On renormalizability of the quantum electrodynamics on noncommutative space

Maja Burić[1] and Voja Radovanović[2]
Faculty of Physics, University of Belgrade, P.O. Box 368, 11001 Belgrade, Serbia and Montenegro

ABSTRACT

In this paper we calculate the divergent part of the one loop effective action for QED on noncommutative space using the background field method. The effective action is obtained up to the second order in the noncommutativity parameter θ and in the classical fields.

[1]E-mail: majab@ff.bg.ac.yu
[2]E-mail: rvoja@ff.bg.ac.yu

1 Introduction

The discovery of noncommutative (NC) structures in string theory intensified the investigation of noncommutative field theories and noncommutative geometry in the last couple of years. An important aspect of this investigation is a representation-free formulation of the theory - endowing the abstract algebra of noncommuting coordinates with structures like derivations, forms, fields etc. [1, 2, 3]. Various types of noncommutativity can be analyzed in this way. The most often considered one is the canonical structure, defined by

$$[\hat{x}^\mu, \hat{x}^\nu] = i\theta^{\mu\nu} \ , \tag{1}$$

where $\hat{x}^\mu$, $\hat{x}^\nu$ are the elements of the algebra, and $\theta^{\mu\nu} = -\theta^{\nu\mu}$ are constants of dimension length^2 . Other structures like Lie algebra or quantum plane have been discussed as well [1, 2]. The other line of work is to represent NC theory by the fields on commutative space, encoding noncommutativity in the definition of the product. The multiplication which corresponds to the canonical structure (1) is the so-called Moyal-Weyl or $\star$-product:

$$f \star g = e^{\frac{i}{2}\theta^{\mu\nu}\frac{\partial}{\partial x^\mu}\frac{\partial}{\partial y^\mu}} f(x)g(y)|_{y\to x} \ , \tag{2}$$

where f and g are functions of the coordinates x^μ on $\mathbf{R^4}$. Obviously,

$$[x^\mu \stackrel{\star}{,} x^\nu] = x^\mu \star x^\nu - x^\nu \star x^\mu = i\theta^{\mu\nu} \ . \tag{3}$$

It is possible to define $*$-products which correspond to the other types of noncommutativity, too.

The $\star$-product (2) with its properties in integration provides us with a new class of actions characterized by dimensionfull parameter θ and nonlocal lagrangians. In this setting the definition of noncommutative scalar field theories like Φ^3 or Φ^4 is straightforward. However, if one wants to define a gauge theory, the use of noncommutative multiplication rule imposes severe restrictions both on the choice of gauge group and on the choice of representation [4, 5, 6]. For example, in NC electrodynamics the values of charge are quantized and restricted to $\pm 1, 0$.

The result of Seiberg and Witten (SW) [7] on the equivalence of classes of commutative and noncommutative gauge theories shows that noncommutativity is not equivalent to quantization. Noncommutative field theories can be quantized in the conventional perturbative way [8, 9, 10]. By now, a number of properties of NC field theories with constant θ are established. Some novel features appear, e.g. UV and IR sectors are mixed in the perturbative expansion. The UV/IR mixing can be seen in the fact that 'nonplanar' diagrams contain terms proportional to $|\tilde{p}|^{-n}$, with $\tilde{p}^\mu = \theta^{\mu\nu}p_\nu$. In principle, one might expect that the apparent nonrenormalizability of NC theories (seen already in the fact that the 'coupling constant' θ is dimensionfull) disappears after the summation of perturbation series due to some special properties of the lagrangian. This was indeed shown for the NC Φ^4 theory in [13]; however, further investigations seem to question this result [14] and prove that the only renormalizable noncommutative theories are the supersymmetric ones.

An obvious drawback of the perturbative treatment of NC theories is that the results, expressed as $|\tilde{p}|^{-n}$, are nonperturbative in the parameter θ. This means that one cannot make a smooth commutative limit or estimate the effects of noncommutativity in the lowest order (in the sectors where they are small). In order to deal with this problem one uses the complementary approach of [1] defined for the field theories with gauge symmetry. As it is shown in [1], the representation of the gauge symmetry on NC space induces the expansion of noncommutative fields in the parameter θ in terms of fields on commutative space which coincides with the SW expansion. The corresponding commutative 'physical' fields carry the usual representation of gauge symmetry on $\mathbf{R^4}$. The SW map induces θ-expansion in the action as well. This gives the possibility to treat θ-linear and θ-quadratic terms in the action as the first corrections, describing the effects of noncommutativity in the lowest order. Further, θ-expanded action enables to approach the problem of quantization in a different way, i.e. considering the lagrangian order by order in θ. In this context, nonuniqueness of the SW map [15] takes the role of an additional 'symmetry'. Indeed, using the particular properties of SW-expanded action, Bichl et al. [16] succeeded to prove full renormalizability of the photon propagator for the pure U(1) NC gauge theory. However, inclusion of the matter spoils renormalizability [16, 17, 18], in spite of the fact that diagrams in θ-linear order have a high degree of symmetry. In the case of massless fermions only one term breaks renormalizability [19]. This behavior motivates the investigation of the theory in the θ^2-order.

Even if not renormalizable, θ-expanded gauge theories can be treated as effective theories at 'low' energies. Various physical models, like noncommutative generalizations of the standard model, have been proposed so far [20, 21]. In that context also it is of importance to calculate the divergent counterterms, as they describe the possible effective interaction vertices.

This lecture is organized as follows. Since a large part of the results on the θ^2-one-loop corrections of the propagators were presented in [22], we summarize them without entering into details. We show how the additional contributions from θ^2-classical lagrangian are obtained and thus complete the result in the θ^2-order. We also give a somewhat more extensive discussion of the field redefinitions and their implications to renormalizability.

