Strings, p-Branes and Dp-Branes With Dynamical Tension

Eduardo Guendelman and Alexander Kaganovich
Department of Physics,
Ben-Gurion University, Beer-Sheva, Israel
email: guendel@bgumail.bgu.ac.il, alexk@bgumail.bgu.ac.il

Emil Nissimov and Svetlana Pacheva
Institute for Nuclear Research and Nuclear Energy,
Bulgarian Academy of Sciences, Sofia, Bulgaria
email: nissimov@inrne.bas.bg, svetlana@inrne.bas.bg

Abstract

We discuss a new class of brane models (extending both p-brane and Dp-brane cases) where the brane tension appears as an *additional dynamical degree of freedom* instead of being put in by hand as an *ad hoc* dimensionfull scale. Consistency of dynamics naturally involves the appearance of additional higher-rank antisymmetric tensor gauge fields on the world-volume which can couple to charged lower-dimensional branes living on the original Dp-brane world-volume. The dynamical tension has the physical meaning of electric-type field strength of the additional higher-rank world-volume gauge fields. It obeys Maxwell (or Yang-Mills) equations of motion (in the string case $p = 1$) or their higher-rank gauge theory analogues (in the Dp-brane case). This in particular triggers a simple classical mechanism of ("color") charge confinement.

1 Introduction

The crucial relevance of Dirichlet p-branes (Dp-branes) [1], *i.e.*, $p+1$-dimensional extended objects in space-time on which the ends of fundamental open strings can be confined, is largely appreciated and exploited in modern string theory (for reviews of string and brane theories, see [2, 3, 4]). Their importance is primarily due to the following basic properties: providing explicit realization of non-perturbative string dualities, microscopic description of black-hole physics, gauge theory/gravity correspondence, largeradius compactifications of extra dimensions, brane-world scenarios in particle phenomenology, *etc.* .

When building actions in geometrically motivated field theories, which is the class where strings and branes belong, one of the most important ingredients is the consistent generally-covariant integration measure density, *i.e.*, covariant under arbitrary diffeomorphisms (reparametrizations) on the underlying space-time manifold. *A priori* there are no compelling geometric reasons restricting us to the natural choice, which is the standard Riemannian metric density $\sqrt{-g}$ with $g \equiv \det \|g_{\mu\nu}\|$. For instance, introducing additional D scalar fields φ^i ($i = 1, \ldots, D$ where D is the space-time dimension) we may employ the following alternative non-Riemannian measure density $\Phi(\varphi)$:

$$\Phi(\varphi) \equiv \frac{1}{D!} \varepsilon^{\mu_1 \ldots \mu_D} \varepsilon_{i_1 \ldots i_D} \partial_{\mu_1} \varphi^{i_1} \ldots \partial_{\mu_D} \varphi^{i_D} \; . \tag{1}$$

Making use of (1) allows to construct a broad class of new models involving Gravity called *Two-Measure Gravitational Models* [5], whose actions are typically of the form:

$$S = \int d^D x \, \Phi(\varphi) \, L_1 + \int d^D x \, \sqrt{-g} \, L_2 \; , \tag{2}$$

$$L_{1,2} = e^{\frac{\alpha\phi}{M_P}} \Big[-\frac{1}{\kappa} R(g,\Gamma) - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + (\text{Higgs}) + (\text{fermions}) \Big] \; . \tag{3}$$

Here $R(g,\Gamma)$ is the scalar curvature in the first-order formalism (*i.e.*, the connection Γ is independent of the metric), ϕ is the dilaton field, M_P is the Planck mass, *etc.*. Although naively the additional "measure-density" scalars φ^i appear in (2) as pure-gauge degrees of freedom (due to the invariance under arbitrary diffeomorphisms in the φ^i-target space), there is still a remnant – the so called "geometric" field $\chi(x) \equiv \frac{\Phi(\varphi)}{\sqrt{-g}}$, which remains an additional dynamical degree of freedom beyond the standard physical degrees of freedom characteristic to the ordinary gravity models with the standard Riemannian-metric integration measure. The most important property of the "geometric" field $\chi(x)$ is that its dynamics is determined solely through the matter fields locally (*i.e.*, without gravitational interaction). The latter turns out to have a significant impact on the physical properties of the two-measure gravity models which allows them to address various basic problems in cosmology and particle physics phenomenology and provide physically plausible solutions, for instance: (i) the issue of scale invariance and its dynamical breakdown, *i.e.*, spontaneous generation of dimensionfull fundamental scales; (ii) cosmological constant problem; (iii) geometric origin of fermionic families.

In what follows we are going to apply the above ideas to the case of string, p-brane and Dp-brane models. To make the exposition self-contained Sections 2 and 3 below review our earlier works [6, 7] by providing a detailed description of the simplest $p = 1$ case - the class of new modified-measure string models. Then in Section 4 we proceed with the extension of our construction to the physically most interesting case of new modified-measure Dp-brane models. Along the way we elaborate on various important properties of the modified-measure string and brane models with dynamical string/brane tension.

2 Bosonic Strings with a Modified World-Sheet Integration Measure

First let us recall the standard Polyakov-type action for the bosonic string [8]:

$$S_{\rm Pol} = -T \int d^2\sigma\, \frac{1}{2}\sqrt{-\gamma}\gamma^{ab}\partial_a X^\mu \partial_b X^\nu G_{\mu\nu}(X) \; . \tag{4}$$

Here $(\sigma^0, \sigma^1) \equiv (\tau, \sigma)$; $a, b = 0, 1$; $\mu, \nu = 0, 1, \ldots, D-1$; $G_{\mu\nu}$ denotes the Riemannian metric on the embedding space-time; γ_{ab} is the intrinsic Riemannian metric on the $1+1$-dimensional string world-sheet and $\gamma = \det\|\gamma_{ab}\|$; T indicates the string tension – a dimensionfull scale introduced *ad hoc*. The resulting equations of motion w.r.t. γ^{ab} and X^μ read, respectively:

$$T_{ab} \equiv \left(\partial_a X^\mu \partial_b X^\nu - \frac{1}{2}\gamma_{ab}\gamma^{cd}\partial_c X^\mu \partial_d X^\nu\right) G_{\mu\nu}(X) = 0 \; , \tag{5}$$

$$\frac{1}{\sqrt{-\gamma}}\partial_a \left(\sqrt{-\gamma}\gamma^{ab}\partial_b X^\mu\right) + \gamma^{ab}\partial_a X^\nu \partial_b X^\lambda \Gamma^\mu_{\nu\lambda} = 0 \; , \tag{6}$$

where $\Gamma^\mu_{\nu\lambda} = \frac{1}{2}G^{\mu\kappa}\left(\partial_\nu G_{\kappa\lambda} + \partial_\lambda G_{\kappa\nu} - \partial_\kappa G_{\nu\lambda}\right)$ is the connection for the external metric.

