Geometrical Methods in the Theory of Superstrings and Superbranes

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Abstract

We review basic properties of various formulations of the worldvolume dynamics of superbranes and introduce the geometrical foundation of superbrane theory.

1 Introduction

These lectures are intended to give an introduction into a geometrical, so called superembedding, approach to the description of the dynamics of supersymmetric particles and extended supersymmetric objects. By now quite a number of these objects have been found in string theory where they play an important role in the perturbative and non–perturbative structure of the theory.

These include super–p–branes, where p stands for the dimensionality of the object, so that p=0 means a particle, p=1 means a string, p=2 means a membrane and so on. Then there are Dirichlet p–branes which carry gauge fields on their surfaces, an M2–brane and an M5–brane which live in an eleven–dimensional corner of string theory called M–theory, and there are also more exotic objects such as Neveu–Schwarz 5–branes, Kaluza–Klein monopoles and some others.

The worldvolume dynamics of all (or almost all) of these objects in a space–time enlarged with Grassmann–odd spinorial directions (i.e. in target superspace) is described by a well known formulation which I shall conventionally call the Green–Schwarz (GS) formulation [1], though for different branes it has been developed by different people.

For the superstrings there exists an alternative formulation, which was historically the first one, known under the name of Neveu, Schwarz and Ramond, the NSR (fermionic or spinning) string [2]. Analogous formulation exists for spinning particles [3], while it seems not possible to construct in the same way spinning membranes or higher dimensional branes [4].

The GS and the NSR formulations are rather well known, but there are also less conventional formulations of superbrane dynamics (and in particular of the superparticles and superstrings) such as a twistor formulation, which was developed among others [5] by Martin Cederwall, and a Lorentz–harmonic formulations developed, in particular by Emil Nissimov and Svetlana Pacheva [6] with collaborators, and by Igor Bandos [7].

The twistor and the Lorentz–harmonic formulations were proposed as an attempt to solve the problem of the covariant quantization of the superparticles and superstrings. But though some interesting results have been obtained using these methods, the problem has still remained and moreover it has now become even more topical because of the present interest in studying string theory in non–trivial supergravity backgrounds with non–zero antisymmetric gauge fields, such as Anti–de–Sitter spaces, pp–waves and string compactifications with fluxes for which the NSR formulation is not applicable.

The superembedding approach arose in the end of 80ths also from the desire to solve the problem of the covariant quantization of the Green–Schwarz superstring. The idea was to find a more general formulation which would encompass main positive features of the NSR, GS and twistor formulation, thus allowing one to make a progress in quantizing the superstring covariantly. Such a formulation was first realized for superparticles and then for superstrings, and in the end for all known superbranes by efforts of several theoretical groups [8]–[17].

The progress in the covariant quantization of the superstring during the last dozen of years has been mainly maid by Nathan Berkovits [10]. He has recently arrived at a very promising quantization prescription [18] based on the use of pure spinors (see [19] for a review and a detailed list of references) which stimulated Peter van Nieuwenhuizen with collaborators to develop their own version of the quantization procedure [20]. The geometrical roots of these techniques stem from the superembedding approach [21].

So, as I have tried to convince the reader, the superembedding is an elegant, profound and in a sense universal geometrical formulation, and the purpose of these lectures is to explain its main features (see [22] for a detailed review).

The superembedding approach is based on a supersymmetric extension of the classical surface theory to the description of superbrane dynamics by means of embedding worldvolume supersurfaces into target superspaces. Being manifestly doubly supersymmetric (on the worldvolume and in target superspace) the superembedding approach has explained the origin and the nature of the notorious fermionic κ–symmetry of the GS formulation as a manifestation of the conventional local supersymmetry of the worldvolume. In this way it has solved the problem of infinite reducibility of the κ–symmetry by realizing it as an irreducible extended worldvolume supersymmetry [8]. As I have already mentioned, this stimulated progress in the covariant quantization of the Green–Schwarz superstring mainly due to persistent work of N. Berkovits [10, 18, 19].

The superembedding approach established or clarified a classical relationship between various formulations of the dynamics of superparticles and superstrings, such as the NSR and the GS formulation [9], the twistor [5] and harmonic [6, 7] descriptions.

This approach has proved to be a universal and powerful method applicable to the description of all known supersymmetric branes, in particular, to those of them for which standard methods encountered problems because of their specific structure, such as the 5–brane of M–theory. The superembedding methods allowed, for the first time, to derive the complete set of covariant equations of motion of the M5–brane [17]. And only later these equations were obtained [23] from the M5–brane action [24] based on a different technique adapted to deal with self–dual fields [25].

The superembedding formulation has also proved to be useful for studying the "brany" mechanism of partial supersymmetry breaking [26] by giving a geometrical recipe [27, 28, 29] for the construction of covariant worldvolume supersymmetric actions for superbranes. Upon gauge fixing worldvolume superdiffeomorphisms these actions become those of effective field theories with non–linearly realized spontaneously broken supersymmetries. This has demonstrated an intrinsic link of the superembedding approach and the method of nonlinear realizations developed in application to supersymmetric theories in [30, 31] (see [32] for a review and references).