2 Classical theory

The noncommutative space which we use is $\mathbf{R^4}$ with the canonical structure

$$[x^\mu \stackrel{\star}{,} x^\nu] = i\theta^{\mu\nu} \ , \tag{4}$$

where $\mu, \nu = 0, \ldots 3$ and $\star$ is the Moyal-Weyl product (2). The classical action for electrodynamics on this space is given by

$$S = \int d^4x \, \bar{\hat{\psi}} \star (i\gamma^\mu \hat{D}_\mu - m)\hat{\psi} - \frac{1}{4}\int d^4x \hat{F}_{\mu\nu} \star \hat{F}^{\mu\nu} \ . \tag{5}$$

Here, $\hat{\psi}$ is the noncommutative fermionic matter field while $\hat{A}_\mu$ is the gauge potential. The corresponding field strength $\hat{F}_{\mu\nu}$ is defined as

$$\hat{F}_{\mu\nu} = \partial_\mu \hat{A}_\nu - \partial_\nu \hat{A}_\mu - i(\hat{A}_\mu \star \hat{A}_\nu - \hat{A}_\nu \star \hat{A}_\mu) \ , \tag{6}$$

and the covariant derivative $\hat{D}_\mu \hat{\psi}$

$$\hat{D}_\mu \hat{\psi} = \partial_\mu \hat{\psi} - i\hat{A}_\mu \star \hat{\psi} \ . \tag{7}$$

The fields $\hat{\psi}$, $\hat{A}_\mu$, $\hat{F}_{\mu\nu}$ which give a representation of noncommutative electrodynamics can be, via the SW map, mapped into the representation of ordinary U(1). To the first orders in θ the map is given by [2]:

$$\begin{aligned}
\hat{A}_\mu &= A_\mu^{(0)} + A_\mu^{(1)} + A_\mu^{(2)} + \ldots \qquad (8)\\
A_\mu^{(0)} &= A_\mu \\
A_\mu^{(1)} &= -\frac{1}{2}\,\theta^{\rho\sigma} A_\rho \left[(\partial_\sigma A_\mu) + F_{\sigma\mu}\right] \\
A_\mu^{(2)} &= \frac{1}{2}\,\theta^{\alpha\beta}\theta^{\rho\sigma}\left[A_\alpha A_\rho (\partial_\sigma F_{\beta\mu}) - (\partial_\beta A_\mu)(\partial_\rho A_\alpha) A_\sigma + A_\alpha F_{\beta\rho} F_{\sigma\mu}\right] \\
\hat{\psi} &= \psi^{(0)} + \psi^{(1)} + \psi^{(2)} + \ldots \qquad (9)\\
\psi^{(0)} &= \psi \\
\psi^{(1)} &= -\frac{1}{2}\,\theta^{\rho\sigma} A_\rho (\partial_\sigma \psi) \\
\psi^{(2)} &= \frac{1}{32}\,\theta^{\alpha\beta}\theta^{\rho\sigma}\big[-4i(\partial_\alpha A_\rho)(\partial_\beta \partial_\sigma \psi) \\
&+ 4A_\alpha A_\rho (\partial_\beta \partial_\sigma \psi) + 8A_\alpha (\partial_\beta A_\rho)(\partial_\sigma \psi) \\
&+ 4A_\alpha F_{\beta\rho}(\partial_\sigma \psi) - 2(\partial_\alpha A_\rho)(\partial_\beta A_\sigma)\psi + 8iA_\alpha A_\sigma (\partial_\rho A_\beta)\psi\big]
\end{aligned}$$

$$
\begin{aligned}
\hat{F}_{\mu\nu} &= F^{(0)}_{\mu\nu} + F^{(1)}_{\mu\nu} + F^{(2)}_{\mu\nu} + \ldots \qquad (10)\\
F^{(0)}_{\mu\nu} &= F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \\
F^{(1)}_{\mu\nu} &= \theta^{\rho\sigma}\left[F_{\mu\rho}F_{\nu\sigma} - A_\rho(\partial_\sigma F_{\mu\nu})\right] \ , \text{ etc.}
\end{aligned}
$$

Inserting (8) and (9) into the action (5), we get the classical θ-expanded action [2, 16]

$$
S = S^{(0)} + S^{(1)}_A + S^{(1)}_\psi + S^{(2)}_A + S^{(2)}_\psi + \ldots \ , \qquad (11)
$$

with

$$
\begin{aligned}
S^{(0)} &= \int d^4x \left[\bar\psi(i\gamma^\mu D_\mu - m)\psi - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}\right] , \qquad (12)\\
S^{(1)}_A &= -\frac{1}{2}\,\theta^{\rho\sigma}\int d^4x \left[F_{\mu\rho}F_{\nu\sigma}F^{\mu\nu} - \frac{1}{4}F_{\rho\sigma}F_{\mu\nu}F^{\mu\nu}\right] , \qquad (13)\\
S^{(1)}_\psi &= \frac{1}{2}\,\theta^{\rho\sigma}\int d^4x \Big[-iF_{\mu\rho}\bar\psi\gamma^\mu D_\sigma\psi + \frac{1}{2}F_{\rho\sigma}\bar\psi(-i\gamma^\mu D_\mu + m)\psi\Big] \ , \qquad (14)\\
S^{(2)}_A &= \int d^4x \Big[-\frac{1}{4}F^{(1)}_{\alpha\beta}F^{(1)\alpha\beta} - \frac{1}{2}F^{(0)}_{\alpha\beta}F^{(2)\alpha\beta}\Big]\\
&= \int d^4x\,\frac{1}{4}\,\theta^{\alpha\beta}\theta^{\mu\nu}\Big[-F^\rho{}_\alpha F^\sigma{}_\beta F_{\rho\mu}F_{\sigma\nu} - A_\alpha(\partial^\rho F_{\rho\sigma})F_{\mu\nu}F^\sigma{}_\beta\\
&+ 2A_\mu(\partial^\sigma A_\nu)(\partial_\beta A_\mu)(\partial^\rho F_{\rho\sigma}) - 2A_\alpha(\partial^\rho F_{\rho\sigma})F_{\beta\mu}F^\sigma{}_\nu\\
&+ A_\mu(\partial_\nu F^{\rho\sigma})F_{\rho\alpha}F_{\sigma\beta}\\
&- A_\mu(\partial_\alpha A_\rho)(\partial_\nu F^{\rho\sigma})F_{\sigma\beta} - A_\alpha(\partial_\beta A^\sigma)(\partial_\mu A^\rho)(\partial_\nu F_{\rho\sigma})\\
&+ A_\alpha A_\mu(\partial^\rho F^\sigma{}_\beta)(\partial_\nu F_{\rho\sigma}) + A_\alpha F_{\sigma\beta}F_{\mu\rho}(\partial_\nu F^{\rho\sigma})\Big] \ , \qquad (15)\\
S^{(2)}_\psi &= \int d^4x \Big[\bar\psi^{(2)}i\gamma^\mu(\partial_\mu\psi) + \bar\psi i\gamma^\mu(\partial_\mu\psi^{(2)}) + \bar\psi^{(1)}i\gamma^\mu(\partial_\mu\psi^{(1)})\\
&- m\bar\psi^{(2)}\psi - m\bar\psi\psi^{(2)} - m\bar\psi^{(1)}\psi^{(1)}\\
&+ \bar\psi^{(2)}\gamma^\mu A_\mu\psi + \bar\psi\gamma^\mu A^{(2)}_\mu\psi + \bar\psi\gamma^\mu A_\mu\psi^{(2)}\\
&+ \bar\psi^{(1)}\gamma^\mu A^{(1)}_\mu\psi + \bar\psi\gamma^\mu A^{(1)}_\mu\psi^{(1)} + \bar\psi^{(1)}\gamma^\mu A_\mu\psi^{(1)}\\
&+ \frac{i}{2}\,\theta^{\alpha\beta}[\bar\psi^{(1)}\gamma^\mu(\partial_\alpha A_\mu)(\partial_\beta\psi)\\
&+ \bar\psi\gamma^\mu(\partial_\alpha A^{(1)}_\mu)(\partial_\beta\psi) + \bar\psi\gamma^\mu(\partial_\alpha A_\mu)(\partial_\beta\psi^{(1)})]\\
&- \frac{1}{8}\,\theta^{\alpha\beta}\theta^{\rho\sigma}\bar\psi\gamma^\mu(\partial_\alpha\partial_\rho A_\mu)(\partial_\beta\partial_\sigma\psi)\Big] \ . \qquad (16)
\end{aligned}
$$

