) coincides with the superstring result for the finite case in a certain class of orbifold models [7] if $d\ln C^{ijk}/d\ln g = 1$.

5 Anomaly–mediated supersymmetry breaking scenario

An interesting aspect of the anomaly–mediated supersymmetry breaking scenario [26] is that it renders the origin of supersymmetry breaking to gravity. This is achieved by attributing the presence of soft supersymmetry breaking terms to the auxiliary field of the off-shell supergravity multiplet.

The natural framework to describe this scenario is the Weyl compensator formalism [27]. In this formalism one defines the local super-Weyl transformations of the various fields involved in a generic locally supersymmetric classical Lagrangian. To guarantee the super-Weyl invariance, i.e. the superconformal invariance, of the latter one has to introduce a Weyl compensator Φ which is a chiral superfield appropriately charged under the super-Weyl transformations. The role of Φ is to guarantee the invariance of the locally supersymmetric Lagrangian under the super-Weyl transformations which can be done subject to the condition that various fields have to be redefined accordingly using the compensating field Φ. Note that the compensating field Φ is non-dynamical.

Next, by a suitable choice of

$$< \Phi >= 1 + m_{3/2}\theta^2 \ , \tag{74}$$

the super-Weyl invariance is completely broken spontaneously and the Lagrangian has only super-Poincaré invariance. We find that after the spontaneous breaking of the super-Weyl invariance the Lagrangian has no terms linear in $m_{3/2}$, while there exist terms proportional to $m_{3/2}^2$ comming from the $\bar{\Phi}\Phi$ terms. In fact one can eliminate also the terms proportional to $m_{3/2}^2$ by redefining the matter chiral superfields according to $Q\Phi \rightarrow Q$.

Therefore, if the classical super-Weyl invariance was exact this would imply that all the soft supersymmetry breaking terms would vanish. However the super-Weyl transformations contain an R-symmetry and in the presence of a super-Yang-Mills sector the super-Weyl transformation is anomalous at the quantum level. As a result there is a shift in the Wilsonian gauge couplings which at the quantum level become Φ - dependent, while they were super-Weyl invariant (and therefore Φ independent) classically. The dependence of the leading component of Φ goes away, but there is a nontrivial dependence on the auxiliary component of Φ. Eventually it was found [26] that the induced soft supersymmetry breaking terms have the following form

$$M \ = \ m_{3/2} \, \frac{\beta_g}{g}, \tag{75}$$

$$h^{ijk} = -m_{3/2}\ \beta_C^{ijk}, \tag{76}$$
$$b^{ij} = -m_{3/2}\ \beta_m^{ij}, \tag{77}$$
$$(m^2)^i{}_j = \frac{1}{2}\ |m_{3/2}|^2\ \mu\frac{d\gamma^i{}_j}{d\mu}, \tag{78}$$

which moreover are RGI to all-loops. Lets us now see how the above results can be obtained from Eqs. (64-69) assuming (a) the existence of a RGI surface on which $C = C(g)$ or equivalently that

$$\frac{dC^{ijk}}{dg} = \frac{\beta_C^{ijk}}{\beta_g} \tag{79}$$

holds, and (b) the existence of RGI surface on which eq. (70) holds too. However in both cases we do not rely on specific solutions of these equations. Assuming also that

$$C_a\frac{\partial}{\partial C_a} = C_a^*\frac{\partial}{\partial C_a^*} \tag{80}$$

for a RGI surface $F(g, C^{ijk}, C^{*ijk})$ we have that

$$\frac{d}{dg} = (\frac{\partial}{\partial g} + 2\frac{\partial}{\partial C}dC/dg) \tag{81}$$
$$= (\frac{\partial}{\partial g} + 2\frac{\beta_C}{\beta_g}\frac{\partial}{\partial C}) \tag{82}$$

where eq. (79) has been used. Now let us consider the partial differential operator $\mathcal{O}$ in eq. (67), which assuming eq. (70) and using eq. (82) becomes

$$\mathcal{O} = \frac{1}{2}M\frac{d}{d\ln g} \tag{83}$$

In turn β_M given in eq. (64) becomes

$$\beta_M = M\frac{d}{d\ln g}(\frac{\beta_g}{g})\ , \tag{84}$$

which by integration provides us [28, 29] with the generalized, i.e. including Yukawa couplings, all-loop RGI Hisano - Shifman relation [20]

$$M = \frac{\beta_g}{g}M_0\ , \tag{85}$$

where M_0 is the integration constant which in a supergravity framework can be fixed to be the gravitino mass $m_{3/2}$ and in that case of eq. (85)

becomes the all-loop RGI eq. (75). Note that β_M using Eqs. (84,85) can be written as

$$\beta_M = m_{3/2}\frac{d}{dt}(\beta_g/g) \ . \tag{86}$$

Similarly

$$(\gamma_1)^i{}_j = \mathcal{O}\gamma^i{}_j = \frac{1}{2}\ m_{3/2}\ \frac{d\gamma^i{}_j}{dt} \ . \tag{87}$$

Next from eq. (70) and eq. (85) we obtain

$$h^{ijk} = -m_{3/2}\ \beta_C^{ijk} \ , \tag{88}$$

while β_h^{ijk} given in eq. (51) and using eq. (87) becomes [29]

$$\beta_h^{ijk} = -m_{3/2}\ \frac{d}{dt}\beta_C^{ijk}, \tag{89}$$

which shows that eq. (88) is all-loop RGI. In a similar way eq. (77) can be shown to be all-loop RGI.