To conclude the Introduction, let me mention one more topic where the application of the superembedding techniques allowed one to get new results. It concerns the construction of supersymmetric actions for coincident Dirichlet branes which with the help of superembedding has been realized for a system of N space filling D2–branes [33].

We now pass to a more detailed consideration of general properties of this geometrical formulation.

2 Double Supersymmetry

As it has been mentioned in the Introduction the superembedding description of superbrane dynamics is formulated in such a way that it possesses manifest supersymmetry on the brane worldvolume and in target superspace, and thus has properties of both the NSR and the GS formulation.

2.1 The Neveu–Schwarz–Ramond formulation

In the NSR formulation which describes spinning particles [3] and spinning strings [2] [1] the worldline or worldsheet of the spinning object is a supersurface $\mathcal{M}_w$ parametrized by bosonic coordinates ξ and fermionic coordinates η which we will collectively call $z^M = (\xi, \eta)$. Depending on the model considered $\mathcal{M}_w$ may have a various number of fermionic directions η. For simplicity we here take only one η. The dynamics of the spinning object is described by embedding $\mathcal{M}_w$ into a bosonic target space–time M_T parametrized by coordinates $X^{\underline{m}}$ ($\underline{m} = 0, 1, \cdots, D-1$). In the classical problems the number of space–time dimensions can be arbitrary, but for quantum consistency the spinning string, whose worldsheet has one or two fermionic directions, must live in a ten–dimensional target space.

The motion of the spinning object is described by the image of $\mathcal{M}_w$ in M_T

$$X^{\underline{m}}(z^M) = x^{\underline{m}}(\xi) + i\eta\chi^{\underline{m}}(\xi), \tag{1}$$

where $x^{\underline{m}}(\xi)$ is associated with the bosonic degrees of freedom of the spinning particle or string in M_T, and the Grassmann–odd vector $\chi^{\underline{m}}(\xi)$ is associated with its spin degrees of freedom.

The NSR formulation is invariant under worldsheet superdiffeomorphisms $z^M \rightarrow z'^M(z^M)$, which include bosonic reparametrizations of $\mathcal{M}_w$

$$\delta\xi = a(\xi), \tag{2}$$

and local worldsheet supersymmetry

$$\delta\eta = \kappa(\xi), \qquad \delta\xi = i\kappa(\xi)\eta. \tag{3}$$

[1] As “Green–Schwarz”, the name “NSR” used for this formulation should be regarded as cumulative, since for particles and strings it has been developed by different people.

The presence of the local symmetries (2) and (3) implies that the dynamics of the spinning objects is subject to bosonic and fermionic first–class constraints, respectively, which I will *schematically* write down in the form

$$(\partial x^{\underline{m}})^2 = 0, \qquad \partial x^{\underline{m}} \chi_{\underline{m}} = 0. \tag{4}$$

The bosonic constraint in (4) is the mass shell or the Virasoro conditions, and the fermionic constraint produces, upon quantization, the Dirac equation for the spin wave functions of the dynamical system.

The NSR formulation does not have target–space supersymmetry. The latter appears, in the case of the spinning string, only at the quantum level upon imposing the Gliozzi–Scherk–Olive projection. The merit of this formulation is that it is covariantly quantizable.

2.2 The Green–Schwarz formulation

This formulation is applicable to all known superparticles, superstrings and superbranes. Now the worldvolume $\mathcal{M}_w$ is a (p+1)–dimesnional bosonic surface parametrized by the coordinates ξ^m $(m = 0, 1, \cdots, p)$ and the target space, into which $\mathcal{M}_w$ is embedded, is a superspace M_{TS} parametrized by bosonic coordinates $x^{\underline{m}}$ $(\underline{m} = 0, 1, \cdots, D-1)$ and by an appropriate number of fermionic coordinates $\theta^{\underline{\alpha}}$ $(\underline{\alpha} = 1, \cdots, 2n)$

$$Z^{\underline{M}}_{TS}(\xi) = (x^{\underline{m}}(\xi), \, \theta^{\underline{\alpha}}(\xi)) \, . \tag{5}$$

The GS formulation is manifestly invariant under bosonic $\mathcal{M}_w$ reparametrizations $\xi \to \xi'(\xi)$, and target space superdiffeomorphisms $Z^{\underline{M}} \to Z'^{\underline{M}}(Z^{\underline{M}})$, which in the case of flat target superspace reduce to the translations along $x^{\underline{m}}$ and to the global target–space supersymmetry transformations

$$\delta\theta^{\underline{\alpha}} = \epsilon^{\underline{\alpha}}, \qquad \delta x^{\underline{m}} = i\bar{\theta}\Gamma^{\underline{m}}\delta\theta. \tag{6}$$

There is also another (non–manifest) local worldvolume fermionic symmetry, so–called κ–symmetry, inherent to the GS formulation. This is an important symmetry which implies and reflects the existence of supersymmetric BPS brane–like solutions of corresponding supergravity theories. It is thus responsible for the 'brane scan' (i.e. it prescribes which brane lives in which target superspace). The κ–symmetry was first observed in the case of superparticles [34] and has the following generic form of transformations (in flat target superspace):

$$\delta\theta^{\underline{\alpha}} = \Pi^{\underline{\alpha}}{}_{\underline{\beta}}\kappa^{\underline{\beta}}(\xi), \qquad \delta x^{\underline{m}} = -i\bar{\theta}\Gamma^{\underline{m}}\delta\theta, \tag{7}$$

where $\Pi^{\underline{\alpha}}{}_{\underline{\beta}}$ is a projector matrix ($\det \Pi^{\underline{\alpha}}{}_{\underline{\beta}} = 0$), whose form depends on the object under consideration.