The 'commutative' covariant derivative is $D_\mu\psi = \partial_\mu\psi - iA_\mu\psi$. Notice that $S^{(2)}_A$ is quartic in the gauge potentials, while $S^{(2)}_\psi$ contains terms with 3, 4 and 5 fields. For example, the part of $S_{\psi}^{(2)}$ with 3 fields is

$$\begin{aligned} S_{\psi,3}^{(2)} &= \int d^4x\, \frac{1}{8}\,\theta^{\alpha\beta}\theta^{\rho\sigma}\Big[(\partial_\alpha\partial_\rho A_\sigma)\bar{\psi}\gamma^\mu(\partial_\beta\partial_\mu\psi) + (\partial_\alpha\partial_\mu A_\rho)\bar{\psi}\gamma^\mu(\partial_\beta\partial_\sigma\psi) \\ &- (\partial_\alpha\partial_\rho A_\mu)\bar{\psi}\gamma^\mu(\partial_\beta\partial_\sigma\psi) + im\,(\partial_\alpha\partial_\rho A_\sigma)\bar{\psi}(\partial_\beta\psi)\Big]\ . \end{aligned} \tag{17}$$

For the purpose of functional integration, we express the Dirac spinor in terms of the Majorana spinors. They are introduced as $\psi_{1,2} = \frac{1}{2}(\psi \pm \psi^C)$, where $\psi^C = C\bar{\psi}^T$ is the charge-conjugated spinor. The Dirac spinor is $\psi = \psi_1 + i\psi_2$; the actions in terms of Majorana spinors are of the form

$$\begin{aligned} S^{(0)} &= \int d^4x \Big[\bar{\psi}_1(i\gamma^\mu\partial_\mu - m)\psi_1 + \bar{\psi}_2(i\gamma^\mu\partial_\mu - m)\psi_2 \\ &+ i\bar{\psi}_1\gamma^\mu A_\mu\psi_2 - i\bar{\psi}_2\gamma^\mu A_\mu\psi_1 - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}\Big]\ , \end{aligned} \tag{18}$$

$$\begin{aligned} S_{\psi}^{(1)} &= \frac{1}{2}\,\theta^{\rho\sigma}\int d^4x \Big[\Big(-\frac{i}{2}\bar{\psi}_1\gamma^\mu(F_{\mu\rho}\partial_\sigma + F_{\rho\sigma}\partial_\mu + F_{\sigma\mu}\partial_\rho)\psi_1 + \frac{1}{2}\,mF_{\rho\sigma}\bar{\psi}_1\psi_1 \\ &- \frac{i}{2}\,\bar{\psi}_2\gamma^\mu(F_{\mu\rho}\partial_\sigma + F_{\rho\sigma}\partial_\mu + F_{\sigma\mu}\partial_\rho)\psi_2 + \frac{1}{2}\,mF_{\rho\sigma}\bar{\psi}_2\psi_2 \\ &- \frac{i}{2}\bar{\psi}_1\gamma^\mu(F_{\mu\rho}A_\sigma + F_{\rho\sigma}A_\mu + F_{\sigma\mu}A_\rho)\psi_2 \\ &+ \frac{i}{2}\bar{\psi}_2\gamma^\mu(F_{\mu\rho}A_\sigma + F_{\rho\sigma}A_\mu + F_{\sigma\mu}A_\rho)\psi_1\Big]\ ,\ \text{etc.} \end{aligned} \tag{19}$$

The cyclic combinations which appear in (19) will be in the following written in a compact way introducing the symbol $\Delta^{\alpha\beta\gamma}_{\sigma\rho\mu}$, e.g. $F_{\mu\rho}\partial_\sigma + F_{\rho\sigma}\partial_\mu + F_{\sigma\mu}\partial_\rho = \frac{1}{2}\,\Delta^{\alpha\beta\gamma}_{\sigma\rho\mu}F_{\gamma\beta}\partial_\alpha$. $\Delta^{\alpha\beta\gamma}_{\sigma\rho\mu}$ is cyclic separately in upper and lower indices, and antisymmetric in any pair of upper or of lower indices:

$$\Delta^{\alpha\beta\gamma}_{\sigma\rho\mu} = \delta^\alpha_\sigma\delta^\beta_\rho\delta^\gamma_\mu - \delta^\alpha_\rho\delta^\beta_\sigma\delta^\gamma_\mu + (\text{cyclic } \alpha\beta\gamma) = -\epsilon^{\alpha\beta\gamma\lambda}\epsilon_{\sigma\rho\mu\lambda}\ . \tag{20}$$

The second order actions can be expressed likewise.

3 Background field method

As explained in [22], in order to find the one-loop effective action via the background field method (and hence the divergencies at one loop), one expresses all fields ϕ^i as the sum of the corresponding classical configuration and the quantum fluctuation: $\phi^i = \phi^i_0 + \Phi^i$. The one-loop effective action is a functional supertrace:

$$\Gamma[\phi^i_0] = S[\phi^i_0] - \frac{1}{2i}\mathrm{STr}\Big(\log S^{(2)}[\phi^i_0]\Big)\ , \tag{21}$$

where $S^{(2)}_{ij} = \frac{\delta_L}{\delta\phi^i}\frac{\delta_R}{\delta\phi^j}S\Big|_{\phi^i=\phi^i_0}$, and $\frac{\delta}{\delta\phi^i}$ denotes the functional derivative.