Now, following the same ideas as in the construction of two-measure gravity theories (2)–(3) let us introduce two additional world-sheet scalar fields φ^i $(i = 1, 2)$ and replace $\sqrt{-\gamma}$ with a new reparametrization-covariant world-sheet integration measure density $\Phi(\varphi)$ defined solely in terms of φ^i:

$$\Phi(\varphi) \equiv \frac{1}{2}\varepsilon_{ij}\varepsilon^{ab}\partial_a\varphi^i\partial_b\varphi^j = \varepsilon_{ij}\dot{\varphi}^i\partial_\sigma\varphi^j \; . \tag{7}$$

However, the naively generalized string action:

$$S_1 = -\frac{1}{2}\int d^2\sigma\, \Phi(\varphi)\gamma^{ab}\partial_a X^\mu \partial_b X^\nu G_{\mu\nu}(X)$$

has a problem: the equations of motion w.r.t. γ^{ab} lead to an unacceptable condition $\Phi(\varphi)\,\partial_a X^\mu \partial_b X^\nu G_{\mu\nu}(X) = 0$, *i.e.*, vanishing of the induced metric on the world-sheet.

To remedy the above situation let us consider topological (total derivative) terms $\int \Omega$ w.r.t. the standard Riemannian world-sheet integration measure. Upon measure replacement $\sqrt{-\gamma} \to \Phi(\varphi)$, *i.e.*, $\int \Omega \;\to \int \Omega \,\frac{\Phi(\varphi)}{\sqrt{-\gamma}}$, the former are *not any more* topological – they will contribute nontrivially to the equations of motion. For instance:

$$\int d^2\sigma \sqrt{-\gamma}\, R \to \int d^2\sigma\, \Phi(\varphi)\, R \quad , \quad R = \frac{\varepsilon^{ab}}{2\sqrt{-\gamma}} \left(\partial_a \omega_b - \partial_b \omega_a\right) \; , \tag{8}$$

where R is the scalar curvature w.r.t. $D=2$ spin-connection $\omega_a^{\bar{a}\bar{b}} = \omega_a \varepsilon^{\bar{a}\bar{b}}$ (here $\bar{a}, \bar{b}$ denote tangent space indices). Note that the vector field ω_a behaves as world-sheet Abelian gauge field.

Eq.(8) prompts us to construct the following consistent modified bosonic string action[1]:

$$S = -\int d^2\sigma\, \Phi(\varphi) \Big[\frac{1}{2}\gamma^{ab} \partial_a X^\mu \partial_b X^\nu G_{\mu\nu} - \frac{\varepsilon^{ab}}{2\sqrt{-\gamma}} F_{ab}(A)\Big] \; , \tag{9}$$

where $\Phi(\varphi)$ is given by (7) and $F_{ab}(A) \equiv \partial_a A_b - \partial_b A_a$ is the field-strength of an auxiliary Abelian gauge field A_a. The action (9) is reparametrization-invariant as its ordinary string analogue (4). Furthermore, (9) is invariant under diffeomorphisms in φ-target space supplemented with a special conformal transformation of γ_{ab} :

$$\varphi^i \longrightarrow \varphi'^{\,i} = \varphi'^{\,i}(\varphi) \quad , \quad \gamma_{ab} \longrightarrow \gamma'_{ab} = J \gamma_{ab} \;\; , \;\; J \equiv \det \Big\| \frac{\partial \varphi'^{\,i}}{\partial \varphi^j} \Big\| \; . \tag{10}$$

The latter symmetry, which we will call *"Φ-extended Weyl symmetry"*, is the counterpart of the ordinary Weyl conformal symmetry of the standard string action (4).

The equations of motion of the action (9) w.r.t. φ^i :

$$\varepsilon^{ab} \partial_b \varphi^i \partial_a \Big(\gamma^{cd} \partial_c X^\mu \partial_d X^\nu G_{\mu\nu}(X) - \frac{\varepsilon^{cd}}{\sqrt{-\gamma}} F_{cd}\Big) = 0 \tag{11}$$

imply (provided $\Phi(\varphi) \neq 0$) :

$$\gamma^{cd} \partial_c X^\mu \partial_d X^\nu G_{\mu\nu}(X) - \frac{\varepsilon^{cd}}{\sqrt{-\gamma}} F_{cd} = M \; \Big(= \text{const}\Big) \; . \tag{12}$$

[1]In ref.[9] another interesting geometric modification of the standard bosonic string model has been proposed, which is based on dynamical world-sheet metric and torsion.

The equations of motion w.r.t. γ^{ab} are:

$$T_{ab} \equiv \partial_a X^\mu \partial_b X^\nu G_{\mu\nu}(X) - \frac{1}{2}\gamma_{ab}\frac{\varepsilon^{cd}}{\sqrt{-\gamma}}F_{cd} = 0 \; . \tag{13}$$

Both Eqs.(12)–(13) yield $M = 0$ and:

$$\Bigl(\partial_a X^\mu \partial_b X^\nu - \frac{1}{2}\gamma_{ab}\gamma^{cd}\partial_c X^\mu \partial_d X^\nu\Bigr) G_{\mu\nu}(X) = 0 \; , \tag{14}$$

which is the same as in standard Polyakov-type formulation (5).

The equations of motion w.r.t. X^μ read:

$$\partial_a \left(\Phi \gamma^{ab}\partial_b X^\mu\right) + \Phi \gamma^{ab}\partial_a X^\nu \partial_b X^\lambda \Gamma^\mu_{\nu\lambda} = 0 \; , \tag{15}$$

where again $\Gamma^\mu_{\nu\lambda}$ is the connection corresponding to the external space-time metric $G_{\mu\nu}$ as in the standard string case (6).

Now, let us consider the equations of motion w.r.t. A_a resulting from (9) :

$$\varepsilon^{ab}\partial_b \Bigl(\frac{\Phi(\varphi)}{\sqrt{-\gamma}}\Bigr) = 0 \; . \tag{16}$$

The latter can be integrated to yield a *spontaneously induced* string tension:

$$\frac{\Phi(\varphi)}{\sqrt{-\gamma}} = \mathrm{const} \equiv T \; .$$

Since the modified-measure string model (9) naturally requires the presence of the auxiliary Abelian world-sheet gauge field A_a, we may extend it by introducing a coupling of A_a to some world-sheet charge current j^a :

$$S = -\int d^2\sigma \, \Phi(\varphi) \Bigl[\frac{1}{2}\gamma^{ab}\partial_a X^\mu \partial_b X^\nu G_{\mu\nu} - \frac{\varepsilon^{ab}}{2\sqrt{-\gamma}}F_{ab}(A)\Bigr] + \int d^2\sigma \, A_a j^a \; . \tag{17}$$

In particular, we may take j^a to be the current of point-like charges on the string, so that in the "static" gauge:

$$\int d^2\sigma \, A_a j^a = -\sum_i e_i \int d\tau A_0(\tau, \sigma_i) \; , \tag{18}$$

where σ_i ($0 < \sigma_1 < \ldots < \sigma_N \leq 2\pi$) are the locations of the charges. Now, instead of (16) the action (17) produces the following A_a-equations of motion:

$$\varepsilon^{ab}\partial_b E + j^a = 0 \quad , \quad E \equiv \frac{\Phi(\varphi)}{\sqrt{-\gamma}} \; . \tag{19}$$

Note that Eqs.(19) look exactly as $D = 2$ Maxwell equations where the *variable* dynamical string tension $E \equiv \Phi(\varphi)/\sqrt{-\gamma}$ is identified as world-sheet electric field strength, *i.e.*, canonically conjugated momentum w.r.t. A_1.