Because of the presence of the projector in the κ–transformations they are infinite reducible, so only half of $\kappa^{\underline{\alpha}}$, i.e. n of the $2n$ Grassmann spinor components, effectively contribute to the variation of the worldvolume fields. This causes the problem of the covariant quantization of the GS formulation.

To solve the infinite reducibility problem of the κ–symmetry it is natural to try to find its irreducible realization which is covariant in target superspace. A natural assumption is that this should be an extended local worldvolume supersymmetry (3) with the number of independent parameters equal to the number of independent (irreducible) κ–symmetries [8]. This reasoning brings us to

2.3 The doubly–supersymmetric formulation

In this formulation the dynamics of superbranes is described by embedding a worldvolume *super*surface $\mathcal{M}_{sw}$ parametrized by the coordinates $z^M = (\xi^m, \eta^\alpha)$ $(m = 0, 1, \cdots, p)$, $(\alpha = 1, \cdots, n)$ into a target *super*space M_{TS} parametrized by the coordinates $Z^{\underline{M}} = (X^{\underline{m}}, \Theta^{\underline{\alpha}})$ $(\underline{m} = 0, 1, \cdots, D-1)$, $(\underline{\alpha} = 1, ..., 2n)$. Note that the number of the Grassmann directions of $\mathcal{M}_{sw}$ is half the number of the Grassmann directions of M_{TS}. Such a choice of the supermanifolds for superembedding is caused by our desire to identify n local supersymmetries on $\mathcal{M}_{sw}$ with n independent κ–symmetries of the GS formulation. But for some applications [9], in particular, as far as quantization is concerned [10, 12, 18, 21], one also considers an embedding of supersurfaces $\mathcal{M}_{sw}$ with less number of Grassmann directions. In these cases the models contain residual non–manifest κ–symmetry.

Thus in the doubly supersymmetric formulation the degrees of freedom of the superbranes are described by worldvolume superfields

$$X^{\underline{m}}(z^M) = x^{\underline{m}}(\xi) + i\eta^\alpha \chi^{\underline{m}}_\alpha(\xi) + \cdots, \quad \Theta^{\underline{\alpha}}(z^M) = \theta^{\underline{\alpha}}(\xi) + \eta^\alpha \lambda^{\underline{\alpha}}_\alpha(\xi) + \cdots, \tag{8}$$

where $\cdots$ stand for the terms of higher order in η^α. These terms contain auxiliary fields and in addition, for example in the case of the D–branes and the M5–brane, include the gauge fields propagating on the worldvolumes of these branes.

¿From eq. (8) we see that in the doubly supersymmetric construction the number of degrees of freedom of the superbrane roughly speaking doubles. We now have $x^{\underline{m}}(\xi)$ describing the bosonic oscillations of the brane, the Grassmann 'spin'–vectors $\chi^{\underline{m}}_\alpha(\xi)$ as in the NSR formulation, the

Green–Schwarz fermionic spinor degrees of freedom $\theta^{\underline{\alpha}}(\xi)$ and their bosonic counterparts $\lambda^{\underline{\alpha}}_{\alpha}(\xi)$. So if all these worldvolume fields are independent the corresponding models will not describe conventional superbranes. In this case one will get, for instance, so called spinning superparticles and spinning superstrings [35] which have more degrees of freedom than the conventional NSR and GS dynamical systems. They also have infinite reducible κ–symmetry as a fermionic symmetry independent of the local worldvolume supersymmetry.

To reach our goal of interpreting κ–symmetry as a manifestation of the local worldvolume supersymmetry we should find an appropriate doubly–supersymmetric description of the *conventional* superbranes. For this we should impose constraints on the superfields (8) which relate their components in such a way that the independent physical degrees of freedom described by these superfields will correspond to the standard GS formulation. The geometrical meaning of these constraints is that they cause the worldvolume supersurface $\mathcal{M}_{sw}$ to be imbedded into the target superspace M_{TS} in a specific way.

