SUPERSYMMETRIC GRANDUNIFICATION AND PROTON DECAY

Borut Bajc
Jožef Stefan Institute, 1001 Ljubljana, Slovenia

ABSTRACT

This is an introduction to the idea of supersymmetric grandunification with particular emphasis on one of the few potential experimental signatures for it, i.e. proton decay. The lecture notes are intended for people with some basics of particle physics like the standard model and field theory, but do not assume any preliminary knowledge of supersymmetry and/or grandunification.

1 Introduction

There are several good reviews in supersymmetry (SUSY) and grand unification theories (GUT) that are much better and more complete than the one I am going to present in the following pages. The only good reason to read this introduction is however its shortness: one can easily enter into the main ideas of this vast and well studied area. A good student is however urged to deepen his/her knowledge reading at least some of the reviews (among others) [1, 2, 3, 4, 5, 6, 7, 8] for SUSY and [9, 10, 11] for GUTs. In these reviews one can find also all the relevant bibliography, which I skipped for space reasons.

2 Why supersymmetry?

There are many theoretical and phenomenological hints favouring supersymmetry. I will describe three of them, which are usually mentioned as the reasons to seriously consider supersymmetric theories.

2.1 The hierarchy problem

Although there are many different ways of describing this problem, one can approximately say, that the issue here is the stability (or better, instability) of the electroweak scale under quantum corrections. This is visible only in the quantum corrections of some lower dimensional operators, more precisely, in the coefficient (mass squared) of the bilinear product of scalar fields. The problem is easily seen using a sharp cutoff regularization. This regularization is however not often used in field theory, on top of this it can give also wrong results, especially because it explicitly breaks Lorentz and gauge invariance. On the contrary, dimensional regularization is the correct tool in the regularization and renormalization procedure, and so we will use it here.

To have a feeling of what the hierarchy problems is about, let us consider a simple toy model with a charged scalar field H, which I will call the Higgs boson for future use, another charged boson field ϕ and a Dirac fermion field ψ. The tree order masses of the three fields will be denoted by m_{H0}, m_{B0} and m_{F0}, while the interaction terms relevant for our discussion are

$$\mathcal{L}_{int} = -\lambda \left|H\right|^2 \left|\phi\right|^2 - yH\bar{\psi}\psi + h.c. \; . \tag{1}$$

Now let us calculate the 1-loop correction to the Higgs mass: there will be one Feynman diagram with the boson ϕ and one with the fermion ψ, giving after dimensional regularization and $\overline{MS}$ renormalization

$$m_H^2 = m_{H0}^2 - c_B \frac{\lambda}{16\pi^2} m_{B0}^2 + c_F \frac{y^2}{16\pi^2} m_{F0}^2 + \ldots \; , \tag{2}$$

where the dots denote the eventual log terms coming after the integration and are not relevant for our discussion. What is relevant is the fact that the second and third terms on the righthandside of (2) are corrections to the mass not proportional to itself, i.e. they correct the mass m_{H0}, but are not dependent on m_{H0}. In particular, they can give a nonzero correction, even if the original mass m_{H0} were zero. In other words, there is no multiplicative renormalization for the scalar (Higgs) mass.

So we have the following two possibilities:
(1) $m_{H0} \approx 100$ GeV (electroweak scale); if there are heavy particles, $m_{B0,F0} \gg m_{H0}$, it follows that $m_H \gg m_{H0}$. The Higgs mass got destabilized in this case by quantum corrections.
(2) $m_H \approx 100$ GeV (this is almost an experimental fact, although, admittedly, the Higgs boson has not been found yet). In this case, if $m_{B0,F0} \gg m_H$, one needs also a very large tree approximation $m_{H0} \gg m_H$ to cancel the large contributions due to $m_{B0,F0}$, again a strange and somewhat unnatural situation.

Of course, strictly speaking, in the standard model alone there is no problem at all. In fact:
- there is no extra boson;
- $m_{F0} \approx y < H >= \mathcal{O}(m_{H0})$.

However, as soon as we have some new physics (at M_{GUT}, M_{Planck}), eq. (2) becomes

$$\mathcal{O}(M_{GUT}^2 \;\text{ or }\; M_{Planck}^2) + m_{H0}^2 \approx m_H^2 = \mathcal{O}((100\text{GeV})^2) \; , \tag{3}$$

which gives

$$\mathcal{O}((10^{16}\mathrm{GeV})^2 \;\text{ or }\; (10^{19}\mathrm{GeV})^2) + m_{H0}^2 \approx m_H^2 = \mathcal{O}((100\mathrm{GeV})^2)\ . \qquad (4)$$

But now we clearly need to fine-tune m_{H0}! And not only approximately, but all the 16 or so decimal points! This is very unnatural and this is what we call the hierarchy problem.