In our case the fields are the real vector gauge field A_μ and the Dirac spinor ψ, and they are coupled. In order to perform the functional integration we have to put them into one 'multiplet' field. However, A_μ is real-number valued while ψ is complex-Grassmann (if they were independent they would have entered the effective action with different coefficients $-\frac{1}{2}$ and 1). To make all fields 'real' we need to express Dirac spinor ψ in terms of two Majorana spinors ψ_1 and ψ_2. Denoting the quantum corrections by $\mathcal{A}^\mu$ and Ψ and splitting

$$A^\mu \to A^\mu + \mathcal{A}^\mu \quad , \ \psi \to \psi + \Psi \ , \tag{22}$$

we obtain for the quadratic part of the action the expression of the type

$$S^{(2)} = \int d^4x \begin{pmatrix} \mathcal{A}_\alpha & \bar{\Psi}_1 & \bar{\Psi}_2 \end{pmatrix} \mathcal{B} \begin{pmatrix} \mathcal{A}_\beta \\ \Psi_1 \\ \Psi_2 \end{pmatrix} , \tag{23}$$

where the matrix $\mathcal{B}$ contains only classical fields. We have to include in $\mathcal{B}$ the gauge fixing term

$$S_{\rm GF} = -\frac{1}{2}\int d^4x (\partial_\mu \mathcal{A}^\mu)^2 \ , \tag{24}$$

while the ghost action will not contribute. The one-loop effective action is then

$$\Gamma_1 = \frac{i}{2}\log \mathrm{Sdet}\,\mathcal{B} = \frac{i}{2}\,\mathrm{STr}\,\log\mathcal{B} \ .$$

Matrix $\mathcal{B}$ can be written as 3×3 block-matrix

$$\mathcal{B} = \begin{pmatrix} \mathcal{B}_{11} & \mathcal{B}_{12} & \mathcal{B}_{13} \\ \mathcal{B}_{21} & \mathcal{B}_{22} & \mathcal{B}_{23} \\ \mathcal{B}_{31} & \mathcal{B}_{32} & \mathcal{B}_{33} \end{pmatrix} ,$$

where the submatrices $\mathcal{B}_{12}$, $\mathcal{B}_{13}$, $\mathcal{B}_{21}$ and $\mathcal{B}_{31}$ are Grassmann-odd while the rest are Grassmann-even. The supertrace of $\mathcal{B}$ is defined by $\mathrm{STr}\,\mathcal{B} = \mathrm{Tr}\,\mathcal{B}_{11} - \mathrm{Tr}\,\mathcal{B}_{22} - \mathrm{Tr}\,\mathcal{B}_{33}$, and $\mathcal{B}$ can be written as

$$\mathcal{B} = \begin{pmatrix} \frac{1}{2}g_{\alpha\beta}\Box & 0 & 0 \\ 0 & i\partial\!\!\!/ & 0 \\ 0 & 0 & i\partial\!\!\!/ \end{pmatrix} + \mathcal{M} \ .$$

In order to expand $\log\mathcal{B}$ around identity, we multiply it by $\mathcal{C}\mathcal{C}^{-1}$ [25], with

$$\mathcal{C} = \begin{pmatrix} 2 & 0 & 0 \\ 0 & -i\partial\!\!\!/ & 0 \\ 0 & 0 & -i\partial\!\!\!/ \end{pmatrix} .$$

Then

$$\begin{aligned}\Gamma_1 &= \frac{i}{2}\,\mathrm{STr}\,\log(\mathcal{B}\mathcal{C}) + \frac{i}{2}\,\mathrm{STr}\,\log\mathcal{C}^{-1} \\ &= \frac{i}{2}\,\mathrm{STr}\,\log(\mathcal{I}+\Box^{-1}\mathcal{M}\mathcal{C}) + \frac{i}{2}\,\mathrm{STr}\,\log\mathcal{C}^{-1} + \frac{i}{2}\,\mathrm{STr}\,\log\Box,\end{aligned} \tag{25}$$

where $\mathcal{I} = \mathrm{diag}(g_{\mu\nu},1,1)$. As usual, the second and the third terms, being independent on the fields, can be included in infinite renormalization. Note that the propagator for all fields is now $\Box^{-1}$, while the massive fermionic terms are in the interaction part, $\mathcal{M}$.

Performing the transformations described above, for NC QED we obtain the effective action in the following form:

$$\Gamma = S_0 + \frac{i}{2}\,\mathrm{STr}\,\log\Big(\mathcal{I}+\Box^{-1}N_0+\Box^{-1}N_1+\Box^{-1}T_1+\Box^{-1}T_2+\Box^{-1}\Pi\Big)\ . \tag{26}$$

Matrices N_0, N_1, T_1 are given by

$$N_0 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & im\partial\!\!\!/ & 0 \\ 0 & 0 & im\partial\!\!\!/ \end{pmatrix}\ , \tag{27}$$

$$N_1 = \begin{pmatrix} 0 & -i\bar\psi\gamma^\alpha\partial\!\!\!/ & \bar\psi\gamma^\alpha\partial\!\!\!/ \\ 2\gamma^\beta\psi & 0 & A\!\!\!/\partial\!\!\!/ \\ -2i\gamma^\beta\psi & -A\!\!\!/\partial\!\!\!/ & 0 \end{pmatrix}\ , \tag{28}$$

$$T_1 = \begin{pmatrix} V & A_1 & A_2 \\ B_1 & C & 0 \\ B_2 & 0 & C \end{pmatrix} \tag{29}$$

with $A_{1,2}, B_{1,2}$ and C defined as

$$\begin{aligned} A_{1,2} &= \frac{i}{4}\theta^{\rho\sigma}\Delta^{\alpha\beta\gamma}_{\sigma\rho\mu}\partial_\gamma\Big(-(\partial_\beta\bar\psi_{1,2})i\gamma^\mu - \frac{m}{2}\delta^\mu_\beta\bar\psi_{1,2}\Big)\partial\!\!\!/ \\ B_{1,2} &= \frac{1}{2}\theta^{\rho\sigma}\Delta^{\alpha\beta\gamma}_{\sigma\rho\mu}\Big(-i\gamma^\mu(\partial_\alpha\psi_{1,2}) + \frac{m}{2}\delta^\mu_\alpha\psi_{1,2}\Big)\partial_\gamma \\ C &= -\frac{i}{4}\theta^{\rho\sigma}\Big(-\frac{i}{2}\Delta^{\alpha\beta\gamma}_{\sigma\rho\mu}\gamma^\mu F_{\gamma\beta}\partial_\alpha + mF_{\rho\sigma}\Big)\partial\!\!\!/\ . \end{aligned}$$