3 The Superembedding Condition

Before discussing in more detail the geometrical meaning of the superembedding condition let us consider its dynamical consequences in the simplest case of a superparticle propagating in a flat $N=1$, $D=3$ target superspace. Then the supersurface $\mathcal{M}_{sw}$ of the previous subsection is associated with the superparticle "worldline" having one bosonic (time) and one fermionic coordinate (ξ,η), and the target superspace is parametrized by bosonic three–vector coordinates $X^{\underline{m}}$ ($\underline{m}=0,1,2$) and Grassmann Majorana two–spinor coordinates $\Theta^{\underline{\alpha}}$ ($\underline{\alpha}=1,2$). The superembedding condition relates the worldline superfields $X^{\underline{m}}(z^M)$ and $\Theta^{\underline{\alpha}}(z^M)$ in the following way (for simplicity we assume the target superspace to be flat)

$$DX^{\underline{m}} - iD\bar{\Theta}\Gamma^{\underline{m}}\Theta = 0, \tag{9}$$

where D is a Grassmann covariant derivative on $\mathcal{M}_{sw}$ which in the case of the superparticles can be chosen to be flat

$$D = \frac{\partial}{\partial\eta} + i\eta\frac{\partial}{\partial\xi}, \qquad \{D,D\} = 2i\partial_{\xi}. \tag{10}$$

Using the η–expansion (8) which in the case under consideration does not contain "$\cdots$"–terms we obtain the following relation between the components of the superfields $X^{\underline{m}}(z^M)$ and $\Theta^{\underline{\alpha}}(z^M)$

$$\partial_\xi x^{\underline{m}} - i\partial_\xi \bar{\theta}\Gamma^{\underline{m}}\theta = \bar{\lambda}\Gamma^{\underline{m}}\lambda, \tag{11}$$

$$\chi^{\underline{m}} = \bar{\theta}\Gamma^{\underline{m}}\lambda. \tag{12}$$

¿From eq. (11) we see that $\lambda^{\underline{\alpha}}(\xi)$ are not independent fields but are expressed in terms of the derivatives of $x^{\underline{m}}$ and $\theta^{\underline{\alpha}}$. Moreover in the l.h.s. of (11) one can recognize the canonical momentum of the superparticle $P^{\underline{m}} = \frac{1}{e(\tau)}(\partial_\xi x^{\underline{m}} - i\partial_\xi \bar{\theta}\Gamma^{\underline{m}}\theta)$ [2] whose square is identically zero

$$P^{\underline{m}}P_{\underline{m}} = 0, \tag{13}$$

because of its so called Cartan–Penrose (or twistor) representation as a bilinear combination of commuting spinor components and Γ–matrix identities. We conclude that the superparticle is massless.

In the case of the superstrings the superembedding condition will produce in a similar way the Virasoro constraints, and it will produce the corresponding constraints for the other superbranes.

Eq. (12) implies the relation between the Grassmann vector and the Grassmann spinor variables, so that only one or another can be taken to describe independent fermionic degrees of freedom. This is a basic relation which allows one to establish a classical correspondence between the NSR and the GS formulation of supersymmetric particles and strings [9].

3.1 Local worldvolume supersymmetry versus κ–symmetry

Let us now demonstrate how κ–symmetry appears in the superembedding formulation as a weird realization of the local worldvolume supersymmetry.

The components (7) of the superfields $X^{\underline{m}}(z^M)$ and $\Theta^{\underline{\alpha}}(z^M)$ transform under the local worldline supersymmetry (3) in the standard way

$$\delta\theta^{\underline{\alpha}} = -\lambda^{\underline{\alpha}}\kappa(\xi), \qquad \delta\lambda^{\underline{\alpha}} = i\partial_\xi\theta^{\underline{\alpha}}\kappa(\xi), \tag{14}$$

$$\delta x^{\underline{m}} = i\chi^{\underline{m}}\kappa(\xi), \qquad \delta\chi^{\underline{m}} = -\partial_\xi x^{\underline{m}}\kappa(\xi). \tag{15}$$

We now substitute into the first equation of (15) the solution (12) of the superembedding condition (9) and observe that, due to the form of the θ–variation (14), the variation of $x^{\underline{m}}$ can be rewritten as follows

$$\delta x^{\underline{m}} = i\bar{\theta}\Gamma^{\underline{m}}\lambda\kappa(\xi) = -i\bar{\theta}\Gamma^{\underline{m}}\delta\theta. \tag{16}$$

[2] $e(\xi)$ is a proportionality coefficient. Its geometrical meaning is to be the worldline gravitational field.

The next step is to replace the Grassmann scalar parameter of the local supersymmetry by the scalar product of the spinor $\lambda_{\underline{\beta}}$ with a Grassmann spinor parameter $\kappa^{\underline{\beta}}(\xi)$, which is always possible,

$$\kappa(\xi) = 2\lambda_{\underline{\beta}}\kappa^{\underline{\beta}}(\xi), \tag{17}$$

and to substitute (17) into the θ–variation (14). We thus get

$$\delta\theta^{\underline{\alpha}} = -2\lambda^{\underline{\alpha}}\lambda_{\underline{\beta}}\kappa^{\underline{\beta}}(\xi). \tag{18}$$

We now note that, due to the superembedding condition (11), the bilinear combination of λ in (18) is nothing but

$$\Pi^{\underline{\alpha}}{}_{\underline{\beta}} = -2\lambda^{\underline{\alpha}}\lambda_{\underline{\beta}} = (\partial_\xi x^{\underline{m}} - i\partial_\xi\bar{\theta}\Gamma^{\underline{m}}\theta)(\Gamma_{\underline{m}})^{\underline{\alpha}}{}_{\underline{\beta}}, \tag{19}$$

which is the projector matrix in the κ–symmetry variation of θ (7). Hence, the local supersymmetry variations (16) and (18) reduce to the κ–variations (7).