This is typical for scalar (spin=0) fields. In fact for fermions (spin=1/2) one can easily find out that the one-loop correction to the mass is

$$m_F = m_{F0} + c\frac{y^2}{16\pi^2}m_{F0} + \ldots\ . \qquad (5)$$

So the correction is proportional to the tree order value, i.e. the mass renormalization for fermi fileds is multiplicative. If for example the tree order mass is zero, then no perturbation will ever generate a nonzero mass (this is not necessary for nonperturbative corrections, but these are out of the scope of these lectures and I will not consider them here), i.e.

$$m_{F0} = 0 \rightarrow m_F = 0\ . \qquad (6)$$

There is a symmetry here that forbids the appearance of a mass term. If $m_{F0} = 0$ then the Lagrangian has a $\mathrm{U}(1)^2$ chiral symmetry and it is invariant under $\psi_{L,R} \rightarrow \exp{(i\alpha_{L,R})}\psi_{L,R}$, which can not be broken at any perturbative order.

Similarly, in the case of spin 1 particles (for example gluons), it is the gauge symmetry that makes the mass renormalization multiplicative.

One of the possible solutions to the hierarchy problem is supersymmetry. The main idea here is to make the boson and fermion loop contributions in (2) cancel each other. Of course this is not easy to achieve, but from what we said we can almost immediately guess that the following must be satisfied:

(a) choose the same number of complex bosons and chiral fermions. This means that for each particle we need a partner with all the internal quantum number the same but with a different spin;

$$\begin{aligned} FERMIONS\ (\psi_i) \quad &<-> \quad SFERMIONS\ (\tilde{\psi}_i)\ , \\ GAUGE\ BOSONS\ (W) \quad &<-> \quad GAUGINOS\ (\tilde{W})\ , \\ HIGGS\ (H) \quad &<-> \quad HIGGSINO\ (\tilde{H})\ . \end{aligned}$$

The particles on the left are our standard model particles, the ones on the right are their superpartners (denoted by tilde).

(b) $m_{Fi} = m_{Bi}$

(c) $\lambda_i = y_i^2$.

These three requirements makes all the non multiplicative corrections to any boson mass vanish, because the fermion and boson contributions exactly cancel each other.

A more precise recipe for constructing a supersymmetric model can be given in the following way:

(1) write down a function of scalar fields

$$W = W(\phi_i) \ , \tag{7}$$

which is called the superpotential, and is holomorphic (i.e. it depends only on ϕ_i, not on ϕ_i^*). If we consider only renormalizable theories, then it can be shown that the superpotential needs to be at most cubic polynomial in the fields.

(2) the Yukawa terms and fermion masses can be found through (definition)

$$-\sum_{i,j} \frac{\partial^2 W}{\partial \phi_i \partial \phi_j} \psi_i \psi_j + h.c. \ , \tag{8}$$

where we use the shorthand two component notation for the fermion bilinear $\psi_i \psi_j \equiv \psi_i^T i\sigma_2 \psi_j$.

(3) the scalar potential can be found from (definition)

$$V = \sum_i \left| \frac{\partial W}{\partial \phi_i} \right|^2 \ . \tag{9}$$

One consequence of the above is that energy in supersymmetric models can never be negative. The minimum ($V = 0$) is achieved when $\partial W / \partial \phi_i = 0$ for all i, and it is true if and only if supersymmetry is not spontaneously broken.

(4) Supersymmetry must be broken. In fact there is no experimental evidence for example for the scalar partner of the electron with the same mass. Although we still need all the superpartners to exist, we will put their mass safely high enough. In doing this we must however be careful not to reintroduce the hierarchy problem. If the masses were too high, we could integrate them out and thus get the standard model back with all its hierarchy problems. In other words, the 1-loop correction to the Higgs mass in a model with $\lambda_i = y_i^2$ is

$$\delta m_H^2 = m_H^2 - m_{H0}^2 \approx \sum_i y_i^2 \frac{(m_{Bi}^2 - m_{Fi}^2)}{16\pi^2} \ . \tag{10}$$

Naturalness constraint (no fine-tuning) requires that this square mass difference should not exceed too much the Higgs mass itself (the expected one), i.e. $(100\mathrm{GeV})^2$, from which it follows that

$$y_i^2 \left| m_{Bi}^2 - m_{Fi}^2 \right| \lesssim (1\mathrm{TeV})^2 \ . \tag{11}$$