$V = \overleftarrow{\partial_\mu}V^{\mu\alpha,\nu\beta}(x)\overrightarrow{\partial_\beta}$ comes from the term $(\partial_\mu\mathcal{A}_\alpha)V^{\mu\alpha,\nu\beta}(\partial_\nu\mathcal{A}_\beta)$ in $S^{(2)}$, with

$$\begin{aligned} V^{\mu\rho,\nu\sigma} &= \frac{1}{2}(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\sigma}g^{\nu\rho})\theta^{\alpha\beta}F_{\alpha\beta} \\ &+ g^{\mu\nu}(\theta^{\alpha\rho}F^\sigma{}_\alpha + \theta^{\alpha\sigma}F^\rho{}_\alpha) + g^{\rho\sigma}(\theta^{\alpha\mu}F^\nu{}_\alpha + \theta^{\alpha\nu}F^\mu{}_\alpha) \\ &- g^{\mu\sigma}(\theta^{\alpha\rho}F^\nu{}_\alpha + \theta^{\alpha\nu}F^\rho{}_\alpha) - g^{\nu\rho}(\theta^{\alpha\sigma}F^\mu{}_\alpha + \theta^{\alpha\mu}F^\sigma{}_\alpha) \\ &+ \theta^{\mu\rho}F^{\nu\sigma} + \theta^{\nu\sigma}F^{\mu\rho} - \theta^{\rho\sigma}F^{\mu\nu} - \theta^{\mu\nu}F^{\rho\sigma} - \theta^{\nu\rho}F^{\mu\sigma} - \theta^{\mu\sigma}F^{\nu\rho}\ . \end{aligned}$$

T_2 is the matrix proportional to

$$\begin{pmatrix} -\frac{1}{2}\bar{\psi}\gamma^\mu\psi\partial_\gamma & \frac{i}{4}(\frac{1}{2}F_{\gamma\beta}+\partial_\gamma A_\beta)\bar{\psi}\gamma^\mu\not{\partial} & -\frac{1}{4}(\frac{1}{2}F_{\gamma\beta}+\partial_\gamma A_\beta)\bar{\psi}\gamma^\mu\not{\partial} \\ -\frac{1}{2}\gamma^\mu\psi(-\frac{1}{2}F_{\gamma\alpha}+A_\alpha\partial_\gamma) & \frac{i}{8}A_\alpha F_{\gamma\beta}\gamma^\mu\not{\partial} & -\frac{1}{8}A_\alpha F_{\gamma\beta}\gamma^\mu\not{\partial} \\ \frac{i}{2}\gamma^\mu\psi(-\frac{1}{2}F_{\gamma\alpha}+A_\alpha\partial_\gamma) & \frac{1}{8}A_\alpha F_{\gamma\beta}\gamma^\mu\not{\partial} & \frac{i}{8}A_\alpha F_{\gamma\beta}\gamma^\mu\not{\partial} \end{pmatrix},$$

with the coefficient of proportionality $\theta^{\rho\sigma}\Delta^{\alpha\beta\gamma}_{\sigma\rho\mu}$

The terms T_1 and T_2 are linear in θ. Π denotes the contribution which comes from the θ^2-classical lagrangian. It can be split as $\Pi = \Pi_1 + \Pi_2 + \Pi_3$; as for T's, index denotes the number of background fields present in the corresponding matrix. E.g. from (17) we get for Π_1 :

$$\Pi_1 = \begin{pmatrix} 0 & -iR_2\not{\partial} & -iR_1\not{\partial} \\ 2S_2 & 0 & -iP\not{\partial} \\ -2S_1 & iP\not{\partial} & 0 \end{pmatrix} \tag{30}$$

with

$$\begin{aligned} R_{1,2} &= \frac{1}{8}\theta^{\mu\nu}\theta^{\rho\sigma}\Big[-i\delta^\alpha_\sigma\partial_\mu\partial_\rho\bar{\psi}_{1,2}\not{\partial}\partial_\nu - i\delta^\alpha_\rho\partial_\mu\partial_\beta\bar{\psi}_{1,2}\gamma^\beta\partial_\sigma\partial_\nu \\ &+ i\delta^\alpha_\beta\partial_\mu\partial_\rho\bar{\psi}_{1,2}\gamma^\beta\partial_\sigma\partial_\nu + m\delta^a_\sigma\partial_\mu\partial_\rho\bar{\psi}_{1,2}\partial_\nu\Big] , \\ S_{1,2} &= \frac{1}{8}\theta^{\mu\nu}\theta^{\rho\sigma}\Big[i\delta^\alpha_\sigma(\partial_\nu\not{\partial}\psi_{1,2})\partial_\mu\partial_\rho + i\delta^\alpha_\rho\gamma^\beta(\partial_\nu\partial_\sigma\psi_{1,2})\partial_\mu\partial_\beta \\ &- i\delta^\alpha_\beta\gamma^\beta(\partial_\nu\partial_\sigma\psi_{1,2})\partial_\mu\partial_\rho - m\delta^\alpha_\sigma(\partial_\nu\psi_{1,2})\partial_\mu\partial_\rho\Big] , \\ P &= \frac{1}{8}\theta^{\mu\nu}\theta^{\rho\sigma}\Big[\frac{1}{2}(\partial_\mu F_{\rho\sigma})(i\not{\partial}-m)\partial_\nu - i(\partial_\mu F_{\rho\beta})\gamma^\beta\partial_\nu\partial_\sigma\Big] . \end{aligned} \tag{31}$$

4 Divergent one-loop effective action

The operator $\mathcal{BC}$ in the formula (26) is split in a way convenient for the analysis of perturbation series. Let us explain the notation again. T-matrices are linear in the parameter θ, Π are quadratic in θ. Index denotes the number of classical fields in a given matrix, i.e. in diagrammatic language, shows the number of 'external legs' of the corresponding diagram. In our calculation we confine to the corrections of linear and quadratic order in θ and of the second order in classical fields. If we consider the expansion of (26)

$$\begin{aligned} \Gamma_1 &= \frac{i}{2}\,\mathrm{STr}\,\log(\mathcal{I} + \Box^{-1}N_0 + \Box^{-1}N_1 \\ &+ \Box^{-1}T_1 + \Box^{-1}T_2 + \Box^{-1}\Pi_1 + \Box^{-1}\Pi_2 + \Box^{-1}\Pi_3) \\ &= \frac{i}{2}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}\,\mathrm{STr}\,(\Box^{-1}N_0 + \Box^{-1}N_1 + \Box^{-1}T_1 + \Box^{-1}T_2 + \Box^{-1}\Pi)^n, \end{aligned} \tag{32}$$