We have thus demonstrated how, in virtue of the superembedding condition (9), the κ–symmetry of the GS formulation of the superbranes shows up from the irreducible local worldvolume supersymmetry.

One might have already noticed the difference in sign in the target–space supersymmetry variations of $x^{\underline{m}}$ (6) and in the worldvolume supersymmetry variations (16) and corresponding κ–variations (7). The target–space supersymmetry and the local worldvolume supersymmetry (or κ–symmetry) can be therefore regarded as, respectively, 'left' and 'right' supertranslations of $x^{\underline{m}}$.

3.2 The worldline superfield action for the $N = 1$, $D = 3$ superparticle

The worldline $n = 1$ superfield action, which produces the superembedding relations (9), (11) and (12), and also the superparticle equations of motion, has the following form

$$S = -i\int d\xi d\eta P_{\underline{m}}\left(DX^{\underline{m}} - iD\bar{\Theta}\Gamma^{\underline{m}}\Theta\right)\ , \tag{20}$$

where $X^{\underline{m}}(\xi,\eta)$ and $\Theta^{\underline{\alpha}}(\xi,\eta)$ are the superfields defined in (8) and

$$P_{\underline{m}}(\xi,\eta) = p_{\underline{m}}(\xi) + i\eta\rho_{\underline{m}}(\xi) \tag{21}$$

is the Lagrange multiplier superfield.

Varying (20) with respect to $P_{\underline{m}}$ we get the superembedding condition (9), and hence (11) and (12).

To see what kind of other equations follow from the action let us integrate it over η using the Berezin rules $\int d\eta = 0$, $\int d\eta\eta = 1$. As a result we get the following action for the components of the superfields (8) and (21)

$$S = \int d\xi\, p_{\underline{m}}(\dot{x}^{\underline{m}} - i\dot{\bar{\theta}}\Gamma^{\underline{m}}\theta - \bar{\lambda}\Gamma^{\underline{m}}\lambda) + i\int d\xi\, \rho_{\underline{m}}(\chi^{\underline{m}} - \bar{\lambda}\Gamma^{\underline{m}}\theta). \qquad (22)$$

We see that $p_{\underline{m}}$ is the particle momentum, and that the second term in (22) reads that $\rho_{\underline{a}}$ and $\chi^{\underline{m}}$ are auxiliary fields satisfying algebraic equations

$$\rho_{\underline{m}} = 0, \quad \chi^{\underline{m}} = \bar{\lambda}\Gamma^{\underline{m}}\theta, \qquad (23)$$

the latter being just part of the superembedding condition. Its another part, namely eq. (11), is obtained from (22) as a variation of $p_{\underline{m}}$.

The equations of motion of $x^{\underline{m}}$ and $\theta^{\underline{\alpha}}$ are, respectively,

$$\partial_\xi p_{\underline{m}} = 0 \quad \text{and} \quad (p_{\underline{m}}\Gamma^{\underline{m}})^{\underline{\alpha}}{}_{\underline{\beta}}\partial_\xi\theta^{\underline{\beta}} = 0. \qquad (24)$$

These are the standard equations of motion of the Green–Schwarz superparticle. What remains to show is that the particle under consideration is massless, i.e. that its momentum is light–like $p^{\underline{m}}p_{\underline{m}} = 0$. To see this one should consider the equation of motion of the commuting spinor variable $\lambda^{\underline{\alpha}}$:

$$(p_{\underline{m}}\Gamma^{\underline{m}})^{\underline{\alpha}}{}_{\underline{\beta}}\,\lambda^{\underline{\beta}} = 0. \qquad (25)$$

The general solution of this equation in $D = 3$, as well as in $D = 4, 6$ and 10 is

$$p_{\underline{m}} = e(\xi)\bar{\lambda}\Gamma_{\underline{m}}\lambda = e(\xi)(\partial_\xi x^{\underline{m}} - i\partial_\xi\bar{\theta}\Gamma^{\underline{m}}\theta)\,, \qquad (26)$$

where $e(\xi)$ is an arbitrary worldline function, and the last equality being the consequence of (11). Hence $p^{\underline{m}}p_{\underline{m}} = 0$ in virtue of the Dirac matrix identity $\Gamma^{\underline{m}}_{\underline{\alpha\beta}}\Gamma_{\underline{m\gamma\delta}} + \Gamma^{\underline{m}}_{\underline{\beta\gamma}}\Gamma_{\underline{m\alpha\delta}} + \Gamma^{\underline{m}}_{\underline{\gamma\alpha}}\Gamma_{\underline{m\beta\delta}} = 0$, and the particle is indeed massless.