If we now take the plausible assumption (at least in nature we expect this to hold) that $m_{Bi} >> m_{Fi}$, it follows that

$$m_{Bi} \lesssim \frac{1\mathrm{TeV}}{y_i} \tag{12}$$

(but bigger than the experimental lower limit which is approximately 100 GeV). The biggest Yukawa is the top one with $y_t \approx 1$. This means that the most constrained is the stop $\tilde{t}$, which is the superpartner of the lefthanded top, and the stop $\tilde{t^c}$, superpartner of the curiously rewritten righthanded top $t^c \equiv C\bar{t}_R^T$ with C the charge conjugation matrix (in this notation the Dirac top quark is written as $\psi_t \equiv \left(t, \overline{t^c}\right)^T$). Also, since we do not want the supersymmetry breaking terms to break the weak SU(2), the sbottom $\tilde{b}$, superpartner of the lefthanded bottom quark, must also have the same mass as its SU(2) partner $\tilde{t}$. So this gives approximately

$$m_{\tilde{t},\tilde{t^c},\tilde{b}} \lesssim 1\mathrm{TeV} \; , \tag{13}$$

while for the other sfermions the constraints are milder (i.e. they can be heavier).

One comment at the end. Although all these requirements for no fine-tuning are amusing and bring into the game some constraints to the otherwise almost completely free parameters of the theory, one should not take them too seriously. After all, who can say which exactly is an admissible fine-tuning and which not? Is a 40% correction to the Higgs mass ok? Is a 7 times bigger tree order mass compared to the 1-loop mass a sign of fine-tuning? Nobody can give a firm answer, so one should have a common sense in judging these issues. Otherwise there is a danger to end in a paradoxical situation, where one minimal theory is erroeously and exaggeratedly claimed to be ruled out, while in reality there is still plenty of space to accommodate data.

(5) Now we can construct the minimal supersymmetric standard model (MSSM). With respect to the standard model we need another Higgs here (so all together two, H and $\bar{H}$), the reasons being

- anomaly cancellation (see for example the lectures of G. Zoupanos);
- since the superpotential W is holomorphic, we can not use both H and $H^\dagger$ as in the standard model to give masses to both up and down quarks.

It is easy to see that we need two Higgs doublets, H and $\bar{H}$, which are in the $(1,2,1/2)$ and $(1,2,-1/2)$ representations respectively under the gauge groups $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$.

Then the gauge invariant superpotential is

$$W = \mu H\bar{H} + HQ^T Y_U u^c + \bar{H}Q^T Y_D d^c + \bar{H}L^T Y_E e^c \; , \tag{14}$$

where Q and L are the lefthanded quark and lepton $\mathrm{SU}(2)_L$ doublets, u^c, d^c and e^c singlets, $Y_{U,D,E}$ are 3×3 matrices in generation space and the notation is symbolical, so that for example the second term should be understood as

$$HQ^T Y_U u^c \equiv H_i \epsilon_{ij} Q^{\alpha}_{aj} \left(Y_U\right)^{ab} \left(u^c\right)_{b\alpha} \ , \tag{15}$$

with the indices $i,j = 1,2$, $\alpha = 1,2,3$ and $a,b = 1,2,3$ running over SU$(2)_L$, SU$(3)_C$ and generation space respectively.

Clearly, having two Higgs doublets, one can have two vacuum expectation values in the symmetry breaking pattern SU$(3)_C\times$SU$(2)_L\times$ U$(1)_Y \to$ SU$(3)_C\times$U$(1)_{em}$:

$$H = (0, v_u) \quad , \quad \bar{H} = (v_d, 0) \quad , \tag{16}$$

so that on top of the SM phenomenological constraint $v_u^2 + v_d^2 \approx (246 \text{ GeV})^2$, one has a new parameter,

$$\tan\beta \equiv \frac{v_u}{v_d} \ . \tag{17}$$

Its numerical value is not known experimentally, but it must lie in the range approximately between 3 and 60.

The fields in (14) (except H and $\bar{H}$) should have tildes on them, being superpartners of our fermions. For simplicity we omitted them.

(6) The explicit supersymmetry breaking terms must be soft, so that they do not spoil the cancellation between the quadratic divergent diagrams (the diagrams that give nonmultiplicative contributions to the boson mass). This means that they can be written as

$$\mathcal{L}_{soft} = -m^2_{Bi} \left|\phi_i\right|^2 + ...(\text{all terms with mass dimension } \leq 3) \ , \tag{18}$$

but still maintain the relations between the bosonic quartic and Yukawa coupling constants $\lambda_i = y_i^2$, which are automatic if one employs the superpotential.