Finally we would like to emphasize that under the same assumptions (a) and (b) the sum rule given in eq. (72) has been proven [25] to be all-loop RGI, which using eq. (85) gives us a generalization of eq. (78) to be applied in considerations of non-universal soft scalar masses, which are necessary in many cases including the MSSM.

Having obtained the Eqs. (75-78) from Eqs. (64-69) with the assumptions (a) and (b) we would like to conclude the present section with some remarks. First it is worth noting the difference, say in first order in g, among the possibilities to consider specific solution of the reduction equations or just assume the existence of a RGI surface, which is a weaker assumption. So in case we consider the reduction equation (79) without relying on a specific solution, the sum rule (72) reads

$$m_i^2 + m_j^2 + m_k^2 = |M|^2\frac{d\ln C^{ijk}}{d\ln g}, \tag{90}$$

and we find that

$$\frac{d\ln C^{ijk}}{d\ln g} = \frac{g}{C^{ijk}}\frac{dC^{ijk}}{dg} = \frac{g}{C^{ijk}}\frac{\beta_C^{ijk}}{\beta_g}, \tag{91}$$

which is clearly model dependent. However assuming a specific power series solution of the reduction equation, as in eq. (3), which in first order in g is just a linear relation among C^{ijk} and g, we obtain that

$$d\ln C^{ijk}/d\ln g = 1 \tag{92}$$

and therefore the sum rule (90) becomes model independent. Then we should also emphasize that in order to show [24] that the relation

$$(m^2)^i{}_j = \frac{1}{2}\frac{g^2}{\beta_g}|M|^2\frac{d\gamma^i{}_j}{dg}, \tag{93}$$

which using eq. (85) becomes eq. (78), is RGI to all-loops a specific solution of the reduction equations has to be required. As it has already been pointed out above such a requirement is not necessary in order to obtain the all-loop RG invariance of the sum rule (72).

6 $SU(5)$ Finite Models

From the classification of theories with vanishing one-loop gauge β-function [43], one can easily see that there exist only two candidate possibilities to construct $SU(5)$ GUTs with three generations. These possibilities require that the theory should contain as matter fields the chiral supermultiplets $\mathbf{5}$, $\overline{\mathbf{5}}$, $\mathbf{10}$, $\overline{\mathbf{5}}$, $\mathbf{24}$ with the multiplicities $(6, 9, 4, 1, 0)$ and $(4, 7, 3, 0, 1)$, respectively. Only the second one contains a **24**-plet which can be used to provide the spontaneous symmetry breaking (SB) of SU(5) down to $SU(3) \times SU(2) \times U(1)$. For the first model one has to incorporate another way, such as the Wilson flux breaking mechanism to achieve the desired SB of $SU(5)$ [1]. Therefore, for a self-consistent field theory discussion we would like to concentrate only on the second possibility.

It is clear, at least for the dimensionless couplings, that the matter content of a theory is only a necessary condition for all-order finiteness. Therefore, there exist, in principle, various finite models for a given matter content. However, during the early studies [44, 47], the theorem [16] that guarantees all-order finiteness and requires the existence of power series solution to any finite order in perturbation theory was not known. The theorem introduces new constraints, in particular requires that the solution to the one-loop finiteness conditions should be non-degenerate and isolated. In most previous studies the freedom resulted as a consequence of the degeneracy in the one- and two-loop solutions was used to make specific ansätze that could lead to phenomenologically acceptable predictions. Note that the existence of such freedom is incompatible with the power series solutions [15, 16].

Taking into account the new constraints an all-order finite $SU(5)$ model has been constructed [1], which among others successfully predicted the bottom and the top quark masses [1, 4]. The latter is due to the Gauge and

Yukawa of the third generation Unification [1]-[4] which has been achieved. In general the predictive power of a finite $SU(5)$ model depends on the structure of the superpotential and on the way the four pairs of Higgs quintets and anti-quintets mix to produce the two Higgs doublets of the minimal supersymmetric standard model (MSSM). Given that the finiteness conditions do not restrict the mass terms, there is a lot of freedom offered by this sector of the theory in mixing the four pairs of Higgs fields. As a result it was possible in the early studies (a) to provide the adequate doublet-triplet splitting in the pair of $\mathbf{5}$ and $\overline{\mathbf{5}}$ which couple to ordinary fermions so as to suppress the proton decay induced by the coloured triplets and (b) to introduce angles in the gauge-Yukawa relations suppressing in this way the strength of the Yukawa couplings. Concerning the requirement (b) one has to recall that at that time it was very unpleasant to have a top mass prediction at $O(150-200)$ GeV; the popular top quark mass was at $O(40)$ GeV. The above was most clearly stated in ref. [47] and has been revived [48] taking into account the recent data. However, it is clear that using the large freedom offered by the Higgs mass parameter space in requiring the condition (b) one strongly diminishes the beauty of a finite theory. Consequently, this freedom was abandoned in the recent studies of the all-loop finite SU(5) model [1] and only the condition (a) was kept as a necessary requirement.