Let us now consider constraints on the canonical fermionic momenta $\pi_{\underline{\alpha}} = \frac{\delta S}{\delta\dot{\theta}^{\underline{\alpha}}}$ conjugate to $\theta^{\underline{\alpha}}$. They are

$$D_{\underline{\alpha}} = \pi_{\underline{\alpha}} - ip_{\underline{m}}(\Gamma^{\underline{m}}\theta)_{\underline{\alpha}} = 0\,. \qquad (27)$$

The Poisson brackets of these constraints are

$$\{D_{\underline{\alpha}}, D_{\underline{\beta}}\} = -2ip_{\underline{m}}\Gamma^{\underline{m}}_{\underline{\alpha\beta}}\,. \qquad (28)$$

Since the momentum $p_{\underline{m}}$ is constrained (26) to be light–like, the r.h.s. of (28) is a degenerate 2×2 matrix of rank one. This means that the fermionic constraints (27) are the mixture of a first and a second class constraint. In the conventional Green–Schwarz formulation it is not possible to separate in (27) an *irreducible* set of the first class fermionic constraints from the second class ones in a Lorentz–covariant way, which causes the covariant quantization problem. The availability of auxiliary commuting spinor variables $\lambda^{\underline{\alpha}}$ in the superembedding formulation allows one to do so. One can convince oneself that the constraint $\lambda^{\underline{\alpha}} D_{\underline{\alpha}}$ obtained by projecting $D_{\underline{\alpha}}$ along $\lambda^{\underline{\alpha}}$ is the first class

$$\{\lambda^{\underline{\alpha}} D_{\underline{\alpha}}, \lambda^{\underline{\beta}} D_{\underline{\beta}}\} = -2ip_{\underline{m}} \lambda \Gamma^{\underline{m}} \lambda \simeq 0\,. \tag{29}$$

The r.h.s. of (29) is weakly zero in the Dirac sense due to (25).

The first class bosonic constraint $p_{\underline{m}} \lambda \Gamma^{\underline{m}} \lambda$ and the first class fermionic constraint $\lambda^{\underline{\alpha}} D_{\underline{\alpha}}$ form the one–dimensional (worldline) $n = 1$ superalgebra and generate local worldline reparametrizations and supersymmetry transformations, the latter being the *irreducible* counterpart of the kappa–symmetry of the Green–Schwarz formulation, as we have discussed in Subsection 3.1. This provides us with an algebraic ground for the covariant quantization of superparticles and superstrings. Actually, as has already been realized in earlier papers [10, 12], a version of the superembedding formulation with manifest $n = 2$ worldsheet supersymmetry turns out to be the most appropriate for quantizing $D = 10$ superparticles and superstrings. It is this version, which gives rise to pure spinors and is in the origin of the covariant quantization procedure of [19] and of [20]. So we shall now turn to the consideration of

3.3 The $N = 1$, $D = 10$ superparticle with $n = 2$ worldline supersymmetry and pure spinors

We are now in a ten–dimensional flat superspace parametrized by ten bosonic coordinates $X^{\underline{m}}$ ($\underline{m} = 0, 1, \cdots, 9$) and by sixteen real Majorana–Weyl Grassmann coordinates $\Theta^{\underline{\alpha}}$ ($\underline{\alpha} = 1, \cdots, 16$). The superparticle worldline is assumed to be a supersurface $\mathcal{M}_{sw}$ with one bosonic and two fermionic directions $(\xi, \eta, \bar{\eta})$, where η and $\bar{\eta}$ are complex conjugate to each other. The worldline fermionic covariant derivatives

$$D = \frac{\partial}{\partial \eta} + i\bar{\eta}\frac{\partial}{\partial \xi}, \quad \bar{D} = \frac{\partial}{\partial \bar{\eta}} + i\eta\frac{\partial}{\partial \xi},$$

generate the worldline $n = 2$ superalgebra

$$D^2 = 0, \quad , \bar{D}^2 = 0, \quad \{D, \bar{D}\} = 2i \frac{\partial}{\partial \xi} \,. \tag{30}$$

The $n = 2$ superfield form of the $N = 1$, $D = 10$ superparticle action is

$$S = -i \int d\xi d\eta d\bar{\eta} \left[P_{\underline{m}} \left(DX^{\underline{m}} - iD\Theta\Gamma^{\underline{m}}\Theta \right) + \bar{P}_{\underline{m}} \left(\bar{D}X^{\underline{m}} - i\bar{D}\Theta\Gamma^{\underline{m}}\Theta \right) \right] \,, \tag{31}$$

where $P_{\underline{m}}(\xi, \eta, \bar{\eta})$ and $\bar{P}_{\underline{m}}(\xi, \eta, \bar{\eta})$ are complex conjugate Lagrange multipliers, whose variation produces the superembedding conditions

$$DX^{\underline{m}} - iD\Theta\Gamma^{\underline{a}}\Theta = 0, \quad \bar{D}X^{\underline{m}} - i\bar{D}\Theta\Gamma^{\underline{m}}\Theta = 0 \tag{32}$$

to be satisfied by the superfields

$$X^{\underline{m}}(\xi, \eta, \bar{\eta}) = x^{\underline{m}}(\xi) + i\eta\chi^{\underline{m}}(\xi) - i\bar{\eta}\bar{\chi}^{\underline{m}}(\xi) + i\eta\bar{\eta}y^{\underline{m}}(\xi) \,,$$

$$\Theta^{\underline{\alpha}}(\xi, \eta, \bar{\eta}) = \theta^{\underline{\alpha}}(\xi) + \eta\lambda^{\underline{\alpha}}(\xi) + \bar{\eta}\bar{\lambda}^{\underline{\alpha}}(\xi) + i\eta\bar{\eta}\sigma^{\underline{\alpha}}(\xi),$$