The physical meaning of the dynamical string tension as world-sheet electric field strength can be directly verified in the framework of the canonical Hamiltonian treatment of the modified-measure string model (17). Indeed, from the explicit form of the action (17) we find the canonical momenta to be:

$$\pi_i^\varphi = -\varepsilon_{ij}\partial_\sigma\varphi^j \Big[\frac{1}{2}\gamma^{ab}\partial_a X^\mu \partial_b X^\nu G_{\mu\nu} - \frac{\varepsilon^{ab}}{2\sqrt{-\gamma}} F_{ab}(A)\Big] , \tag{20}$$

$$\pi_{A_1} \equiv E = \frac{\Phi(\varphi)}{\sqrt{-\gamma}} \quad , \quad \mathcal{P}_\mu = -\Phi(\varphi)\left(\gamma^{00}\dot{X}^\nu + \gamma^{01}\partial_\sigma X^\nu\right) G_{\mu\nu} , \tag{21}$$

Using (20)–(21) we obtain the canonical Hamiltonian as a linear combination of first-class constraints only. Part of the latter resemble the constraints in the ordinary string case $\pi_{\gamma^{ab}} = 0$ and

$$\mathcal{T}_\pm \equiv \frac{1}{4} G^{\mu\nu}\Big(\frac{\mathcal{P}_\mu}{E} \pm G_{\mu\kappa}\partial_\sigma X^\kappa\Big)\Big(\frac{\mathcal{P}_\nu}{E} \pm G_{\nu\lambda}\partial_\sigma X^\lambda\Big) = 0 ,$$

where in the last Virasoro constraints the dynamical string tension E appears instead of the *ad hoc* constant tension.

The rest of the Hamiltonian constraints are $\pi_{A_0} = 0$ and

$$\partial_\sigma E - \sum_i e_i \delta(\sigma - \sigma_i) = 0 , \tag{22}$$

which is precisely the $D = 2$ "Gauss law" constraint for the dynamical string tension coinciding with the 0-th component of the Maxwell-type Eq.(19). Finally, we have constraints involving only the measure-density fields:

$$\partial_\sigma \varphi^i \pi_i^\varphi = 0 \quad , \quad \frac{\pi_2^\varphi}{\partial_\sigma \varphi^1} = 0 . \tag{23}$$

The last two constraints span a closed Poisson-bracket algebra:

$$\Big\{ \partial_\sigma\varphi^i\pi_i^\varphi(\sigma) \, , \, \partial_{\sigma'}\varphi^i\pi_i^\varphi(\sigma') \Big\} =$$
$$2\partial_\sigma\varphi^i\pi_i^\varphi(\sigma)\partial_\sigma\delta(\sigma-\sigma') + \partial_\sigma\left(\partial_\sigma\varphi^i\pi_i^\varphi\right)\delta(\sigma-\sigma') ,$$

(a centerless Virasoro algebra), and:

$$\Big\{ \partial_\sigma\varphi^i\pi_i^\varphi(\sigma) \, , \, \frac{\pi_2^\varphi}{\partial_\sigma\varphi^1}(\sigma') \Big\} = -\partial_\sigma\Big(\frac{\pi_2^\varphi}{\partial_\sigma\varphi^1}\Big)\delta(\sigma-\sigma') .$$

Therefore, the constraints (23) imply that the measure-density scalars φ^i are pure-gauge degrees of freedom. The only physical remnant of the latter is the specific combination $\frac{\Phi(\varphi)}{\sqrt{-\gamma}}$ (see first Eq.(21)) – the world-sheet electric field-strength simultaneously playing the role of dynamical string tension.

3 Non-Abelian Generalization

We will now show that it is possible to introduce an alternative form of topological term necessary to make consistent the modified-measure string model which is built in terms of a *non-Abelian* auxiliary world-sheet gauge field.

First, let us notice the following identity in $D = 2$ involving Abelian gauge field A_a:

$$\frac{1}{2\sqrt{-\gamma}}\varepsilon^{ab}F_{ab}(A) = \sqrt{\frac{1}{2}F_{ab}(A)F_{cd}(A)\gamma^{ac}\gamma^{bd}}\ . \tag{24}$$

This suggests the following proper extension of the modified-measure bosonic string action (9) involving a non-Abelian (*e.g.*, $SU(\mathcal{N})$) auxiliary gauge field A_a (here we take for simplicity flat external metric $G_{\mu\nu} = \eta_{\mu\nu}$) :

$$\begin{aligned} S &= -\int d^2\sigma\, \Phi(\varphi)\Big[\frac{1}{2}\gamma^{ab}\partial_a X^\mu \partial_b X_\mu - \sqrt{\frac{1}{2}\,\mathrm{Tr}(F_{ab}(A)F_{cd}(A))\gamma^{ac}\gamma^{bd}}\Big] \\ &= -\int d^2\sigma\, \Phi(\varphi)\Big[\frac{1}{2}\gamma^{ab}\partial_a X^\mu \partial_b X_\mu - \frac{1}{\sqrt{-\gamma}}\sqrt{\mathrm{Tr}(F_{01}(A)F_{01}(A))}\Big]\ , \end{aligned} \tag{25}$$

where $F_{ab}(A) = \partial_a A_b - \partial_b A_c + i[A_a,\, A_b]$.

The action (25) is again invariant under the Φ*-extended Weyl (conformal)* symmetry (10).

Notice that the *"square-root" Yang-Mills* action (with the regular Riemannian metric integration measure):

$$\int d^2\sigma\, \sqrt{-\gamma}\sqrt{\frac{1}{2}\,\mathrm{Tr}(F_{ab}(A)F_{cd}(A))\gamma^{ac}\gamma^{bd}} = \int d^2\sigma\, \sqrt{\mathrm{Tr}(F_{01}(A)F_{01}(A))} \tag{26}$$

is a "topological" action similarly to the $D = 3$ Chern-Simmons action, *i.e.*, it is *metric-independent*.

Similarly to the Abelian case (17) we can also add a coupling of the auxiliary non-Abelian gauge field A_a to an external "color"-charge world-sheet current j^a:

$$S = -\int d^2\sigma\, \Phi(\varphi)\Big[\frac{1}{2}\gamma^{ab}\partial_a X^\mu \partial_b X_\mu - \frac{1}{\sqrt{-\gamma}}\sqrt{\mathrm{Tr}(F_{01}(A)F_{01}(A))}\Big]$$

$$+\int d^2\sigma\ \mathrm{Tr}\,(A_a j^a)\ . \tag{27}$$

In particular, for a current of "color" point-like charges on the world-sheet in the "static" gauge :

$$\int d^2\sigma\ \mathrm{Tr}\,(A_a j^a) = -\sum_i \mathrm{Tr}\, C_i \int d\tau A_0(\tau,\sigma_i)\ , \tag{28}$$

where σ_i $(0 < \sigma_1 < \ldots < \sigma_N \leq 2\pi)$ are the locations of the charges.