A predictive Finite unified SU(5) model which is finite to all orders, in addition to the requirements mentioned already, should also have the following properties.

1. One-loop anomalous dimensions are diagonal, i.e., $\gamma_i^{(1)\,j} \propto \delta_i^j$, according to the assumption (61).

2. Three fermion generations, $\overline{\mathbf{5}}_i \quad (i = 1, 2, 3)$, obviously should not couple to $\mathbf{24}$. This can be achieved for instance by imposing $B - L$ conservation.

3. The two Higgs doublets of the MSSM should mostly be made out of a pair of Higgs quintet and anti-quintet, which couple to the third generation.

In the following we discuss two versions of the all-order finite model.

A: The model of ref. [1], whose matter content we just described above.
B: A slight variation of the model **A**, which can also be obtained from the class of the models suggested by Kazakov *et al.* [22] with a modification to suppress non-diagonal anomalous dimensions.

The quark mixing can be accommodated in these models, but for simplicity we neglect the intergenerational mixing and postpone the interesting problem of predicting the mixings to a future publication.

The superpotential which describe the two models takes the form [1, 22]

$$
\begin{aligned}
W &= \sum_{i=1}^{3}[\,\frac{1}{2}g_i^u\,\mathbf{10}_i\mathbf{10}_iH_i + g_i^d\,\mathbf{10}_i\overline{\mathbf{5}}_i\,\overline{H}_i\,] + g_{23}^u\,\mathbf{10}_2\mathbf{10}_3H_4 \\
&\quad + g_{23}^d\,\mathbf{10}_2\overline{\mathbf{5}}_3\,\overline{H}_4 + g_{32}^d\,\mathbf{10}_3\overline{\mathbf{5}}_2\,\overline{H}_4 + \sum_{a=1}^{4} g_a^f\,H_a\,\mathbf{24}\,\overline{H}_a + \frac{g^\lambda}{3}\,(\mathbf{24})^3
\end{aligned} \tag{94}
$$

where H_a and $\overline{H}_a$ $(a = 1, \ldots, 4)$ stand for the Higgs quintets and anti-quintets. Given the superpotential W, we can compute now the γ-functions of the model, from which we then compute the β-functions. We find:

$$
\begin{aligned}
\gamma^{(1)}_{\mathbf{10}_1} &= \frac{1}{16\pi^2}[-\frac{36}{5}\,g^2 + 3\,(g_1^u)^2 + 2\,(g_1^d)^2\,]\,, \\
\gamma^{(1)}_{\mathbf{10}_2} &= \frac{1}{16\pi^2}[-\frac{36}{5}\,g^2 + 3\,(g_2^u)^2 + 2\,(g_2^d)^2 + 3\,(g_{23}^u)^2 + 2\,(g_{23}^d)^2\,]\,, \\
\gamma^{(1)}_{\mathbf{10}_3} &= \frac{1}{16\pi^2}[-\frac{36}{5}\,g^2 + 3\,(g_3^u)^2 + 2\,(g_3^d)^2 + 3\,(g_{23}^u)^2 + 2\,(g_{32}^d)^2\,]\,, \\
\gamma^{(1)}_{\overline{\mathbf{5}}_1} &= \frac{1}{16\pi^2}[-\frac{24}{5}\,g^2 + 4\,(g_1^d)^2\,]\,, \\
\gamma^{(1)}_{\overline{\mathbf{5}}_2} &= \frac{1}{16\pi^2}[-\frac{24}{5}\,g^2 + 4\,(g_2^d)^2 + 4\,(g_{32}^d)^2\,]\,, \\
\gamma^{(1)}_{\overline{\mathbf{5}}_3} &= \frac{1}{16\pi^2}[-\frac{24}{5}\,g^2 + 4\,(g_3^d)^2 + 4\,(g_{23}^d)^2\,]\,, \\
\gamma^{(1)}_{H_i} &= \frac{1}{16\pi^2}[-\frac{24}{5}\,g^2 + 3\,(g_i^u)^2 + \frac{24}{5}(g_i^f)^2\,]\quad i = 1,2,3\,, \\
\gamma^{(1)}_{\overline{H}_i} &= \frac{1}{16\pi^2}[-\frac{24}{5}\,g^2 + 4\,(g_i^d)^2 + \frac{24}{5}(g_i^f)^2\,]\quad i = 1,2,3\,, \\
\gamma^{(1)}_{H_4} &= \frac{1}{16\pi^2}[-\frac{24}{5}\,g^2 + 6\,(g_{23}^u)^2 + \frac{24}{5}(g_4^f)^2\,]\,, \\
\gamma^{(1)}_{\overline{H}_4} &= \frac{1}{16\pi^2}[-\frac{24}{5}\,g^2 + 4\,(g_{23}^d)^2 + 4\,(g_{32}^d)^2 + \frac{24}{5}(g_4^f)^2\,]\,, \\
\gamma^{(1)}_{\mathbf{24}} &= \frac{1}{16\pi^2}[-10\,g^2 + \sum_{a=1}^{4}(g_a^f)^2 + \frac{21}{5}\,(g^\lambda)^2\,]\,.
\end{aligned} \tag{95}
$$