(7) In the SM baryon and lepton number are perturbatively (accidentally ?) conserved just as a consequence of renormalizability and gauge symmetry. This is no more true in the supersymmetric version, where terms like

$$\delta W = \lambda_{ijk} L_i L_j e^c_k + \lambda'_{ijk} L_i Q_j d^c_k + \lambda''_{ijk} u^c_i d^c_j d^c_k + ... \ , \tag{19}$$

which explicitly break baryon and/or lepton number, could be in principle present. These terms would be a disaster and would cause an extremely fast proton decay. The coefficients λ should then be very small, which makes many people in the field believe that they are zero. This could be even achieved by imposing a discrete Z_2 symmetry, the so called R-parity: $R = (-1)^{3(B-L)+2S}$, where S is the spin. The SM particles are even under it, while the superpartners are odd. Equivalently one can define the matter parity $M = (-1)^{3(B-L)}$, under which the SM model fermions and their superpartners change sign, while the Higgses and Higgsinos are invariant.

It is not known whether this symmetry is exact in nature or not. If it is exact, an interesting consequence follows: the lightest supersymmetric partner ($R = -1$) is stable, which could explain the dark matter in the universe.

2.2 Radiative electroweak symmetry breaking

In the SM the potential is

$$V = m_H^2 \left|H\right|^2 + \lambda \left|H\right|^4 \ . \tag{20}$$

To have the electroweak symmetry breaking, one needs $m_H^2 < 0$. There is no explanation why this is so.

In supersymmetric extensions of the SM there are also other bosons with masses $m_{\tilde{t}}^2$, $m_{\tilde{t}^c}^2$, etc. Clearly they must be positive (only H and $\bar{H}$ need negative square masses), otherwise we would for example break too much gauge symmetry, as for example the colour. So here one would like to understand why is the Higgs special with respect to the other bosons, for example the stop. In other words, one would like to see why the Higgs mass square is negative, while the masses square for the other bosons are positive. It is a remarkable fact that supersymmetry combined with gauge symmetry can give a possible explanation of this behaviour.

To see this in detail, remember first that dimensional regularization among other consists in changing the integration over loop momenta:

$$\int \frac{d^4k}{(2\pi)^4} \to \mu^{4-D} \int \frac{d^Dk}{(2\pi)^D} \ . \tag{21}$$

Calculating for example the boson loop for the Higgs mass in the toy model (1) one gets after renormalization for the physical mass (the pole of the propagator)

$$m_{PHYS}^2 = m_H^2 - y^2 \frac{m_B^2}{16\pi^2} \left(1 + \log\left(\mu^2/m_B^2\right)\right) \ . \tag{22}$$

Clearly a physical parameter must not depend on this mass parameter μ, which is possible only if the other parameters exactly cancel this dependence. To see how these, running, parameters depend on the scale μ, one first demands that

$$\mu \frac{dm_{PHYS}^2}{d\mu} = 0 \ , \tag{23}$$

from which it is easy to derive the so called renormalization group equations (RGE). Applying this idea to the Higgs and the two stop masses, one gets the following system of differential equations:

$$\mu \frac{dm_H^2}{d\mu} = 3\frac{y_t^2}{16\pi^2}\left(m_H^2 + m_{\tilde{t}^c}^2 + m_{\tilde{t}}^2\right) + \ldots , \quad (24)$$

$$\mu \frac{dm_{\tilde{t}^c}^2}{d\mu} = 2\frac{y_t^2}{16\pi^2}\left(m_H^2 + m_{\tilde{t}^c}^2 + m_{\tilde{t}}^2\right) + \ldots , \quad (25)$$

$$\mu \frac{dm_{\tilde{t}}^2}{d\mu} = 1\frac{y_t^2}{16\pi^2}\left(m_H^2 + m_{\tilde{t}^c}^2 + m_{\tilde{t}}^2\right) + \ldots , \quad (26)$$

where the dots denote other contributions not necessary for the understanding of the phenomenon. We see that the righthandsides are the same except for the numerical factor in front of them. These numerical factors are just consequences of gauge symmetry, i.e. of the representation for the Higgs and stops under the gauge groups, which is not difficult to read from the relevant Feynman diagrams.

So we see that all the three masses increase with the scale (the righthandsides are positive) and that the Higgs mass increase faster than the other two. So one can think that the three masses are for example equal at a large scale, for example at the Planck scale, $\mu = M_{Planck}$. Then one evolves the three masses down using the RGE (24)-(26). Since m_H^2 decreases faster than the other two masses squared with decreasing scale μ, one could finish with a negative $m_H^2 < 0$ at small scales, but positive $m_{\tilde{t}^c,\tilde{t}}^2 > 0$. This is indeed what happens when one chooses realistic values for the parameters, giving a very appealing explanation for the electroweak symmetry breaking in the MSSM.