The action (27) produces the following equations of motion w.r.t. φ^i and γ^{ab}, respectively:

$$\frac{1}{2}\gamma^{cd}\partial_c X^\mu \partial_d X_\mu - \frac{1}{\sqrt{-\gamma}}\sqrt{\mathrm{Tr}(F_{01}F_{01})} = M\ \Big(= \mathrm{const}\Big)\ , \tag{29}$$

$$T_{ab} \equiv \partial_a X^\mu \partial_b X_\mu - \frac{1}{\sqrt{-\gamma}}\gamma_{ab}\sqrt{\mathrm{Tr}(F_{01}F_{01})} = 0\ . \tag{30}$$

As in the Abelian case the above Eqs.(29)–(30) imply $M = 0$ and the Polyakov-type equation (5).

Similarly to the Abelian case (19), the equations of motion of (27) w.r.t. the auxiliary gauge field A_a resemble the $D = 2$ non-Abelian Yang-Mills equations:

$$\varepsilon^{ab}\nabla_b \mathcal{E} + j^a = 0\ , \tag{31}$$

where:

$$\nabla_a \mathcal{E} \equiv \partial_a \mathcal{E} + i[A_a, \mathcal{E}] \quad , \quad \mathcal{E} \equiv \pi_{A_1} \equiv \frac{\Phi(\varphi)}{\sqrt{-\gamma}}\frac{F_{01}}{\sqrt{\mathrm{Tr}(F_{01}F_{01})}}\ . \tag{32}$$

Here $\mathcal{E}$ is the non-Abelian electric field-strength – the canonically conjugated momentum π_{A_1} of A_1, whose norm is the dynamical string tension $T \equiv |\mathcal{E}| = \Phi(\varphi)/\sqrt{-\gamma}$.

The equations of motion for the dynamical string tension following from (31) is:

$$\partial_a\Big(\frac{\Phi(\varphi)}{\sqrt{-\gamma}}\Big) + \varepsilon_{ab}\frac{\mathrm{Tr}\,\Big(F_{01}j^b\Big)}{\sqrt{\mathrm{Tr}\,(F_{01}^2)}} = 0\ . \tag{33}$$

In particular, in the absence of external charges $(j^a = 0)$: $T \equiv \Phi(\varphi)/\sqrt{-\gamma} = T_0 \equiv \mathrm{const}$

Finally, the X^μ-equations of motion $\partial_a\Big(\Phi(\varphi)\gamma^{ab}\partial_b X^\mu\Big) = 0$ resulting from the action (27) can be rewritten in the conformal gauge $\sqrt{-\gamma}\gamma^{ab} = \eta^{ab}$

as:

$$\frac{\Phi(\varphi)}{\sqrt{-\gamma}}\partial^a\partial_a X^\mu - \widetilde{j}^a\partial_a X^\mu = 0 \quad , \quad \text{where } \widetilde{j}_a \equiv \varepsilon_{ab}\frac{\mathrm{Tr}\left(F_{01}j^b\right)}{\sqrt{\mathrm{Tr}\left(F_{01}^2\right)}} \; . \tag{34}$$

For static charges $\widetilde{j}_1 = -\sum_i \widetilde{e}_i\delta(\sigma-\sigma_i)$:

$$T \equiv \Phi(\varphi)/\sqrt{-\gamma} = T_0 + \sum_i \widetilde{e}_i\theta(\sigma-\sigma_i) \quad , \quad \widetilde{e}_i \equiv \frac{\mathrm{Tr}\left(F_{01}C_i\right)}{\sqrt{\mathrm{Tr}\left(F_{01}^2\right)}}\Bigg|_{\sigma=\sigma_i} \; ; \tag{35}$$

$$T\partial^a\partial_a X^\mu + \Big(\sum_i \widetilde{e}_i\delta(\sigma-\sigma_i)\Big)\partial_\sigma X^\mu = 0 \;\; \to \left\{ \begin{array}{l} \partial^a\partial_a X^\mu = 0 \\ \partial_\sigma X^\mu \Big|_{\sigma=\sigma_i} = 0 \end{array} \right. . \tag{36}$$

Let us return to the $D=2$ Yang-Mills-like Eqs.(31) whose 0-th component $\partial_\sigma\mathcal{E} + i\Big[A_1\,,\,\mathcal{E}\Big] + j^0 = 0$ is the "Gauss law" constraint for the dynamical string tension ($T \equiv |\mathcal{E}| = \Phi(\varphi)/\sqrt{-\gamma}$). For point-like "color" charges and using the gauge $A_1 = 0$ (*i.e.*, $\mathcal{E} \to \widetilde{\mathcal{E}} = G\mathcal{E}G^{-1}$ where $A_1 = -iG^{-1}\partial_\sigma G$), the latter reads:

$$\partial_\sigma\widetilde{\mathcal{E}} - \sum_i \widetilde{C}_i\delta(\sigma-\sigma_i) = 0 \quad , \quad \widetilde{C}_i \equiv GC_iG^{-1}\Bigg|_{\sigma=\sigma_i} \; . \tag{37}$$

Let us consider the case of *closed* modified string with positions of the "color" charges at $0 < \sigma_1 < \ldots < \sigma_N \leq 2\pi$. Then, integrating the "Gauss law" constraint (37) along the string (at fixed proper time) we obtain:

$$\sum_i \widetilde{C}_i = 0 \quad , \quad \widetilde{\mathcal{E}}_{i,i+1} = \widetilde{\mathcal{E}}_{i-1,i} + \widetilde{C}_i \; , \tag{38}$$

where $\widetilde{\mathcal{E}}_{i,i+1} = \widetilde{\mathcal{E}}$ in the interval $\sigma_i < \sigma < \sigma_{i+1}$.

The implications of Eqs.(35)–(38) can be summarize as follows:

- Eqs.(35)–(36) tell us that the modified-measure (closed) string with N point-like ("color") charges on it ((17) or (27)) is equivalent to N chain-wise connected regular open string segments (stretching from σ_i to σ_{i+1}, $i = 0, 1, \ldots, N-1$) which obey Neumann boundary conditions.

- Each of the above open string segments, with end-points at the charges e_i and e_{i+1} (in the Abelian case) or C_i and C_{i+1} (in the non-Abelian case), has *different* constant string tension $T_{i,i+1}$ such that $T_{i,i+1} = T_{i-1,i} + \overset{(\sim)}{e}_i$ (the non-Abelian $\widetilde{e}_i$ are defined in (35)).

- Eq.(38) shows that the only (classically) admissable configuration of "color" point-like charges coupled to a modified-measure closed bosonic string is the one with *zero* total "color" charge, *i.e.*, the model (27) provides a classical mechanism of "color" charge confinement.