The non-degenerate and isolated solutions to $\gamma_i^{(1)} = 0$ for the models $\{\mathbf{A}\ ,\ \mathbf{B}\}$ are:

$$
(g_1^u)^2 = \{\frac{8}{5}\ ,\ \frac{8}{5})\}g^2\ ,\ (g_1^d)^2 = \{\frac{6}{5}\ ,\ \frac{6}{5}\}g^2\ ,\ (g_2^u)^2 = (g_3^u)^2 = \{\frac{8}{5}\ ,\ \frac{4}{5}\}g^2\ ,
$$

$$
\begin{aligned}
(g_2^d)^2 &= (g_3^d)^2 = \{\frac{6}{5}\ ,\ \frac{3}{5}\}g^2\ ,\ (g_{23}^u)^2 = \{0\ ,\ \frac{4}{5}\}g^2\ , \\
&\quad (g_{23}^d)^2 = (g_{32}^d)^2 = \{0, \frac{3}{5}\}g^2, \qquad (96) \\
(g^\lambda)^2 &= \frac{15}{7}g^2\ ,\ (g_2^f)^2 = (g_3^f)^2 = \{0\ ,\ \frac{1}{2}\}g^2\ ,\ (g_1^f)^2 = 0\ ,\ (g_4^f)^2 = \{1\ ,\ 0\ \}g^2\ .
\end{aligned}
$$

We have explicitly checked that these solutions (96) are also the solutions of the reduction equation (2) and that they can be uniquely extended to the corresponding power series solutions (3) [2]. Consequently, these models are finite to all orders.

After the reduction of couplings (96) the symmetry of W (94) is enhanced: For the model **A** one finds that the superpotential has the $Z_7 \times Z_3 \times Z_2$ discrete symmetry

$$
\begin{aligned}
\overline{\mathbf{5}}_1 &: (4,0,1)\ ,\ \overline{\mathbf{5}}_2 : (1,0,1)\ ,\ \overline{\mathbf{5}}_3 : (2,0,1)\ , \\
\mathbf{10}_1 &: (1,1,1)\ ,\ \mathbf{10}_2 : (2,2,1)\ ,\ \mathbf{10}_3 : (4,0,1)\ , \\
H_1 &: (5,1,0)\ ,\ H_2 : (3,2,0)\ ,\ H_3 : (6,0,0)\ , \qquad (97) \\
\overline{H}_1 &: (-5,-1,0)\ ,\ \overline{H}_2 : (-3,-2,0)\ ,\ \overline{H}_3 : (-6,0,0)\ , \\
H_4 &: (0,0,0)\ ,\ \overline{H}_4 : (0,0,0)\ ,\ \mathbf{24} : (0,0,0)\ ,
\end{aligned}
$$

while for the model **B** one finds $Z_4 \times Z_4 \times Z_4$ defined as

$$
\begin{aligned}
\overline{\mathbf{5}}_1 &: (1,0,0)\ ,\ \overline{\mathbf{5}}_2 : (0,1,0)\ ,\ \overline{\mathbf{5}}_3 : (0,0,1)\ , \\
\mathbf{10}_1 &: (1,0,0)\ ,\ \mathbf{10}_2 : (0,1,0)\ ,\ \mathbf{10}_3 : (0,0,1)\ , \\
H_1 &: (2,0,0)\ ,\ H_2 : (0,2,0)\ ,\ H_3 : (0,0,2)\ , \qquad (98) \\
\overline{H}_1 &: (-2,0,0)\ ,\ \overline{H}_2 : (0,-2,0)\ ,\ \overline{H}_3 : (0,0,-2)\ , \\
H_4 &: (0,3,3)\ ,\ \overline{H}_4 : (0,-3,-3)\ ,\ \mathbf{24} : (0,0,0)\ ,
\end{aligned}
$$

where the numbers in the parenthesis stand for the charges under the discrete symmetries.

The main difference of the models **A** and **B** is that three pairs of Higgs quintets and anti-quintets couple to the **24** for **B** so that it is not necessary [22] to mix them with H_4 and $\overline{H}_4$ in order to achieve the triplet-doublet splitting after SB of $SU(5)$. This enhances the predicitivity, because then the mixing of the three pairs of Higgsess are strongly constrained to fit the phenomenology of the first two generations [22].

[2]The coefficients in (96) are slightly different from those in models considered in refs. [22].

7 Predictions of Low Energy Parameters

In this section we will refine the predictions of the two finite models for the third family quark masses, taking into account certain corrections and we will compare them with the experimental data.