2.3 Unification of couplings

One can write down the RGE also for the gauge couplings g_i:

$$\mu \frac{dg_i}{d\mu} = \frac{1}{16\pi^2} b_i g_i^3 , \quad (27)$$

where $i = 1, 2, 3$ denotes the gauge group and b is the so called β function coefficient. It can be straightforwardly calculated in each theory via (G,F,B stay for gauge bosons, fermions, bosons)

$$b = -\frac{11}{3} C_2(G) + \frac{2}{3} T_F + \frac{1}{3} T_B . \quad (28)$$

The Dynkin index

$$T(R)\delta_{ab} = Tr\left(T_a(R) T_b(R)\right) \quad (29)$$

and the second Casimir

$$C_2(R)\delta^{ij} = \sum_a (T_a(R)T_b(R))^{ij} \tag{30}$$

depend on the choice of the gauge group and on the representation involved. The indices a, b run over the generators of the group ($N^2 - 1$ in SU(N)), while i, j run from 1 to the dimension of the representation. The normalization usually chosen is $T = 1/2$ for the fundamental representation (quarks, leptons). Then one has in the SU(N) group for the fundamental representation $C_2 = (N^2 - 1)/(2N)$, and for the adjoint $T = C_2 = N$. The dimension of the representation is N for fundamentals and $N^2 - 1$ for adjoint. To remember also that in SU(2) the generators in the fundamental are the Pauli matrices $T_a^{ij} = \tau_a^{ij}/2$, while in the adjoint representations are the Levi-Civita antisymmetric tensor $T_a^{ij} = -i\epsilon_{aij}$.

For supersymmetric theories we know that for each fermion (boson) there is a boson (fermion) in the same group representation, so (28) can be written more compactly as

$$b = -3C_2(G) + T\ . \tag{31}$$

It is a simple exercise to show that the beta coefficients are $b_i = (33/5, 1, -3)$ in the MSSM and $b_i = (41/10, -19/6, -7)$ in the SM. Negative coefficients here mean asymptotic freedom. One knows the experimental values of g_i at M_Z and can evolve them towards larger scales μ using (27). It is then surprising that in the supersymmetric case the three couplings unify in a single point at $\mu \approx 10^{16}$ GeV. To appreciate this fact one should notice that this unification fails badly in the nonsupersymmetric case. So, if we have supersymmetric partners at M_Z or close to 1 TeV as required by naturalness (hierarchy problem), then we have unification of gauge couplings for free!

3 Why Grand Unification?

We have just seen that supersymmetry predicts the unification of couplings. This is a very strong argument to believe in supersymmetry, but it is also another argument in favour of the beautiful idea, that all known interactions (except gravity) are really low energy remnants of one single interaction. On top of this already remarkable idea, I want to point out some other bonuses one gets assuming unification of couplings.

3.1 Unification (partial) of matter

In the standard model (supersymmetric or not) we have not only many interactions, we have also several representations for matter (quarks and leptons). It is difficult to believe that a fundamental theory has so many representations (5 for each generation, Q, L, u^c, d^c and e^c). It is reassuring to find out that the number of these representation drastically diminish going to grand unification groups, as we will see in the following.

The minimal unification gauge group, which has as a subgroup the SM $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$, is the Georgi-Glashow SU(5). In it we need only two representations for each generation:

$$10 = \begin{pmatrix} 0 & u_3^c & -u_2^c & u_1 & d_1 \\ -u_3^c & 0 & u_1^c & u_2 & d_2 \\ u_2^c & -u_1^c & 0 & u_3 & d_3 \\ -u_1 & -u_2 & -u_3 & 0 & e^c \\ -d_1 & -d_2 & -d_3 & -e^c & 0 \end{pmatrix} \quad , \quad \bar{5} = \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \\ \nu \\ e \end{pmatrix} \quad . \tag{32}$$

In the next to minimal grand unified group, SO(10), we have an even better situation, all the fermions of one generation are in the same representation, 16 ($= 10 + \bar{5} + 1$), predicting an SU(5) singlet, the righthanded neutrino. Being a SM singlet, its mass is naturally very high, but its presence gives a tiny mass and mixing to the light neutrinos through the see-saw mechanism. In this respect the SO(10) grand unified theory is superior to SU(5), since it is also a theory of the neutrino mass.

There are many other advantages of the SO(10) theory: for example, using the 126 dimensional Higgs one can naturally explain the low-energy R-parity of the MSSM, improve the fit of fermion masses and mixings and even connect the large atmospheric neutrino mixing angle with the $b - \tau$ unification.

3.2 Electric charge quantization

In a U(1) gauge theory there are no reasons for the charges to be quantized. There are however two known exceptions:
(a) anomaly cancellation; it can be shown that already in the standard model charges must be quantized, otherwise the theory would be anomalous, i.e. at the quantum level gauge currents would not be conserved, thus spoiling the Ward identities needed to prove renormalizability.
(b) magnetic monopole; already Dirac has shown long ago that the existence of one single magnetic monopole in the whole universe would cause all the electric charges to be quantized, due to the requirement of single-valuedness of the wave functions.