it seems as we have to include only the powers $n = 1, 2$. But due to the nonvanishing fermionic mass m (i.e. the existence of the term N_0), in principle we will have to take into account also higher powers of n. n is finite and determined by the fact that we want to calculate only the divergent part. Analyzing the structure of $\Box^{-1}N_0, \ldots, \Box^{-1}T_2$ in some detail we conclude that the following terms (from θ-linear action) in the expansion (32) may be divergent: $(\Box^{-1}N_0)^k\Box^{-1}T_2$ for $k = 2, 3, 4$; $(\Box^{-1}N_0)^k\Box^{-1}N_1\Box^{-1}T_1$ for $k = 1, 2, 3$; $(\Box^{-1}N_0)^k(\Box^{-1}N_1)^2$ for $k = 1, 2$ and $(\Box^{-1}N_0)^k(\Box^{-1}T_1)^2$ for $k = 1, 2, 3, 4$. (Here, of course, terms are written symbolically, i.e. without the exact order of the operators.) From θ-quadratic action the contribution comes from $\Box^{-1}N_1\Box^{-1}\Pi_1$ and $\Box^{-1}N_0\Box^{-1}\Pi_2$; Π_3 contributes only to vertices. It is also clear that in the massless fermionic case the absence of N_0 brings a considerable simplification.

In order to compare with the known result [17], we first calculate the divergencies in the purely bosonic case: we put $N_0 = 0$, $N_1 = 0$, $T_2 = 0$ and $\Pi_1 = 0$. The contribution coming from Π_2 vanishes in all cases. Assuming all this, T_1 reduces to $\mathcal{T}_1$:

$$\mathcal{T}_1 = \begin{pmatrix} V & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} . \tag{33}$$

Denoting

$$\Gamma_b = \frac{i}{2}\,\mathrm{STr}\,\log\left(1 + \Box^{-1}\mathcal{T}_1\right) = \frac{i}{2}\Big[\mathrm{Tr}\,\Box^{-1}\mathcal{T}_1 - \frac{1}{2}\mathrm{Tr}\,(\Box^{-1}\mathcal{T}_1)^2 + \ldots\Big] \tag{34}$$

we get, after dimensional regularization of the traces, extraction of the divergent parts and multiple use of partial integration and Bianchi identities

$$\begin{aligned} \Gamma_b \;&=\; \frac{1}{64\pi^2\epsilon}\int d^4x \Big[\Box\tilde{F}^{\mu\nu}\Box\tilde{F}_{\mu\nu} + \frac{2}{15}\,\Box\tilde{F}^{\mu\rho}\partial_\mu\partial^\nu\tilde{F}_{\nu\rho} \\ &+\; \frac{1}{5}\,\Box F^{\mu\nu}\tilde{\partial}_a\tilde{\partial}^\alpha F_{\mu\nu} - \frac{1}{4}\,\theta^2\Box F^{\mu\nu}\Box F_{\mu\nu}\Big] . \end{aligned} \tag{35}$$

We use the notation $\tilde{F}^{\mu\nu} = \theta^{\mu}{}_{\alpha}F^{\alpha\nu}$, $\tilde{F} = \theta_{\mu\nu}F^{\mu\nu}$, $\tilde{\partial}^\mu = \theta^{\mu}{}_{\alpha}\partial^\alpha$, $\theta^2 = \theta^{\rho\sigma}\theta_{\rho\sigma}$.

For the full NC QED case, from the supertraces containing T_1 and T_2, we obtain

$$\begin{aligned} \Gamma_1^{(1)} \;&=\; \frac{1}{(4\pi)^2\epsilon}\int d^4x\Big[4i\bar{\psi}\partial\!\!\!/\psi - 16m\bar{\psi}\psi - \frac{2}{3}F_{\mu\nu}F^{\mu\nu} \\ &+\; \theta^{\alpha\beta}\Big(\frac{1}{3}\,\bar{\psi}\gamma_\alpha\partial_\beta(\Box - im\partial\!\!\!/)\psi + \frac{1}{6}\bar{\psi}\sigma_{\alpha\beta}\Box(i\partial\!\!\!/ - m)\psi \end{aligned} \tag{36}$$

$$
\begin{aligned}
&+\quad m^2\bar\psi\gamma_\alpha\partial_\beta\psi+\frac{m^2}{2}\,\bar\psi\sigma_{\alpha\beta}(i\partial\!\!\!/-m)\psi\Big)\\
&-\quad \frac{1}{120}\,\tilde F^{\rho\sigma}\Box^2\tilde F_{\sigma\rho}+\frac{1}{30}\,\tilde F^{\rho\sigma}\Box^2\tilde F_{\rho\sigma}-\frac{1}{30}\,\tilde F^{\rho\sigma}\Box\partial_\sigma\partial^\nu\tilde F_{\rho\nu}\\
&+\quad \frac{m^2}{6}\,\tilde F^{\rho\sigma}\Box\tilde F_{\rho\sigma}-\frac{m^2}{12}\,\tilde F^{\rho\sigma}\Box\tilde F_{\sigma\rho}-\frac{m^2}{6}\,\tilde F^{\rho\mu}\partial_\mu\partial^\nu\tilde F_{\rho\nu}-\frac{m^4}{4}\,\tilde F_{\rho\sigma}\tilde F^{\sigma\rho}\\
&+\quad \frac{i}{48}\,\theta^2\bar\psi\Box^2\partial\!\!\!/\psi-\frac{i}{24}\,\theta^{\alpha\mu}\theta^\beta{}_\mu\bar\psi\Box\partial\!\!\!/\partial_\alpha\partial_\beta\psi-\frac{i}{12}\,\theta^{\alpha\mu}\theta^\beta{}_\mu\bar\psi\Box^2\gamma_\alpha\partial_\beta\psi\\
&+\quad \frac{m}{12}\,\theta^{\alpha\mu}\theta^b{}_\mu\bar\psi\partial_\rho\partial_\sigma\Box\psi\\
&+\quad \frac{5im^2}{48}\,\theta^2\bar\psi\Box\partial\!\!\!/\psi-\frac{im^2}{24}\,\theta^{\alpha\mu}\theta^\beta{}_\mu\bar\psi\partial\!\!\!/\partial_\alpha\partial_\beta\psi-\frac{7im^2}{24}\,\theta^{\alpha\mu}\theta^\beta{}_\mu\bar\psi\gamma_\alpha\partial_\beta\Box\psi\\
&+\quad \frac{m^3}{4}\,\theta^{\alpha\mu}\theta^\beta{}_\mu\bar\psi\partial_\alpha\partial_\beta\psi+\frac{5im^4}{24}\,\theta^2\bar\psi\partial\!\!\!/\psi\\
&-\quad \frac{im^4}{3}\theta^{\alpha\mu}\theta^\beta{}_\mu\bar\psi\gamma_\alpha\partial_\beta\psi-\frac{m^5}{8}\,\theta^2\bar\psi\psi\Big]+\Gamma_b\;. \qquad (37)
\end{aligned}
$$