7.1 Top Mass Predictions

We recall that $\tan\beta$ is usually determined in the Higgs sector. However, it has turned out that in the case of GYU models it is convenient to define $\tan\beta$ by using the matching condition at M_s [59],

$$\begin{aligned} \alpha_t^{\rm SM} &= \alpha_t \sin^2\beta \ , \ \alpha_b^{\rm SM} = \alpha_b \cos^2\beta \ , \ \alpha_\tau^{\rm SM} = \alpha_\tau \cos^2\beta \ , \\ \alpha_\lambda &= \frac{1}{4}(\frac{3}{5}\alpha_1 + \alpha_2) \cos^2 2\beta \ , \end{aligned} \tag{99}$$

where $\alpha_i^{\rm SM}$ $(i = t, b, \tau)$ are the SM Yukawa couplings and α_λ is the Higgs coupling $(\alpha_I = g_I^2/4\pi^2)$. With a given set of the input parameters [60],

$$M_\tau = 1.777 \text{ GeV} \ , \ M_Z = 91.188 \text{ GeV} \ , \tag{100}$$

with [61]

$$\begin{aligned} \alpha_{\rm EM}^{-1}(M_Z) &= 127.9 + \frac{8}{9\pi} \log\frac{M_t}{M_Z} \ , \\ \sin^2\theta_{\rm W}(M_Z) &= 0.2319 - 3.03 \times 10^{-5} T - 8.4 \times 10^{-8} T^2 \ , \\ T &= M_t/[\text{GeV}] - 165 \ , \end{aligned} \tag{101}$$

the matching condition (99) and the GYU boundary condition at $M_{\rm GUT}$ can be satisfied only for a specific value of $\tan\beta$. Here M_τ, M_t, M_Z are pole masses, and the couplings above are defined in the $\overline{\rm MS}$ scheme with six flavors. Under the assumptions specified above, it is possible without knowing the details of the scalar sector of the MSSM to predict various parameters such as the top quark mass [2]-[4]. We present them for the models **FUTA**, **FUTB** in tables 1 and 2 respectively.

Comparing, for instance, the M_t predictions above with the most recent experimental value

$$M_t = (173.8 \pm 5.2) \ \text{ GeV} \ , \tag{102}$$

and recalling that the theoretical values for M_t given in the tables may suffer from a correction of less than ~ 4 % [4], we see that they are consistent with the experimental data. (For more details, see ref. [4], where various

Table 1: The predictions for different M_s for **A**

M_s [GeV]	$\alpha_{3(5f)}(M_Z)$	$\tan\beta$	$M_{\rm GUT}$ [GeV]	M_b [GeV]	M_t [GeV]
300	0.123	54.1	2.2×10^{16}	5.3	183
500	0.122	54.2	1.9×10^{16}	5.3	183
10^3	0.120	54.3	1.5×10^{16}	5.2	184

Table 2: The predictions for different M_s for **B**

M_s [GeV]	$\alpha_{3(5f)}(M_Z)$	$\tan\beta$	$M_{\rm GUT}$ [GeV]	M_b [GeV]	M_t [GeV]
800	0.120	48.2	1.5×10^{16}	5.4	174
10^3	0.119	48.2	1.4×10^{16}	5.4	174
1.2×10^3	0.118	48.2	1.3×10^{16}	5.4	174

corrections on the predictions of GYU models such as the MSSM threshold corrections are estimated[3].)

7.2 Implications of the Sum Rule

Here we will discuss the structure of the sum rule for the soft scalar masses for each of the finite models. According to (61), we recall that they are supposed to be diagonal. From the one-loop finiteness for the soft scalar masses, we obtain (there are $\{10\ ,\ 13\}$ equations for 15 unknown $\kappa^{(0)}$'s):

$$\begin{aligned}
\kappa^{(0)}_{H_i} &= 1-2\kappa^{(0)}_{\mathbf{10}_i}\ ,\ \kappa^{(0)}_{\overline{H}_i} = 1-\kappa^{(0)}_{\mathbf{10}_i}-\kappa^{(0)}_{\overline{\mathbf{5}}_i}\ \ (i=1,2,3)\ , \\
\kappa^{(0)}_{H_4} &= \frac{2}{3}-\kappa^{(0)}_{\overline{H}_4}\ ,\ \kappa^{(0)}_{\mathbf{24}} = \frac{1}{3}\ \ \text{for}\ \ \mathbf{A}\ ,
\end{aligned} \tag{103}$$

and

$$\begin{aligned}
\kappa^{(0)}_{H_1} &= 1-2\kappa^{(0)}_{\mathbf{10}_1}\ ,\ \kappa^{(0)}_{H_2}=\kappa^{(0)}_{H_3}=\kappa^{(0)}_{H_4}=1-2\kappa^{(0)}_{\mathbf{10}_3}\ , \\
\kappa^{(0)}_{\overline{H}_1} &= 1-\kappa^{(0)}_{\mathbf{10}_1}-\kappa^{(0)}_{\overline{\mathbf{5}}_1}\ ,\ \kappa^{(0)}_{\overline{H}_2}=\kappa^{(0)}_{\overline{H}_3}=\kappa^{(0)}_{\overline{H}_4}=-\frac{1}{3}+2\kappa^{(0)}_{\mathbf{10}_3}\ , \\
\kappa^{(0)}_{\overline{\mathbf{5}}_2} &= \kappa^{(0)}_{\overline{\mathbf{5}}_3}=\frac{4}{3}-3\kappa^{(0)}_{\mathbf{10}_3}\ ,\ \kappa^{(0)}_{\mathbf{10}_2}=\kappa^{(0)}_{\mathbf{10}_3}\ ,\ \kappa^{(0)}_{\mathbf{24}}=\frac{1}{3}\ \ \text{for}\ \ \mathbf{B}\ ,
\end{aligned} \tag{104}$$

[3]The GUT threshold corrections in the $SU(5)$ finite model are given in ref, [48].

where we have defined

$$\frac{m_i^2}{|M|^2} \;=\; \kappa_i^{(0)} + \frac{g^2}{16\pi^2}\kappa_i^{(1)} + \cdots \;,\; i = \mathbf{10}_1, \mathbf{10}_2, \ldots, \mathbf{24}\;. \tag{105}$$

We then use the solution (96) to calculate the actual value for S', which express the two-loop correction to the sum rule. Surprisingly, it turns out for both models that

$$S' \;=\; 0\;. \tag{106}$$

That is, the one-loop sum rule in the present models is not corrected in two-loop order.