It is interesting to notice that both (a) and (b) are just consequences of the embedding of the U(1) in a larger grandunified theory (with a simple or semisimple group).

In fact in this case the U(1) generator can be written as a linear combination of the other generators of the grand unified (semi)simple group, $\hat{Q} = \sum_i c_i \hat{T}_i$. But then, for each multiplet, $Tr\hat{Q} = 0$, since for a (semi)simple group the generators are traceless. It is of course trivial to check the above statement for the electric charge in the 10 and $\bar{5}$ dimensional representations of SU(5).

4 Some properties of the minimal SUSY SU(5)

Let us briefly describe the main characteristic of the SU(5) theory.

4.1 The Higgs sector

To spontaneously break the gauge symmetry SU(5) into the standard model $SU(3)_C \times SU(2)_L \times U(1)_Y$ we will employ the 24-dimesional adjoint representation Higgs Σ. The most general renormalizable superpotential for such a Higgs is

$$W_\Sigma = \frac{\lambda}{3} Tr\ \Sigma^3 + \frac{m_\Sigma}{2} Tr\ \Sigma^2 + cTr\ \Sigma\ , \tag{33}$$

where c is the Lagrange multiplier that takes care of the tracelessness of the 5×5 complex matrix Σ. The minimum of the potential $V = |\partial W / \partial \Sigma|^2$ is achieved when

$$\frac{\partial W_\Sigma}{\partial \Sigma_{ji}} = \lambda \left(\Sigma^2\right)_{ij} + m_\Sigma \left(\Sigma\right)_{ij} + c\delta_{ij} = 0\ \ ,\ \ \frac{\partial W_\Sigma}{\partial c} = Tr\ \Sigma = 0\ \ . \tag{34}$$

We have the gauge freedom to diagonalize Σ. Since (34) is quadratic in the field (this is a consequence of the assumption of renormalizability), there are just two possible solutions to it. These two vevs (σ_i) can be present different number of times as diagonal elements of Σ:
(1) $\Sigma = diag(\sigma_1, \sigma_1, \sigma_1, \sigma_2, \sigma_2)$, i.e. we break SU(5) into the SM SU(3)$\times$ SU(2)$\times$U(1);
(2) $\Sigma = diag(\sigma'_1, \sigma'_1, \sigma'_1, \sigma'_1, \sigma'_2)$, i.e. the final gauge group is SU(4)$\times$U(1). Notice that $\sigma'_i \neq \sigma_i$.

Tracelessness of Σ requires
(1) $\Sigma = \sigma\ diag(2, 2, 2, -3, -3)$;
(2) $\Sigma = \sigma'\ (1, 1, 1, 1, -4)$.

One calculates now σ (or σ', which is however physically not relevant) from (34), using for the Lagrange multiplier c

$$\delta_{ij} \frac{\partial W}{\partial \Sigma_{ji}} = 0 = \lambda Tr \Sigma^2 + 5c \ \ \to \ \ c = -\frac{\lambda}{5} Tr \Sigma^2\ . \tag{35}$$

The rest is left as an exercise.

To have the right low energy (MSSM) degrees of freedom, one must include also a fundamental 5_H and an antifundamental Higgs representations $\bar{5}_H$ with for example the following contribution to the superpotential:

$$W_H = \bar{5}_H\,(\eta\Sigma + m_H)\,5_H\ . \tag{36}$$

One needs a fine tuning of the parameters to get the MSSM term $\mu\bar{H}H$, with $\mu = \mathcal{O}(100\text{GeV})$ instead of the huge $M_{GUT} = \mathcal{O}(10^{16}\text{GeV})$. This, the so-called doublet-triplet problem, is the remnant of the gauge hierarchy problem in supersymmetric theories. Although there exist some interesting and elegant solutions to this problem, it is not clear yet, which one is chosen by nature. In any case the doublet-triplet problem in supersymmetric theories is better than the hierarchy problem in ordinary nonsupersymmetric theories: once we fine-tune the parameters in supersymmetry, they do not get spoiled by higher orders in perturbation theory, since the loop corrections are only logarithmic and thus under better control.

We have described up to now the renormalizable minimal supersymmetric SU(5). We must keep in mind however, that the requirement of renormalizabilty is just an assumption and it may well not be true. In fact, the GUT scale is quite close to the Planck mass (in comparison with the electroweak scale), so that one could add terms proportional to Σ/M_{Pl} which are guage invariant and could be important. Suppose for example that the coupling λ is small, so that the next interaction term, $Tr\Sigma^4$ dominates the superpotential. Although the breaking can be similar to the one described, the two theories give dramatically different predictions, so that one should in principle allow also these nonrenormalizable operators, at least until there is an experimental sign that show they are negligible.