4 Dp-Branes With Dynamical Tension

Our construction from the previous two sections can be extended to the case of higher-dimensional extended objects with dynamical tension embracing both (ordinary) p-branes and Dp-branes. First, let us recall the standard formulation of Dp-branes given in terms of the Dirac-Born-Infeld (DBI) action [4] :

$$S_{DBI} = -T \int d^{p+1}\sigma \left[e^{-\alpha U} \sqrt{-\det \|G_{ab} - \mathcal{F}_{ab}\|} + \frac{\varepsilon^{a_1 \ldots a_{p+1}}}{p+1} \mathcal{C}_{a_1 \ldots a_{p+1}} \right] , \tag{39}$$

with the following short-hand notations:

$$G_{ab} \equiv \partial_a X^\mu \partial_b X^\nu G_{\mu\nu}(X) \quad , \quad \mathcal{F}_{ab} \equiv \partial_a X^\mu \partial_b X^\nu B_{\mu\nu}(X) - F_{ab}(A) \; , \tag{40}$$

$$\mathcal{C}_{a_1 \ldots a_{p+1}} \equiv \partial_{a_1} X^{\mu_1} \ldots \partial_{a_{p+1}} X^{\mu_{p+1}} C_{\mu_1 \ldots \mu_{p+1}}(X) \; . \tag{41}$$

Here $G_{\mu\nu}(X)$, $U(X)$, $B_{\mu\nu}(X)$, and $C_{\mu_1 \ldots \mu_{p+1}}(X)$ are the background metric, the dilaton, 2-form Neveu-Schwarz and $(p{+}1)$-form Ramond-Ramond gauge fields, respectively, whereas $F_{ab}(A) = \partial_a A_b - \partial_b A_b$ is the field-strength of the Abelian world-volume gauge field A_a. All world-volume indices take values $a, b = 0, 1, \ldots, p$ and $\varepsilon^{a_1 \ldots a_{p+1}}$ is the $(p+1)$-dimensional totally antisymmetric tensor ($\varepsilon^{01 \ldots p} = 1$).

Similarly to the string case we now introduce a modified world-volume integration measure density in terms of $p+1$ auxiliary scalar fields φ^i ($i = 1, \ldots, p+1$) :

$$\Phi(\varphi) \equiv \frac{1}{(p+1)!} \varepsilon_{i_1 \ldots i_{p+1}} \varepsilon^{a_1 \ldots a_{p+1}} \partial_{a_1} \varphi^{i_1} \ldots \partial_{a_{p+1}} \varphi^{i_{p+1}} \; , \tag{42}$$

and use it to construct the following new p-brane-type action (coupling to the Ramond-Ramond background field is omitted for simplicity)[2]:

$$S = -\int d^{p+1}\sigma\, \Phi(\varphi) \Big[e^{-\beta U} \frac{1}{2} \zeta^{ab} \left(G_{ba} - \mathcal{F}_{ba} \right) + \frac{1}{\sqrt{-\zeta}} \Omega(\mathcal{A}) \Big] + \int d^{p+1}\sigma\, \mathcal{L}(\mathcal{A}) \; . \tag{43}$$

Here apart from (40) the following new notations are used. The $(p+1) \times (p+1)$ matrix ζ_{ab} of auxiliary variables is an arbitrary world-volume 2-tensor, ζ^{ab} denotes the corresponding inverse matrix ($\zeta^{ac}\zeta_{cb} = \delta^a_b$) and $\zeta \equiv \det\|\zeta_{ab}\|$. The term $\Omega(\mathcal{A})$ indicates a topological density given in terms of some additional auxiliary gauge (or matter) fields $\mathcal{A}^I$ living on the world-volume, where "topological" means:

$$\frac{\partial \Omega}{\partial \mathcal{A}^I} - \partial_a \left(\frac{\partial \Omega}{\partial \partial_a \mathcal{A}^I} \right) = 0 \;\; \text{identically} \quad , \quad \text{i.e.} \;\; \delta\Omega(\mathcal{A}) = \partial_a \left(\frac{\partial \Omega}{\partial \partial_a \mathcal{A}^I} \delta\mathcal{A}^I \right) \; . \tag{44}$$

$\mathcal{L}(\mathcal{A})$ describes possible coupling of the auxiliary fields A^I to external "currents" on the brane world-volume.

The requirement for $\Omega(\mathcal{A})$ to be a topological density is dictated by the requirement that the new brane action (43) (in the absence of the last gauge/matter term $\int d^{p+1}\sigma\, \mathcal{L}(\mathcal{A})$) reproduces the standard Dp-brane equations of motion resulting from the DBI action (39) apart from the fact that the Dp-brane tension $T \equiv \Phi(\varphi)/\sqrt{-\zeta}$ becomes now an *additional dynamical degree of freedom* (note that no *ad hoc* dimensionfull tension factor T has been introduced in (43)). Let us stress that, similarly to the string case, the modified-measure brane model (43) naturally requires (through the necessity to introduce topological density $\Omega(\mathcal{A})$) the existence on the world-volume of an additional (higher-rank tensor) gauge field $\mathcal{A}$ apart from the standard world-volume Abelian vector gauge A_a.

Splitting the auxiliary tensor variable $\zeta^{ab} = \gamma^{ab} + \zeta^{[ab]}$ into symmetric and anti-symmetric parts and setting $\zeta^{[ab]} = 0$, the action (43) reduces to the action of the modified-measure model of ordinary p-branes [7] with Neveu-Schwarz field $B_{\mu\nu}$ and world-volume gauge field A_a disappearing and γ_{ab} assuming the role of world-volume Riemannian metric.

The most obvious example of a topological density $\Omega(\mathcal{A})$ for the addi-

[2]Some time ago a Polyakov-type action classically equivalent to the original DBI-action of the Dp-brane (39) has been proposed in [10] : $S = -T \int d^{p+1}\sigma \sqrt{-\zeta} \Big[e^{-\beta U} \frac{1}{2} \zeta^{ab} \left(G_{ba} - \mathcal{F}_{ba} \right) - (p-1) \Big]$. The advantage of the present more general action (43) is that not only it yields variable dynamical brane tension and naturally introduces additional higher-rank world-volume gauge fields, but also it does not need a "cosmological" term.

tional auxiliary gauge/matter world-volume fields in (43) is:

$$\Omega(\mathcal{A}) = -\frac{\varepsilon^{a_1\ldots a_{p+1}}}{p+1} F_{a_1\ldots a_{p+1}}(\mathcal{A}) \quad , \quad F_{a_1\ldots a_{p+1}}(\mathcal{A}) = (p+1)\partial_{[a_1}\mathcal{A}_{a_2\ldots a_{p+1}]} \; , \tag{45}$$

where $\mathcal{A}_{a_1\ldots a_p}$ denotes rank p antisymmetric tensor (Abelian) gauge field on the world-volume. Further, as a physically interesting example let us consider the following natural coupling of the auxiliary p-form gauge field:

$$\int d^{p+1}\sigma\, \mathcal{L}(\mathcal{A}) = \int d^{p+1}\sigma\, \mathcal{A}_{a_1\ldots a_p} j^{a_1\ldots a_p} \tag{46}$$

to an external world-volume current:

$$j^{a_1\ldots a_p} = \sum_i e_i \int_{\mathcal{B}_i} d^p u\, \frac{1}{p!}\varepsilon^{\alpha_1\ldots\alpha_p} \frac{\partial\sigma_i^{a_1}}{\partial u^{\alpha_1}} \ldots \frac{\partial\sigma_i^{a_p}}{\partial u^{\alpha_p}} \delta^{(p+1)}(\underline{\sigma} - \underline{\sigma}_i(\underline{u})) \; . \tag{47}$$

Here $j^{a_1\ldots a_p}$ is a current of charged $(p-1)$-sub-branes $\mathcal{B}_i$ embedded into the original p-brane world-volume via $\sigma^a = \sigma_i^a(\underline{u})$ with parameters $\underline{u} \equiv (u^\alpha)_{\alpha=0,\ldots,p-1}$. For simplicity we assume that the $\mathcal{B}_i$ sub-branes do not intersect each other.