where in addition to the component fields already known to the reader, which obey the relations similar to (11) and (12), there appear auxiliary fields $y^{\underline{m}}(\xi)$ and $\sigma^{\underline{\alpha}}(\xi)$ which are expressed through other components and/or their derivatives via the superembedding conditions (32). We shall not present the explicit form of these expressions here but just show how the pure spinor conditions follow from (32). To this end let us hit the first expression in (32) by D and the second one by $\bar{D}$. In view of the $n = 2$ superalgebra commutation relations (30) we have

$$D\Theta\Gamma^{\underline{m}}D\Theta = 0, \qquad \bar{D}\Theta\Gamma^{\underline{m}}\bar{D}\Theta = 0,$$

and hence

$$\lambda\Gamma^{\underline{m}}\lambda = 0, \quad \bar{\lambda}\Gamma^{\underline{m}}\bar{\lambda} = 0. \tag{33}$$

By definition complex commuting spinors satisfying eqs. (33) are called pure spinors.

As in the previous case of the $N = 1$, $D = 3$ superparticle one can convince oneself that the following fermionic constraints are of the first class

$$\lambda^{\underline{\alpha}}D_{\underline{\alpha}} = \lambda^{\underline{\alpha}}(\pi_{\underline{\alpha}} - ip_{\underline{m}}(\Gamma^{\underline{m}}\theta)_{\underline{\alpha}}) = 0 \,, \tag{34}$$

$$\bar{\lambda}^{\underline{\alpha}}D_{\underline{\alpha}} = \bar{\lambda}^{\underline{\alpha}}(\pi_{\underline{\alpha}} - ip_{\underline{m}}(\Gamma^{\underline{m}}\theta)_{\underline{\alpha}}) = 0.$$

In the pure spinor approach to the covariant quantization [18, 19] the spinors $\bar{\lambda}$ never appear, the spinors λ are ghosts and the constraint $Q =$

$\lambda^{\underline{\alpha}} D_{\underline{\alpha}}$ plays the role of the BRST charge whose cohomology has been shown to reproduce the correct physical spectrum of quantized superparticles and superstrings.

We have thus shown that ingredients of the pure spinor quantization procedure appear in the superembedding formulation. The relationship between the two approaches has been established in [21] with the example of the heterotic string. We refer the reader to this article for further details and pass to the explanation of

4 The geometrical meaning of the superembedding condition

To understand the superembedding condition from the geometrical point of view let us note that the left hand side of (9) is the Grassmann component of the pull–back onto the superworldline of the target–space vector supervielbein one–form $E^{\underline{a}} = dX^{\underline{a}} - id\bar{\Theta}\Gamma^{\underline{a}}\Theta$

$$E^{\underline{a}}|_{\mathcal{M}_{sw}} = (d\xi + i\eta d\eta)(\partial_\xi X^{\underline{a}} - i\partial_\xi \bar{\Theta}\Gamma^{\underline{a}}\Theta) + d\eta(DX^{\underline{a}} - iD\bar{\Theta}\Gamma^{\underline{a}}\Theta), \quad (35)$$

where $d\xi + i\eta d\eta$ and $d\eta$ form a basis of the supercovariant one–forms (supervielbeins) on the superworldline.

We see that the superembedding condition (9) requires that the pullback of the target–space *vector* supervielbein $E^{\underline{a}}$ vanishes along the *Grassmann* directions of the superworldvolume.

This is the generic requirement for the superembedding to be appropriate to the description of the dynamics of the superbranes.

In general, if we take a supersurface $\mathcal{M}_{sw}$, whose geometry is described by supervielbein one–forms $e^a(z^M)$ $(a = 0, 1, \cdots, p)$ and $e^\alpha(z^M)$ $(\alpha = 1, \cdots n)$, and a curved target superspace M_{TS}, whose geometry is described by supervielbein one–forms $E^{\underline{a}}(Z^{\underline{M}})$ $(\underline{a} = 0, 1, \cdots, D-1)$ and $E^{\underline{\alpha}}(Z^{\underline{M}})$ $(\underline{\alpha} = 1, \cdots 2n)$[3], and consider the embedding of $\mathcal{M}_{sw}$ into M_{TS}, then for the superembedding to describe a super–p–brane propagating in M_{TS} the pull–back of E^a along the Grassmann directions of $\mathcal{M}_{sw}$ must vanish, i.e. in

$$E^{\underline{a}}|_{\mathcal{M}_{sw}} = e^{\underline{a}} E_a{}^{\underline{a}} + e^{\underline{\alpha}} E_\alpha{}^{\underline{a}} \quad (36)$$

the Grassmann components are zero

$$E_\alpha{}^{\underline{a}}(Z(z)) = 0. \quad (37)$$

[3]Note that the supergeometries of $\mathcal{M}_{sw}$ and M_{TS} should be that of corresponding supergravities, which implies that the torsions and curvatures of $\mathcal{M}_{sw}$ and M_{TS} are subject to appropriate supergravity constraints.