Next we would like to address the question of whether the sum rule (62) is the unique solution to the two-loop finiteness. The two-loop finiteness for the soft scalar masses follows if the following condition

$$\begin{aligned}\rho_{ipq(0)}\rho^{jpq}_{(0)}(\kappa_i^{(1)} + \kappa_p^{(1)} + \kappa_q^{(1)}) &= -8C(i)\sum_l [\kappa_p^{(0)} - (1/3)]\; T(R_l) \\ &= -8C(i)S'\;,\end{aligned} \tag{107}$$

is satisfied. There are 15 equations for 15 unknown $\kappa^{(1)}$'s. We find that the solution is not unique; it can be parameterized by $\{7,4\}$ parameters for a given S' which is zero for the present models. For instance,

$$\begin{aligned}\kappa_{H_i}^{(1)} &= -2S' - 2\kappa_{\mathbf{10}_i}^{(1)}\;,\; \kappa_{\overline{H}_i}^{(1)} = -2S' - \kappa_{\overline{\mathbf{5}}_i}^{(1)} - \kappa_{\mathbf{10}_i}^{(1)}\;(i=1,2,3)\;, \\ \kappa_{H_4}^{(1)} &= -\frac{4S'}{3} - \kappa_{\overline{H}_4}^{(1)}\;,\; \kappa_{24}^{(1)} = -\frac{2S'}{3}\;\text{ for }\;\mathbf{A}\;,\end{aligned} \tag{108}$$

and

$$\begin{aligned}\kappa_{H_1}^{(1)} &= -2S' - 2\kappa_{\mathbf{10}_1}^{(1)}\;,\; \kappa_{24}^{(1)} = -\frac{2S'}{3}\;,\; \kappa_{\mathbf{10}_2}^{(1)} = \kappa_{\mathbf{10}_3}^{(1)}\;, \\ \kappa_{H_2}^{(1)} &= \kappa_{H_3}^{(1)} = \kappa_{H_4}^{(1)} = -2S' - 2\kappa_{\mathbf{10}_3}^{(1)}\;,\; \kappa_{\overline{H}_2}^{(1)} = \kappa_{\overline{H}_3}^{(1)} = \kappa_{\overline{H}_4}^{(1)} = \frac{2S'}{3} + 2\kappa_{\mathbf{10}_3}^{(1)}\;, \\ \kappa_{\overline{H}_1}^{(1)} &= -2S' - \kappa_{\overline{\mathbf{5}}_1}^{(1)} - \kappa_{\mathbf{10}_1}^{(1)}\;,\; \kappa_{\overline{\mathbf{5}}_2}^{(1)} = \kappa_{\overline{\mathbf{5}}_3}^{(1)} = -\frac{8S'}{3} - 3\kappa_{\mathbf{10}_3}^{(1)}\;\text{ for }\;\mathbf{B}\;.\end{aligned} \tag{109}$$

As one can easily see the $\kappa^{(1)}$'s satisfy

$$\kappa_i^{(1)} + \kappa_j^{(1)} + \kappa_k^{(1)} \;=\; -2S' = 0\;, \tag{110}$$

which shows that the sum rule (62) in the present models is the unique solution to two-loop finiteness.

Thus, the sum rule gives us the following boundary conditions at the GUT scale [7]:

$$m_{H_u}^2 + 2m_{\mathbf{10}}^2 = m_{H_d}^2 + m_{\bar{\mathbf{5}}}^2 + m_{\mathbf{10}}^2 = M^2 \quad \text{for} \quad \mathbf{A} \ , \tag{111}$$

$$\begin{aligned} m_{H_u}^2 + 2m_{\mathbf{10}}^2 &= M^2 \ , \ m_{H_d}^2 - 2m_{\mathbf{10}}^2 = -\frac{M^2}{3} \ , \\ m_{\bar{\mathbf{5}}}^2 + 3m_{\mathbf{10}}^2 &= \frac{4M^2}{3} \quad \text{for} \quad \mathbf{B}, \end{aligned} \tag{112}$$

where we use as free parameters $m_{\bar{\mathbf{5}}} \equiv m_{\bar{\mathbf{5}}_3}$ and $m_{\mathbf{10}} \equiv m_{\mathbf{10}_3}$ for the model **A**, and $m_{\mathbf{10}}$ for **B**, in addition to M.