With the choices (45) and (46)–(47) the action (43) becomes:

$$S = -\int d^{p+1}\sigma\, \Phi(\varphi)\Big[e^{-\beta U}\frac{1}{2}\zeta^{ab}\left(G_{ba} - \mathcal{F}_{ba}\right) - \frac{\varepsilon^{a_1\ldots a_{p+1}}}{(p+1)\sqrt{-\zeta}} F_{a_1\ldots a_{p+1}}(\mathcal{A})\Big] \\ - \int d^{p+1}\sigma\, \mathcal{A}_{a_1\ldots a_p}\varepsilon^{a a_1\ldots a_p}\widetilde{j}_a \; , \tag{48}$$

with $\widetilde{j}_a = \sum_i e_i \mathcal{N}_a^{(i)}$, where $\mathcal{N}_a^{(i)}$ is the normal vector w.r.t. world-hyper-surface of the $(p-1)$-sub-brane $\mathcal{B}_i$:

$$\mathcal{N}_a^{(i)} \equiv \frac{1}{p!}\varepsilon_{ab_1\ldots b_p} \int_{\mathcal{B}_i} d^p u\, \frac{1}{p!}\varepsilon^{\alpha_1\ldots\alpha_p} \frac{\partial\sigma_i^{a_1}}{\partial u^{\alpha_1}} \ldots \frac{\partial\sigma_i^{a_p}}{\partial u^{\alpha_p}} \delta^{(p+1)}(\underline{\sigma} - \underline{\sigma}_i(\underline{u})) \; . \tag{49}$$

Let us note that some time ago a modified p-brane model has been proposed in ref.[11] which also contains world-volume p-form gauge field $\mathcal{A}_{a_1\ldots a_p}$. However, the latter model is significantly different from (48) since it is not of Polyakov-type and, moreover, $\mathcal{A}_{a_1\ldots a_p}$ appears there quadratically rather than linearly.

More generally, for $p+1 = rs$ we can have a more general type of topological density entering (43) :

$$\Omega(\mathcal{A}) = \frac{1}{rs}\varepsilon^{a_{11}\ldots a_{1r}\ldots a_{s1}\ldots a_{sr}} F_{a_{11}\ldots a_{1r}}(\mathcal{A}) \ldots F_{a_{s1}\ldots a_{sr}}(\mathcal{A}) \tag{50}$$

with rank $r-1$ (smaller than p) auxiliary world-volume gauge fields.

We may also employ *non-Abelian* auxiliary world-volume gauge fields as in the string case. For instance, when $p=3$ we may take:

$$\Omega(\mathcal{A}) = \frac{1}{4}\varepsilon^{abcd}\,\mathrm{T}r\,(F_{ab}(\mathcal{A})F_{cd}(\mathcal{A})) \tag{51}$$

or, more generally, for $p+1=2q$:

$$\Omega(\mathcal{A}) = \frac{1}{2q}\varepsilon^{a_1 b_1 \ldots a_q b_q}\,\mathrm{T}r\,\Big(F_{a_1 b_1}(\mathcal{A})\ldots F_{a_q b_q}(\mathcal{A})\Big)\ , \tag{52}$$

where $F_{ab}(\mathcal{A}) = \partial_a\mathcal{A}_b - \partial_b\mathcal{A}_a + i[\mathcal{A}_a,\mathcal{A}_b]$.

The modified-measure brane action (48) yields the following equations of motion w.r.t. φ^i and ζ^{ab} :

$$e^{-\beta U}\frac{1}{2}\zeta^{ab}\left(G_{ba}-\mathcal{F}_{ba}\right) + \frac{1}{\sqrt{-\zeta}}\Omega(\mathcal{A}) = M \equiv \mathrm{const}\ , \tag{53}$$

$$e^{-\beta U}\left(G_{ab}-\mathcal{F}_{ab}\right) + \zeta_{ab}\frac{1}{\sqrt{-\zeta}}\Omega(\mathcal{A}) = 0\ . \tag{54}$$

Both Eqs.(53)–(54) imply:

$$\zeta^{ab}\left(G_{ba}-\mathcal{F}_{ba}\right) = 2M\frac{p+1}{p-1}e^{\beta U} \quad , \quad \frac{1}{\sqrt{-\zeta}}\Omega(\mathcal{A}) = -\frac{2M}{p-1} \tag{55}$$

which when substituted in (54) give:

$$G_{ab}-\mathcal{F}_{ab} = \frac{2M}{p-1}e^{\beta U}\,\zeta_{ab} \tag{56}$$

We now consider the equations of motion of the modified brane action (43) w.r.t. auxiliary (gauge) fields $\mathcal{A}^I$ – these are the equations determining the dynamical brane tension $\mathcal{T} \equiv \Phi(\varphi)/\sqrt{-\zeta}$:

$$\partial_a\Big(\frac{\Phi(\varphi)}{\sqrt{-\zeta}}\Big)\,\frac{\partial\Omega}{\partial\partial_a\mathcal{A}^I} + j_I = 0\ , \tag{57}$$

where $j_I \equiv \frac{\partial\mathcal{L}}{\partial\mathcal{A}^I} - \partial_a\Big(\frac{\partial\mathcal{L}}{\partial\partial_a\mathcal{A}^I}\Big)$ is the corresponding "current" coupled to $\mathcal{A}^I$. Note that the topological nature of $\Omega(\mathcal{A})$ (Eq.(44)) is crucial in the derivation of (57). With the choice (46)–(47), Eq.(57) becomes:

$$\partial_a\Big(\frac{\Phi(\varphi)}{\sqrt{-\zeta}}\Big) + \tilde{j}_a = 0 \quad , \quad \tilde{j}_a \equiv \sum_i e_i\mathcal{N}^{(i)}_a\ . \tag{58}$$

It is straightforward to deduce from the action (48) that, similarly to the modified string case (19), the dynamical brane tension is identified as the electric-type field strength, *i.e.*, the canonical momentum corresponding to the p-form gauge field: $\pi_{A_{1\ldots p}} \equiv E = \frac{\Phi(\varphi)}{\sqrt{-\zeta}}$. Accordingly, Eqs.(58) are of the same form as the Maxwell-type equations of motion for the p-form gauge field in $p+1$ dimensions.