4.2 The Yukawa sector

A general rule in finding invariants in SU(N) is just have the same number of indices up (representations 5, 10,...) and number of indices down ("antirepresentations" $\bar{5}$, $\overline{10}$,...), where also the use of the completely antisymmetric Levi-Civita tensor $\epsilon_{\alpha_1\ldots\alpha_N}$ is allowed. In our, SU(5) case, we get the following renormalizable terms

$$W_Y = \epsilon^{ijklm} 5_{Hi} 10^a_{jk}\,(Y_U)_{ab}\,10^b_{lm} + \bar{5}^i_H 10^a_{ij}\,(Y_D)_{ab}\,\bar{5}^{bj}\ , \tag{37}$$

where $i, j, ... = 1, ...5$ are SU(5) indices, while $a, b = 1, 2, 3$ are generation indices.

In order not to get terms like for example $\bar{5}$ 10 5, which badly break baryon and lepton number, we had to assume the R-parity with R(10,$\bar{5}$)$=$ -1 and R(5_H,$\bar{5}_H$,Σ)$=$ 1.

We find that SU(5) symmetry predicts some relations between MSSM Yukawa terms, noting that

$$5_H = \begin{pmatrix} T \\ H \end{pmatrix} \quad , \quad \bar{5}_H = \begin{pmatrix} \bar{T} \\ \bar{H} \end{pmatrix} \quad , \tag{38}$$

with T ($\bar{T}$) are $\mathrm{SU}(3)_C$ (anti)triplets and H, $\bar{H}$ the SM $\mathrm{SU}(2)_L$ Higgs doublets. From (37) using (32)

$$Y_U = Y_U^T \quad , \quad Y_D = Y_E \quad . \tag{39}$$

Among other the last relation predicts

$$y_b = y_\tau \ , \tag{40}$$

which is true at M_{GUT}. One of course has to use then the RGE and at $\mu \approx M_Z$ this comes out to be consistent with the experimental values.

At the same time however relations like $y_s = y_\mu$ and $y_d = y_e$ at $\mu = M_{GUT}$ come out to be wrong (ideally one would need something like $y_s \approx y_\mu/3$ and $y_d \approx 3y_e$). Possible improvements to these predictions could come out from the introduction of nonrenormalizable operators like

$$\Delta W_Y = \epsilon^{ijklm} 5_{Hi} \frac{\Sigma_j^p}{M_{Pl}} 10^a_{pk} \left(Y'_U\right)_{ab} 10^b_{lm} + \dots \ . \tag{41}$$

The ratio Σ/M_{Pl} is numerically $10^{-2} - 10^{-3}$, so it could be important to the first and second generation Yukawas, which are also of this order or smaller.

4.3 The Gauge Sector

The gauge multiplet is in the adjoint representation, $24 = 5 \times \bar{5}$ traceless:

$$A_\mu = \begin{pmatrix} & & & X^1 & Y^1 \\ & G & & X^2 & Y^2 \\ & & & X^3 & Y^3 \\ X_1^* & X_2^* & X_3^* & W_3 & W^+ \\ Y_1^* & Y_2^* & Y_3^* & W^- & -W_3 \end{pmatrix}_\mu + c\, B_\mu\, diag(2,2,2,-3,-3) \quad , \tag{42}$$

where G are the 8 gluons, the W's are the weak gauge bosons, B is the hypercharge gauge boson, while X and Y are colour triplets. The electric charges are $Q_E(X) = -4/3$, $Q_E(Y) = -1/3$. The X and Y gauge bosons get a large $\mathcal{O}(M_{GUT})$ mass through the Higgs mechanism:

$$|\partial_\mu \Sigma + i g_U \left[A_\mu, \Sigma\right]|^2 \to g_U^2 \sigma^2 \left|\left[A_\mu, diag(2,2,2,-3,-3)\right]\right|^2 \ , \tag{43}$$

from which it follows that $m_{X,Y} \approx g_U \sigma$.

Gluons, W's and B remain massless at this stage. The final (SM) symmetry breaking $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \to \mathrm{U}(1)_{em}$ takes place only when H and $\bar{H}$ get their small (order 100 GeV) vevs.

5 Proton decay

Supersymmetry itself (MSSM) predicts the existence of superpartners at the TeV energy. Grand unified theories on the other hand have essentially two smoking guns: the existence of magnetic monopoles and proton decay. Although they are generic in both ordinary and supersymmetric versions, they can differ numerically. Due to lack of space I will limit myself just to a short explanation of the proton decay.