7.3 Predictions for the Higgs mass and the S-spectrum

Since the gauge symmetry is spontaneously broken below $M_{\rm GUT}$, the finiteness and gauge-Yukawa unification conditions do not restrict the renormalization property at low energies, and all it remains are boundary conditions on the gauge and Yukawa couplings (96), the $h = -MC$ relation (60) and the soft scalar-mass sum rule (62) at $M_{\rm GUT}$, as applied in the various models. So we examine the evolution of these parameters according to their renormalization group equations at two-loop for dimensionless parameters and at one-loop for dimensionful ones with the relevant boundary conditions. Below $M_{\rm GUT}$ their evolution is assumed to be governed by the MSSM. We further assume a unique supersymmetry breaking scale M_s so that below M_s the SM is the correct effective theory.

In the SSB sector, besides the constraints imposed by reduction of couplings and finiteness, we also look for solutions which are compatible with radiative electroweak symmetry breaking. Besides the gaugino mass we have two and one more free parameters respectively in models **A** and **B**, as previously mentioned. Thus, we look for the parameter space in which the lighter $\tilde{\tau}$ mass squared $m_{\tilde{\tau}}^2$ is larger than the lightest neutralino mass squared m_{χ}^2 (which is the LSP). In the case where all the soft scalar masses are universal at the unfication scale, there is no region of $M_s = M$ below O(few) TeV in which $m_{\tilde{\tau}}^2 > m_{\chi}^2$ is satisfied. But once the universality condition is relaxed this problem can be solved naturally (provided the sum rule). More specifically, using the sum rule (62) and imposing the conditions a) successful radiative electroweak symmetry breaking b) $m_{\tilde{\tau}^2} > 0$ and c) $m_{\tilde{\tau}^2} > m_{\chi^2}$, we find a comfortable parameter space for both models (although model **B** requires large $M \sim 1$ TeV).

In fig. 1 we present the $m_{\mathbf{10}}$ dependence of m_h for $M = 0.8$ (dashed) 1.0 (solid) TeV for the finite Model **B**, which shows that the value of m_h

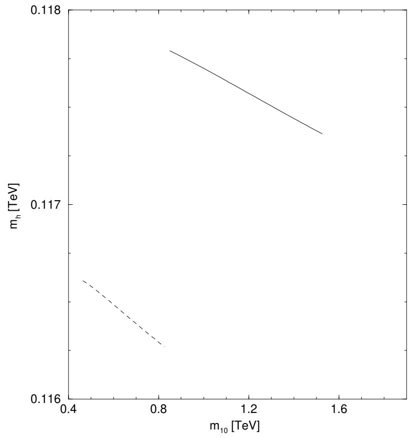

Figure 1: m_h as function of $m_{\mathbf{10}}$ for $M = 0.8$ (dashed) 1.0 (solid) TeV for the finite model **B**.

is rather stable. In fig. 2 we show the m_5 dependence of m_h for different values of M for Model **A**, when the LSP is either a neutralino or the stau. It can be seen that the m_h value can be as low as 111 GeV.

In Tables 3 and 4 we present representative examples of the values obtained for the sparticle spectra in each of the models. The value of the lightest Higgs physical mass m_h has already the one-loop radiative corrections included [66, 67]. The prediction for the Higgs mass for the two models is

$$m_h = 111 - 130 \ \ \text{GeV}, \tag{113}$$

where one uncertainty comes from variations of the gaugino mass M and the soft scalar masses, and is smaller than above in the case of the FUTB, due to the more restricted parameter space. Another uncertainty comes from the finite (i.e. not logarithmically divergent) corrections in going from the $\overline{DR}$ scheme to the pole scheme [67].

Finally, we calculate $BR(b \to s\gamma)$ [68], whose experimental value is $1 \times 10^{-4} < BR(b \to s\gamma) < 4 \times 10^{-4}$. The SM predicts $BR(b \to s\gamma) = 3.1 \times 10^{-4}$. This imposes a further restriction in our parameter space, namely $M \sim 1$ TeV if $\mu < 0$ for the two models. This restriction is less strong in the case that $\mu > 0$.