In particular, in the absence of coupling of external world-volume currents to the auxiliary (gauge) fields $\mathcal{A}^I$ Eq.(57) (or (58)) imply:

$$\mathcal{T} \equiv \Phi(\varphi)/\sqrt{-\zeta} = C \equiv \text{const} \tag{59}$$

Now, using Eqs.(53) and (56) it is straightforward to show that the modified brane action (43) with $\mathcal{L}(\mathcal{A}) = 0$ classically reduces to the standard Dp-brane DBI-action (39) :

$$S'_{DBI} = -T' \int d^{p+1}\sigma \, e^{-\beta' U} \sqrt{-\det ||G_{ab} - \mathcal{F}_{ab}||} \; , \tag{60}$$

$$T' \equiv \frac{1}{2} C (2M)^{-\frac{p-1}{2}} (p-1)^{\frac{p+1}{2}} \quad , \quad \beta' \equiv \frac{p+1}{2} \beta \; , \tag{61}$$

where, however, the Dp-brane tension T' is dynamically generated according to (59) and (61).

More generally, recalling the definition (49) of $\mathcal{N}^{(i)}_a$ we find from (58) that the dynamical brane tension $\mathcal{T} \equiv \Phi(\varphi)/\sqrt{-\gamma}$ is piece-wise constant on the world-volume with jumps when crossing the world-hypersurface of each charged $(p-1)$-sub-brane $\mathcal{B}_i$. The corresponding jump being equal to the charge magnitude $\pm e_i$ (the overall sign depending on the direction of crossing w.r.t. the normal $\mathcal{N}^{(i)}_a$).

Finally, let us consider the equations of motion of (43) w.r.t. the Abelian Dp-brane vector gauge field A_a :

$$\partial_b \left(\Phi(\varphi) \zeta^{[ab]} e^{-\beta U} \right) = 0 \; , \tag{62}$$

and the equations of motion w.r.t. the embedding coordinates X^μ (taking into account Eqs.(62)) :

$$\partial_a \left(\Phi(\varphi) \zeta^{(ab)} \partial_b X^\mu \right) + \\ \Phi(\varphi) \partial_a X^\lambda \partial_b X^\nu \left[\zeta^{(ab)} \Gamma^\mu_{\lambda\nu}(\bar{G}) + \frac{1}{2} \zeta^{[ab]} G^{\mu\kappa} \left(\beta \partial_\kappa U B_{\lambda\nu} - \mathcal{H}_{\kappa\lambda\nu}(B) \right) \right] = 0 \, . \tag{63}$$

where $\Gamma^\mu_{\lambda\nu}(\bar{G})$ is the connection of the rescaled background metric $\bar{G}_{\mu\nu} = e^{-\beta U} G_{\mu\nu}$:

$$\Gamma^\mu_{\lambda\nu}(\bar{G}) = \frac{1}{2} \bar{G}^{\mu\kappa} \left(\partial_\lambda \bar{G}_{\kappa\nu} + \partial_\nu \bar{G}_{\kappa\lambda} - \partial_\kappa \bar{G}_{\lambda\nu} \right) \tag{64}$$

and $\mathcal{H}_{\kappa\lambda\nu}(\bar{B})$ is the field-strength of the background Neveu-Schwarz two-form gauge field:

$$\mathcal{H}_{\kappa\lambda\nu}(B) = \partial_\kappa B_{\lambda\nu} + \partial_\nu B_{\kappa\lambda} + \partial_\lambda B_{\nu\kappa} \tag{65}$$

Using (58), both Eqs.(62)–(63) can be rewritten in the form:

$$\frac{\Phi(\varphi)}{\sqrt{-\zeta}}\partial_b \widetilde{H}^{ab} - \sum_i e_i \mathcal{N}_b^{(i)} \widetilde{H}^{ab} = 0 \quad , \quad \widetilde{H}^{ab} \equiv \sqrt{-\zeta}\,\zeta^{[ab]} e^{-\beta U} \; , \tag{66}$$

and:

$$\frac{\Phi(\varphi)}{\sqrt{-\zeta}}\left[\partial_a\left(\sqrt{-\zeta}\zeta^{(ab)}\partial_b X^\mu\right) + \ldots\right] - \sum_i e_i \mathcal{N}_a^{(i)} \sqrt{-\zeta}\zeta^{(ab)}\partial_b X^\mu = 0 \; . \tag{67}$$

The dots in the l.h.s. of (67) indicate the same expression as in the second line of Eq.(63) with $\Phi(\varphi)$ substituted by $\sqrt{-\zeta}$.

Recalling again the definition (49) of the normal $\mathcal{N}_b^{(i)}$ w.r.t. the charged $(p-1)$-sub-branes $\mathcal{B}_i$ and the resulting property of the dynamical tension $\mathcal{T} \equiv \Phi(\varphi)/\sqrt{-\gamma}$ being piece-wise constant on the world-volume with jumps when crossing each sub-brane $\mathcal{B}_i$, we find that Eqs.(66) imply:

$$\partial_b \widetilde{H}^{ab} = 0 \quad , \quad \widetilde{H}^{a\perp}\Big|_{\mathcal{B}_i} = 0 \; , \tag{68}$$

where the superscript $\perp$ indicates projection along the normal $N_a^{(i)}$ w.r.t. the world-hypersurface of sub-brane $\mathcal{B}_i$. The first Eq.(68) can be viewed as Bianchi identity for the field-strength of a new $p-2$-form world-volume gauge field $B_{a_1 \ldots a_{p-2}}$ (which can be viewed as dual variable w.r.t. anti-symmetric part of ζ^{ab}) :

$$H_{a_1\ldots a_{p-1}} = (p-1)\partial_{[a_1} B_{a_2\ldots a_{p-1}]} \quad , \quad \widetilde{H}^{ab} \equiv \frac{1}{(p-1)!}\varepsilon^{abc_1\ldots c_{p-1}} H_{c_1\ldots c_{p-1}} \tag{69}$$

with $\widetilde{H}^{ab}$ as defined in (66). The second Eq.(68) shows that the restrictions of the dual field strength H on each $(p-1)$-sub-brane $\mathcal{B}_i$ vanish:

$$H_{\alpha_1\ldots\alpha_{p-1}}\Big|_{\mathcal{B}_i} = 0 \tag{70}$$

Here the $\alpha_1, \ldots, \alpha_{p-1}$ indicate p-dimensional world-hypersurface tensorial indices on the sub-brane $\mathcal{B}_i$. Let us note that the objects $\widetilde{H}^{ab}$ and $B_{a_1\ldots a_{p-2}}$

already appeared in the Polyakov-type formulation of the standard (non-modified) Dp-brane [10]. The new feature in the present modified-measure Dp-brane model is the condition (70) on the dual field strength.