The generic predictions for proton decay come through the exchange of heavy X and Y gauge bosons. Their interaction with matter does not depend on supersymmetry. The X and Y gauge bosons interactions can break B and/or L number, but maintain $B-L$. It is not difficult to show, that the heavy propagator gives a suppression factor $g_U^2/M_{X,Y}^2 \approx 1/M_{GUT}^2$. The effective operator connects four fermions, so it is called dimension 6 ($d=6$) proton decay operator.

The experimental limit on proton lifetime is $\tau_p \gtrsim 10^{33}$ years. This gives a limit for $M_{GUT} \gtrsim 10^{15.5}$ GeV just from the contribution of $d=6$ operators.

¿From the running in the MSSM with the assumption of desert $M_{GUT} \approx 2.10^{16}$ GeV, and so the corresponding proton lifetime due to $d=6$ operators would be $\tau_p(d=6) \approx 10^{36}$ yrs, which is too much for present day experiments. We can not hope to find an experimental sign of $d=6$ operators today.

There is however another possibility in supersymmetric GUTs. This comes from the exchange of heavy colour triplets from 5_H, $\bar{5}_H$. ¿From (37) one can derive using (32) and (38)

$$\begin{aligned} W &= HQ^T Y_U u^c + \bar{H} Q^T Y_D d^c + \bar{H} L^T Y_D e^c \\ &+ TQ^T Y_U Q + T u^{cT} Y_U e^c + \bar{T} Q^T Y_D L + \bar{T} u^{cT} Y_D d^c \,, \end{aligned} \tag{44}$$

which however gives wrong predictions already in the light Yukawa sector as we saw in section 4.2. One then needs to include nonrenormalizable corrections as in (41). So the fermion masses can be corrected, but then we can also have a more general superpotential

$$\begin{aligned} W &= HQ^T Y_U u^c + \bar{H} Q^T Y_D d^c + \bar{H} L^T Y_E e^c \\ &+ TQ^T \underline{A} Q + T u^{cT} \underline{B} e^c + \bar{T} Q^T \underline{C} L + \bar{T} u^{cT} \underline{D} d^c \,, \end{aligned} \tag{45}$$

with now $\underline{A}$, $\underline{B}$, $\underline{C}$ and $\underline{D}$ that can have some freedom.

Anyway, the exchange of heavy triplets T and $\bar{T}$ can also mediate baryon and lepton number violating interactions. Since it gives an effective operator between two fermions and two sfermions, it is called dimension 5 contribution to proton decay. The two sfermions must then exchange a gaugino or higgsino to give two fermions in the final state. The important point here is that being $d=5$ operators, they are suppressed only by one power of the large triplet mass $m_T \approx M_{GUT}$. This could give in principle a far too large contribution, but fortunately other suppression factors make the lifetime almost compatible with experiment even in the nonrealistic case of completely renormalizable superpotential (37), (33) and (36).

The freedom in the value of m_T due to nonrenormalizable contributions in (33) and the freedom in $\underline{A}$, $\underline{B}$, $\underline{C}$ and $\underline{D}$ are large enough to make the experiments still compatible with SUSY SU(5). On top of this, the SUSY breaking terms have too many unknowns. It is however true that in the near future we should be able to measure proton decay or rule out the minimal superymmetric SU(5).

Acknowledgments

It is a pleasure to thank Branko Dragovich and the other organizers of the school as well as the participants for the stimulating and pleasant environment. I thank also my friends and collaborators Charan Aulakh, Pavel Fileviez Perez, Ilia Gogoladze, Ramon Guevara, Alejandra Melfo, Andrija Rašin, Francesco Vissani and especially Goran Senjanović for teaching me much of the physics presented here. This work has been supported by the Ministry of Education, Science and Sport of the Republic of Slovenia.

References

[1] J. P. Derendinger, *Prepared for 3rd Hellenic School on Elementary Particle Physics, Corfu, Greece, 13-23 Sep 1989*

[2] M. Dine, arXiv:hep-ph/9612389.

[3] S. P. Martin, arXiv:hep-ph/9709356.

[4] M. F. Sohnius, Phys. Rept. **128** (1985) 39.

[5] H. E. Haber and G. L. Kane, Phys. Rept. **117** (1985) 75.

[6] H. P. Nilles, Phys. Rept. **110** (1984) 1.

[7] J. D. Lykken, arXiv:hep-th/ 9612114.

[8] J. Wess and J. Bagger, "Supersymmetry And Supergravity", Princeton University Press (1992).

[9] R. N. Mohapatra, arXiv:hep-ph/9911272.

[10] R. N. Mohapatra, "Unification And Supersymmetry. The Frontiers Of Quark - Lepton Physics", Springer-Verlag (1986).

[11] G. G. Ross, “Grand Unified Theories”, Benjamin/Cummings (1985) .