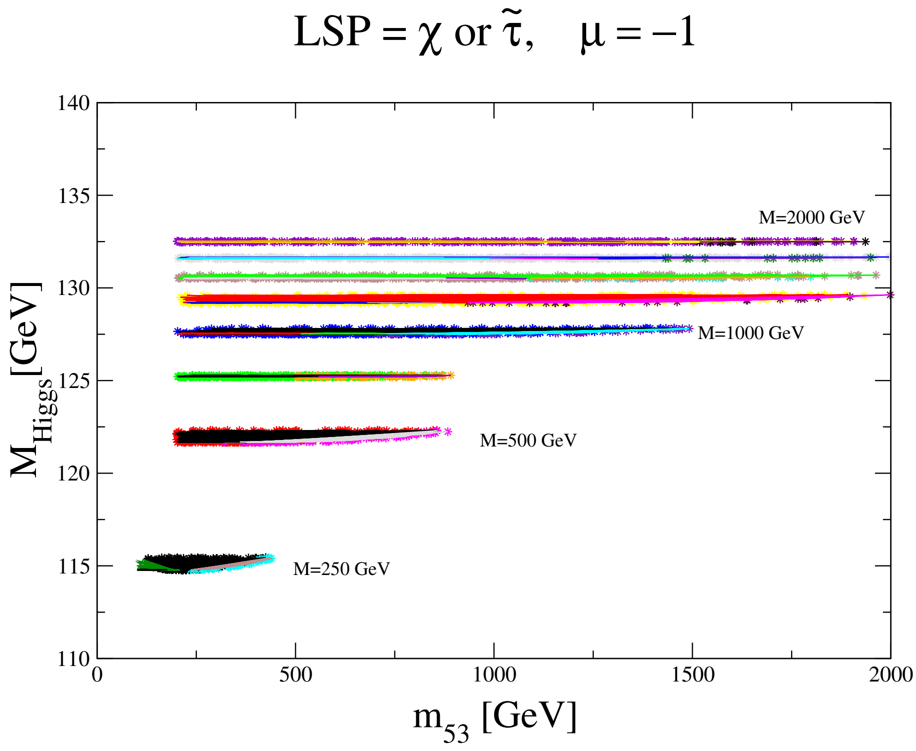

Figure 2: m_h as function of $m_{\mathbf{5}}$ for different values of M for the finite model **A**. The LSP can be either the neutralino or the $\tilde{\tau}$.

Table 3: A representative example of the predictions for the s-spectrum for the finite model **A** with $M = 1.0$ TeV, $m_{\overline{\mathbf{5}}} = 0.8$ TeV and $m_{\mathbf{10}} = 0.6$ TeV.

$m_\chi = m_{\chi_1}$ (TeV)	0.45	$m_{\tilde{b}_2}$ (TeV)	1.76
m_{χ_2} (TeV)	0.84	$m_{\tilde{\tau}} = m_{\tilde{\tau}_1}$ (TeV)	0.63
m_{χ_3} (TeV)	1.49	$m_{\tilde{\tau}_2}$ (TeV)	0.85
m_{χ_4} (TeV)	1.49	$m_{\tilde{\nu}_1}$ (TeV)	0.88
$m_{\chi_1^\pm}$ (TeV)	0.84	m_A (TeV)	0.64
$m_{\chi_2^\pm}$ (TeV)	1.49	$m_{H^\pm}$ (TeV)	0.65
$m_{\tilde{t}_1}$ (TeV)	1.57	m_H (TeV)	0.65
$m_{\tilde{t}_2}$ (TeV)	1.77	m_h (TeV)	0.122
$m_{\tilde{b}_1}$ (TeV)	1.54		

Table 4: A representative example of the predictions of the s-spectrum for the finite model **B** with $M = 1$ TeV and $m_{\mathbf{10}} = 0.65$ TeV.

$m_\chi = m_{\chi_1}$ (TeV)	0.45	$m_{\tilde{b}_2}$ (TeV)	1.70
m_{χ_2} (TeV)	0.84	$m_{\tilde{\tau}} = m_{\tilde{\tau}_1}$ (TeV)	0.47
m_{χ_3} (TeV)	1.30	$m_{\tilde{\tau}_2}$ (TeV)	0.67
m_{χ_4} (TeV)	1.31	$m_{\tilde{\nu}_1}$ (TeV)	0.88
$m_{\chi_1^\pm}$ (TeV)	0.84	m_A (TeV)	0.73
$m_{\chi_2^\pm}$ (TeV)	1.31	$m_{H^\pm}$ (TeV)	0.73
$m_{\tilde{t}_1}$ (TeV)	1.51	m_H (TeV)	0.73
$m_{\tilde{t}_2}$ (TeV)	1.73	m_h (TeV)	0.118
$m_{\tilde{b}_1}$ (TeV)	1.56		

8 Conclusions

The programme of searching for exact RGI relations among dimensionless couplings in supersymmetric GUTs, started few years ago, has now supplemented with the derivation of similar relations involving dimensionful parameters in the SSB sector of these theories. In the earlier attempts it was possible to derive RGI relations among gauge and Yukawa couplings of supersymmetric GUTs, which could lead even to all-loop finiteness under certain conditions. These theoretically attractive theories have been shown not only to be realistic but also to lead to a successful prediction of the top quark mass. The new theoretical developments include the existence of a RGI sum rule for the soft scalar masses in the SSB sector of $N = 1$ supersymmetric gauge theories exhibiting gauge-Yukawa unification. The all-loop sum rule substitutes now the universal soft scalar masses and overcomes its phenomenological problems. Of particular theoretical interest is the fact that the finite unified theories, which could be made all-loop finite in the supersymmetric sector can now be made *completely finite*. In addition it is interesting to note that the sum rule coincides with that of a certain class of string models in which the massive string modes are organized into $N = 4$ supermultiplets. Last but not least, in ref. [25] the exact β-function for the soft scalar masses in the NSVZ scheme was obtained for the first time. On the other hand the above theories have a remarkable predictive power leading to testable predictions of their spectrum in terms of very few parameters. In addition to the prediction of the top quark mass, which holds unchanged, the characteristic features that will judge the viability of these models in the future are: 1) the lightest Higgs mass is found to be around 111-130 GeV and 2) the s-spectrum starts beyond several hundreds of GeV. Therefore the next important test of Gauge-Yukawa and Finite Unified theories will be given with the measurement of the Higgs mass, for which these models show an appreciable stability, which is alarmingly close to the IR quasi-fixed point prediction of the MSSM for large tan β [69].

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