The same arguments from the previous paragraph applied to (67) imply that X^μ satisfy the standard Dp-brane equations of motion (in the Polyakov-type formulation [10]) :

$$\partial_a \left(\sqrt{-\zeta}\zeta^{(ab)}\partial_b X^\mu\right) + \sqrt{-\zeta}\partial_a X^\lambda \partial_b X^\nu \times \\ \times \Big[\zeta^{(ab)}\Gamma^\mu_{\lambda\nu}(\bar{G}) + \frac{1}{2}\zeta^{[ab]}G^{\mu\kappa}\left(\beta\partial_\kappa U B_{\lambda\nu} - \mathcal{H}_{\kappa\lambda\nu}(B)\right)\Big] = 0 \; . \tag{71}$$

together with Neumann boundary conditions on the world-hypersurfaces of each charged $(p-1)$-sub-brane $\mathcal{B}_i$:

$$\partial_\perp X^\mu \Bigg|_{\mathcal{B}_i} = 0 \quad , \quad \partial_\perp \equiv N^{(i)}_a \zeta^{(ab)}\partial_b \tag{72}$$

(recall that the symmetric part $\gamma^{ab} \equiv \zeta^{(ab)}$ plays the role of (inverse) world-volume Riemannian metric).

Let us consider again Eq.(58) and integrate it along arbitrary smooth closed curve Γ on the Dp-brane world-volume which is transversal to (some or all of) the $(p-1)$-sub-brane $\mathcal{B}_i$, we obtain the following constraints on the possible sub-brane configurations:

$$\sum_i e_i\, n_i(\Gamma) = 0 \; , \tag{73}$$

Here $n_i(\Gamma)$ is the sign-weighted total number of Γ crossing $\mathcal{B}_i$. Eq.(73) is the brane analog of the "color" charge confinement condition (first Eq.(38)) in the modified-measure string model. In the present $p \geq 2$-brane case, however, due to the much more complicated topologies of the pertinent world-volumes Eq.(73) may yield various different types of allowed sub-brane configurations.

As a simple illustration, let us consider the simplest non-trivial case $p = 2$ and take the static gauge for the $p = 1$ sub-branes (strings), *i.e.*, the proper times of the charged strings coincides with the proper time of the bulk membrane. The latter means that the fixed-time world-volume of the bulk *closed* membrane is a Riemann surface with some number g of handles and no holes. Further, we will assume the following simple topology of the attached N charged strings $\mathcal{B}_i$: upon cutting the membrane surface along these attached strings it splits into N *open* membranes $\mathcal{M}_i$

$(i = 1, \ldots, N)$ with Neumann boundary conditions (cf. (72)), each of which being a Riemann surface with g_i handles and 2 holes (boundaries) formed by the strings $\mathcal{B}_{i-1}$ and $\mathcal{B}_i$, respectively[3]. The brane tension of $\mathcal{M}_i$ is a dynamically generated constant T_i where $T_{i+1} = T_i + e_i$. In the present configuration Eq.(73) evidently reduces to the constraint $\sum_i e_i = 0$.

Thus, we conclude that similarly to the string case, modified-measure Dp-brane models describe configurations of charged $(p-1)$-branes with charge confinement. Apart from the latter, in general there exist more complicated configurations allowed by the constraint (73), whose properties deserve further study.

5 Conclusions

Finally, let us summarize the main features of the new class of modified-measure string and brane models. The above discussion shows that:

- There exist natural from physical point of view modifications of world-sheet and world-volume integration measures which may significantly affect string and brane dynamics.
- Consistency of dynamics *naturally* requires the introduction of auxiliary world-sheet vector gauge field (in the string case) and higher-rank world-volume antisymmetric tensor gauge fields in the general brane case beyond the standard Dp-brane $U(1)$ vector gauge field.
- The string/brane tension is *not* anymore a constant scale given *ad hoc*, but rather appears as an *additional dynamical degree of freedom* beyond the ordinary string/brane degrees of freedom.
- The dynamical string/brane tension has physical meaning of an electric field strength for the auxiliary world-sheet/world-volume gauge field.
- The dynamical string/brane tension obeys "Gauss law" constraint equation and may be nontrivially variable in the presence of point-like charges (on the string world-sheet) or charged lower-dimensional branes (on the Dp-brane world-volume).

[3]The Euler characteristics of the bulk membrane Riemann surface is $\chi = 2 - 2g$, whereas for the open brane $\mathcal{M}_i$ it is $\chi_i = 2 - 2g_i - 2$, so that $\chi = \sum_i \chi_i$ or, equivalently, $g = 1 + \sum_i g_i$.

- Modified-measure string/brane models provide simple classical mechanisms for confinement of point-like "color" charges or charged lower-dimensional branes due to variable dynamical tension.

Possible applications of the new class of modified-measure Dp-brane models in the context of modern brane-world theories are currently being studied. As a step in this direction we refer to the recent paper [12] where by employing the modified integration measure density (1) a new conformally invariant brane-world model without (bulk) cosmological constant fine tuning has been constructed.

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References

[1] Polchinski, J. (1996), *"TASI Lectures on D-Branes"* (hep-th/9611050).

[2] Ne'eman, Y. and Eizenberg, E. (1995), *"Membranes and Other Extendons"*, World Scientific.

[3] Green, M., Schwarz, J. and Witten, E. (1987), *"Superstring Theory"*, Vol.1,2, Cambridge Univ. Press;
Polchinksi, J. (1998), *"String Theory"*, Vol.1,2, Cambridge Univ. Press.

[4] Johnson, C. (2002), *"D-Branes"*, Cambridge Univ. Press, and references therein (see also hep-th/0007170).

[5] Guendelman, E.I. and Kaganovich, A. (1996), *Phys. Rev.* **D53** 7020;
Guendelman, E.I. and Kaganovich, A. (1999), *Phys. Rev.* **D60** 065004;
Guendelman, E.I.(1999), *Mod. Phys. Lett.* **A14** 1043;
Guendelman, E.I. (2000), *Class. Quant. Grav.* **17** 261;
Kaganovich, A. (2001), *Phys. Rev.* **D63** 025022;
Guendelman E.I. and Kaganovich, A. (2002), *Int. J. Mod. Phys.* **A17** 417 (*hep-th/0106152*);
Guendelman E.I. and Kaganovich, A. (2002), *Mod. Phys. Lett.* **A17** 1227 (hep-th/0110221).

[6] Guendelman, E.I. (2000), *Class. Quantum Grav.* **17** 3673 (hep-th/0005041);
Guendelman, E.I. (2001), *Phys. Rev.* **D63** 046006 (hep-th/0006079).

[7] Guendelman, E.I., Kaganovich, A., Nissimov, E. and Pacheva, S. (2002), *Phys. Rev.* **D66** 046003 (hep-th/0203024);
Guendelman, E.I., Kaganovich, A., Nissimov, E. and Pacheva, S. (2002), hep-th/0210062.

[8] Polyakov, A. (1981), *Phys. Lett.* **103B** 207,211.

[9] Katanaev, M. and Volovich, I. (1986), *Phys. Lett.* **B175** 413 (hep-th/0209014).

[10] Abou Zeid, M. and Hull, C. (1997), *Phys. Lett.* **404B** 264.

[11] Bergshoeff, E., London, L. and Townsend, P.K. (1992), *Class. Quantum Grav.* **9** 2545 (hep-th/9206026).

[12] Guendelman, E.I. (2003), hep-th/0303048