The traces containing Π's give additionally

$$
\begin{aligned}
\Gamma_1^{(2)}\quad &=\quad \frac{1}{4(4\pi)^2\epsilon}\int d^4x\Big[\frac{1}{240}\tilde F\Box^2\tilde F+\frac{1}{120}\,\tilde F^{\rho\sigma}\Box^2\tilde F_{\sigma\rho}-\frac{1}{15}\,\tilde F^{\rho\sigma}\Box^2\tilde F_{\rho\sigma}\\
&+\quad \frac{1}{15}\,\tilde F^{\rho\sigma}\Box\partial_\sigma\partial^\nu\tilde F_{\rho\nu}+\frac{im}{24}\theta^{\alpha\mu}\theta_\alpha{}^\nu\bar\psi\,\sigma_{\beta\nu}\partial^\beta\partial_\mu\Box\psi\Big]\;. \qquad (38)
\end{aligned}
$$

5 Discussion

Our goal in this lecture was to obtain the divergent part of the one-loop effective action in NC QED in the second order in the noncommutativity parameter θ and the same order in the classical fields, ψ, A^μ. Thus we obtained the second order corrections to the propagators in the theory and therefore the form of the counterterms necessary for renormalization. The method we used is the background field method; the initial point for the perturbative expansion is (26). Expansion is written in such a way that it is easy to sample out the terms contributing to the 2-point, 3-point, 4-point etc. functions.

The main motive of this calculation was to check the renormalizability of θ-expanded NC QED in the first and second order in θ, and the possibilities of generalization to all orders. This was done for the pure NC U(1) in [16]. The trick which was used is that the SW map is not unique and thus does not fix the fields in the θ-expansion fully, but allows for their redefinitions. If the fields are expanded (written symbolically) as

$$
\hat A_\mu=\sum\theta^n A_\mu^{(n)}\;,\quad \hat\psi=\sum\theta^n\psi^{(n)}\;, \qquad (39)
$$

the allowed redefinitions can be of the form

$$A_\mu^{(n)\prime} = A_\mu^{(n)} + \mathbf{A}_\mu^{(n)} \ , \tag{40}$$

$$\psi^{(n)\prime} = \psi^{(n)} + \mathbf{\Psi}^{(n)} \ , \tag{41}$$

where $\mathbf{A}_\mu^{(n)}$, $\mathbf{\Psi}^{(n)}$ are gauge covariant expressions of appropriate dimension with exactly n factors of θ. These field redefinitions produce in the actions the extra-terms of the following forms [16]:

$$\Delta S_A = \int d^4x \, (D_\nu F^{\mu\nu}) \mathbf{A}_\nu^{(n)} \tag{42}$$

$$\Delta S_\psi = \int d^4x \left[\bar\psi (i\not{D} - m) \mathbf{\Psi}^{(n)} + \bar{\mathbf{\Psi}}^{(n)} (i\not{D} - m)\psi \right] . \tag{43}$$

So, if the renormalizability of the theory can be achieved by the field redefinitions, all counterterms have to be of the types (42-43), and in the final theory we will have only the redefined, 'physical' fields and no divergencies.

It is easy to see that in the purely bosonic case the action (35) is of the type (42). The gauge potential can be redefined into physical potential: $A_\alpha \to A_\alpha + \mathbf{A}_\alpha^{(2)}$:

$$\mathbf{A}_\alpha^{(2)} = \frac{-1}{(4\pi)^2\epsilon} \Big[\frac{1}{120} \Box^2 \tilde\partial^\rho \tilde F^{\alpha\rho} - \frac{17}{60} \Box^2 \tilde\partial^\rho \tilde F^{\rho\alpha} + \frac{1}{20} \Box \partial^\rho \tilde\partial^\mu \tilde\partial_\mu F_{\rho\alpha} - \frac{1}{16} \theta^2 \Box^2 \partial^\rho F_{\rho\alpha} \Big] \tag{44}$$

and the divergent term in the effective action will cancel with the one coming from the field redefinition.

Let us discuss what happens when the fermions are present. All bosonic corrections (which come from fermionic parts in the trace) are of the θ^2-order, and all are of allowed type except for the term $-\frac{1}{(4\pi)^2\epsilon}\frac{m^4}{4}\tilde F_{\rho\sigma}\tilde F^{\sigma\rho}$. The fermionic θ-linear correction, on the other hand, is

$$\frac{1}{(4\pi)^2\epsilon}\theta^{\alpha\beta} \Big[\frac{1}{3} \bar\psi \gamma_\alpha \partial_\beta (\Box - im\not\partial)\psi$$
$$+ \frac{1}{6} \bar\psi \sigma_{\alpha\beta} \Box (i\not\partial - m)\psi + m^2 \bar\psi \gamma_\alpha \partial_\beta \psi + \frac{m^2}{2} \bar\psi \sigma_{\alpha\beta} (i\not\partial - m)\psi \Big] . \tag{45}$$

One can check that the pieces $\frac{1}{3}\bar\psi\gamma_\alpha\partial_\beta\Box\psi + \frac{1}{6}\bar\psi\sigma_{\alpha\beta}\Box(i\not\partial - m)\psi$ and $m^2\bar\psi\gamma_\alpha\partial_\beta\psi + \frac{m^2}{2}\bar\psi\sigma_{\alpha\beta}(i\not\partial - m)\psi$ of (45) can be obtained from the field redefinitions, leaving the term $-\frac{im}{3}\bar\psi\gamma_\rho\partial_\sigma\psi$ excessive. Thus we see that renormalizability cannot be achieved in the massive case $m \neq 0$, as there are (at least) two terms which obstruct it.