For the most of the superbranes (with some subtleties for the space filling and codimension one branes [36, 37, 28, 29]), the superembedding condition (37), accompanied by the M_{TS} and/or $\mathcal{M}_{sw}$ supergravity constraints, implies that

- the geometry of the superworldvolume $\mathcal{M}_{sw}$ is induced by its imbedding into M_{TS}, i.e. the $\mathcal{M}_{sw}$ supergravity on the brane is not propagative;
- the dynamics of the superbrane is subject to the standard constraints of the Green–Schwarz formulation, such as the Virasoro constraints and their fermionic counterparts;
- κ–symmetry is a particular form of worldvolume superdiffeomorphisms;
- the consistency of the superembedding condition results in the same 'brane scan' as that of the GS formulation.

In addition, when the number of the Grassmann directions of $\mathcal{M}_{sw}$ is 16 or higher, the integrability of the superembedding condition requires the worldvolume superfields to satisfy the dynamical equations of motion of the superbrane [16] [4]. It is in this way the covariant equations of motion of the M5–brane were obtained for the first time [17].

5 Superembedding actions

In the cases when the superembedding condition does not imply dynamical equations but only the kinematic constraints, one can construct (doubly supersymmetric) worldvolume superfield actions for corresponding superbranes. Several related methods have been proposed so far to construct the superembedding actions [8, 11, 13, 14, 15, 38, 27, 28, 29].

As we have already seen with the examples of the $N = 1$ superparticles in three and ten dimensions, the action for the massless superparticles is simply the integral over the superworldline of the product of the left hand side of the superembedding condition (37) with a Lagrange multiplier $P_{\underline{a}}^{\alpha}(z)$

$$S = \int d\xi d^n\eta P_{\underline{a}}^{\alpha}\ E_{\alpha}^{\underline{a}}(Z(z))\,. \tag{38}$$

In the case of the massive superparticles and superbranes, to the action (38) one must add a second term which governs the dynamics of the superbrane

[4]This is similar to the case of, say, higher dimensional super–Yang–Mills and supergravity theories whose superfield constraints produce the dynamical equations of motion.

and produces both the Nambu–Goto (or Dirac–Born–Infeld) term and the Wess–Zumino term of the GS formulation.

Let me explain the general structure of this second term with the example of a space filling D3–brane in $N = 2$, $D = 4$ superspace. In this case, the geometry of the worldvolume $\mathcal{M}_{sw}$ is that of chiral $N = 1$, $D = 4$ supergravity [28, 29].

The D3–brane couples to supergravity fields via the worldvolume pull–back of the Wess–Zumino four–form

$$\hat{C} = C_4 + F_2 \wedge C_2 + \frac{1}{2} F_2 \wedge F_2 C_0, \qquad (39)$$

where F_2 is the field strength of the BI gauge field propagating on the D3–brane and C_p (p=0,2,4) are 'Ramond-Ramond' p–form fields.

To construct the D3–brane action as an integral over the chiral $N = 1$, $D = 4$ superspace $\mathcal{M}_{sw}$ parametrized in the left–chiral basis by $z_L^M = (\xi_L^m, \eta^\alpha)$ $(\alpha = 1, 2)$ we take a component of the pull–back of (39) onto $\mathcal{M}_{sw}$ which has the appropriate dimensionality. This is $\hat{C}_{\dot{\alpha}\dot{\beta}ab}$. We then contract $\hat{C}_{\dot{\alpha}\dot{\beta}ab}$ with the antisymmetric product of the Pauli matrices $(\sigma^{ab})^{\dot{\alpha}\dot{\beta}}$. The D3–brane action (accompanied by the superembedding term (38)) is

$$S = \int d^4\xi_L d^2\eta \; \mathcal{E}_L (\sigma^{ab})^{\dot{\alpha}\dot{\beta}} \hat{C}_{\dot{\alpha}\dot{\beta}ab} \quad + \quad h.c., \qquad (40)$$

where $\mathcal{E}_L$ is a chiral superspace integration measure (see [28]).

Upon solving for the superembedding condition, integrating over η and eliminating auxiliary fields one can get from this simply–looking superembedding action the D3–brane action of the GS formulation. The relation of the action (40) to the superfield DBI action discussed in [40, 31, 32] has been established in [41].

As a final remark to this section we note that when the superembedding condition (37) produces dynamical equations of motion and the worldvolume superfield actions cannot be constructed there exist geometrically well–grounded recipes [39, 38] of how to construct Green–Schwarz–type actions for corresponding superbranes.

6 Conclusion

We have tried to describe in simple terms basic features of the superembedding approach, which has not only allowed one to explain and clarify various classical and quantum properties of superstring and superbrane theory, but has also found applications in the construction and description of new superbrane models, and of field theories with partially broken supersymmetry, as well as for solving practical problems. For instance, it has been used for calculating vertex operators in (M2–M5)–brane systems [42], and for constructing higher order contributions to brane actions [13, 43].

One may expect the superembedding methods to be also useful for other purposes, such as a unified (S–duality) description of fundamental and solitonic extended objects [44], and, in particular, for a covariant description of N coincident Dp-branes and of corresponding supersymmetric non–Abelian Dirac–Born–Infeld theories [33, 45].

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