One observes further that for $m = 0$ the θ^2 fermionic contribution fits into the field redefinition scheme, too. This contribution reads

$$\frac{1}{(4\pi)^2\epsilon}\Big[\frac{i}{48}\,\theta^2\bar\psi\Box^2\partial\!\!\!/\psi - \frac{i}{24}\,\theta^{\alpha\mu}\theta^{\beta}{}_{\mu}\bar\psi\Box\partial\!\!\!/\partial_\alpha\partial_\beta\psi - \frac{i}{12}\,\theta^{\alpha\mu}\theta^{\beta}{}_{\mu}\bar\psi\Box^2\gamma_\alpha\partial_\beta\psi\Big] \ , \quad (46)$$

and it is of the form (43). The full field redefinition of fermions in the massless case is given by

$$\mathbf{\Psi}^{(1)} = \frac{1}{(4\pi)^2\epsilon}\theta^{\alpha\beta}\Big[\frac{1}{6}\sigma_{\mu\alpha}\partial_\beta\partial^\mu\psi + \frac{1}{12}\sigma_{\alpha\beta}\Box\psi\Big] \ , \quad (47)$$

$$\mathbf{\Psi}^{(2)} = \frac{1}{(4\pi)^2\epsilon}\Big[\frac{1}{96}\,\theta^2\Box^2\psi - \frac{1}{16}\,\theta^{\alpha\mu}\theta^{\beta}{}_{\mu}\Box\partial_\alpha\partial_\beta\psi + \frac{i}{24}\,\theta^{\alpha\mu}\theta^{\beta}{}_{\mu}\sigma_{\mu\alpha}\Box\partial^\mu\alpha\partial_\beta\psi\Big] \quad (48)$$

and it renders the propagators of physical fermions finite to θ^2 order. It is easy to extend this conclusion to higher orders. Namely, the higher-order divergencies for the propagators ($m = 0$) after the partial integration have the form

$$\begin{aligned} S_A^{(n)} &= \underbrace{\theta\ldots\theta}_{n}\, A\, \underbrace{\partial\ldots\partial}_{k} A \\ S_\psi^{(n)} &= \underbrace{\theta\ldots\theta}_{n}\, \bar\psi\,\gamma\ldots \underbrace{\partial\ldots\partial}_{l}\psi \ . \end{aligned} \quad (49)$$

Counting dimensions, we see that $k = 2+2n$, $l = 1+2n$, so there is always at least one derivative in the expressions (49). This in the massless case (along with the fact that everything, being a result of the background field method, is covariant) ensures that they can be interpreted as field redefinitions.

Unfortunately, as also stressed in [19], the full renormalizability is spoiled by vertex corrections. Obviously, θ-expansion introduces new interactions into NC QED (compared to the commutative QED), like photon self-interaction and four-fermion interaction. The divergent part of the 4-fermion vertex, in the linear order in θ, is found to be

$$\begin{aligned} S_{4\psi} &= -\frac{1}{(4\pi)^2\epsilon}\,\frac{1}{2}\,\theta^{\rho\sigma}\Delta^{\alpha\beta\gamma}_{\sigma\rho\mu}\int d^4x\,\bar\psi\gamma^\mu\psi\,\bar\psi\gamma_\alpha\gamma_\beta\gamma_\gamma\psi \\ &= -\frac{1}{(4\pi)^2\epsilon}\,3i\theta^{\rho\sigma}\int d^4x\,\epsilon_{\rho\sigma\mu\nu}\bar\psi\gamma^\mu\psi\,\bar\psi\gamma_5\gamma^\nu\psi \ . \end{aligned} \quad (50)$$

This term is independent of m and cannot be achieved by field redefinition. Thus we see that it spoils renormalizability in the massless fermionic case. Finally, even if all vertex divergencies were of the prescribed form, one would have to check (a quite nontrivial property) that redefinitions would remove the divergencies in all propagators and vertices simultaneously.

So we are left with the conclusion that NC QED is not renormalizable and can only be treated as an effective theory at low energies. The regularization procedure than gives new effective interactions, which have to be included in the cross sections. Many interesting phenomenological questions arise: e.g. can 4ψ vertex be interpreted in terms of some effective (scalar) particles, higgses? In order to investigate this possibility in more details, it would be useful to calculate the 4ψ contribution in the θ^2 order for $U(1)$, or in θ-linear order for some other gauge theory like $SU(2)$.

References

[1] J. Madore, S. Schraml, P. Schupp and J. Wess, Eur. Phys. Jour. C16 (2000) 161

[2] B. Jurčo, L. Moller, S. Schraml, P. Schupp and J. Wess, Eur. Phys. J. C21 (2001) 383

[3] J. Madore, *An Introduction to Noncommutative Differential Geometry and its Physical Applications*, Camb. Univ. Press, Cambridge, 1999

[4] M. Hayakawa, "Perturbative analysis on infrared and ultraviolet aspects on noncommutative QED on R^4", preprint hep-th/9912167

[5] L. Bonora, M. Schnabl, M. M. Sheikh-Jabbari and A. Tomasiello, Nucl. Phys. B589 (2000) 461

[6] M. Chaichian, P. Prešnajder, M. M. Sheikh-Jabbari and A. Tureanu, Phys. Lett. B 526 (2002) 132

[7] N. Seiberg and E. Witten, JHEP 9909, 032 (1999)

[8] T. Krajewski and R. Wulkenhaar, Int. J. Mod.Phys. A 15 (1999) 1011

[9] S. Minwalla, M. Raamsdonk and N. Seiberg, JHEP 0002, (2000) 020

[10] A. Matusis, L. Susskind and N. Toumbas, JHEP 0012 (2000) 002

[11] R. J. Szabo, "Quantum field theory on noncommutative spaces", preprint hep-th/0109162

[12] M. R. Douglas and N. A. Nekrasov, Rev. Mod. Phys. 73 (2002) 977

[13] L. Griguolo and M. Pietroni, JHEP 0105 (2001) 032

[14] H. Grosse and R. Wulkenhaar, preprint hep-th/0305066

[15] T. Asakawa and I. Kishimoto, "Comments on Gauge Equivalencein Noncommutative Theory", preprint hep-th/9909139

[16] A. A. Bichl, J.M. Grimstrup, H. Grosse, L. Popp, M. Schweeda and R. Wulkenhar, JHEP 0106 (2001) 013

[17] A. A. Bichl, J.M. Grimstrup, L. Popp, M. Schweeda and R. Wulkenhar, Int. J. Mod. Phys. A17 (2002) 2219

[18] A. A. Bichl, J.M. Grimstrup, L. Popp, M. Schweeda and R. Wulkenhar, "Deformed QED via Seiberg-Witten Map", preprint hep-th/0102103

[19] R. Wulkenhaar, JHEP 0203 (2002) 024

[20] X. Calmet, B. Jurčo, P. Schupp, J. Wess and M. Wohlgennant, preprint hep-ph/0111115

[21] P. Aschieri, B. Jurčo, P. Schupp and J. Wess, preprint hep-th/0205214

[22] M. Burić and V. Radovanović, JHEP 10 (2002) 074

[23] A. Miković and V. Radovanović, Class. Quant. Grav. 15 (1998) 827

[24] A. Miković and V. Radovanović, Nucl. Phys. B 504 (1997) 511

[25] A. G. Barvinsky and G. A. Vilkovisky , Phys. Rep. 119 (1985) 1