Chromogravity - An Effective Diff(4,R) Gauge for the IR region of QCD

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Abstract

Previous work on the IR regime approximation of QCD in which the dominant contribution comes from a dressed two-gluon effective metric-like field $G_{\mu\nu} = g_{ab}A^a_\mu A^b_\nu$ (g_{ab} a color $SU(3)$ metric) is reviewed. The QCD gauge is approximated by effective "chromodiffeomorphisms", i.e. by a gauge theory based on a pseudo-diffeomorphisms group. The second-quantized $G_{\mu\nu}$ field, together with the Lorentz generators close on the $\overline{SL}(4,R)$ algebra. This algebra represents a spectrum generating algebra for the set of hadron states of a given flavor - hadronic "manifields" transforming w.r.t. $\overline{SL}(4,R)$ (infinite-dimensional) unitary irreducible representations. The equations of motion for the effective pseudo-gravity are derived from a quadratic action describing Riemannian pseudo-gravity in the presence of shear ($\overline{SL}(4,R)$ covariant) hadronic matter currents. These equations yield p^{-4} propagators, i.e. a linearly rising confining potential $H(r) \sim r$, as well as linear $J \sim m^2$ Regge trajectories. The $\overline{SL}(4,R)$ symmetry based dynamical theory for the QCD IR region is successfully applied to hadron resonances. The pseudo-gravity potential reaches over to Nuclear Physics, where its $J^P = 2^+,\ 0^+$ quanta provide for the ground state excitations of the Arima-Iachello Interacting Boson Model.

1 Introduction

One of the main challenges in Particle Physics is the understanding and/or classification of quite a large number of presently known hadronic resonances. Here we are faced with an intriguing situation: In the "horizontal" direction one has flavor symmetries and rather powerful quantitative techniques with practically none understanding of the corresponding underlying fundamental interaction. As for the "vertical" direction (fixed flavor content), the basic interaction is given by the presently widely accepted Quantum Chromodynamics (QCD) theory, however the non-perturbative features of QCD have made it difficult to apply the theory exactly. Quite a number of approaches to deal with this region have been proposed so far with different degree of success. We believe that the merits of the approach described in this paper are both the fact that our starting point is QCD itself and that the predictions fit very well with experiment.

If the hadron lowest ground states are colorless (our assumption) and in the approximation of an external QCD potential, the hadron spectrum above these levels will be generated by color-singlet quanta, whether made of dressed two-gluon configurations, three-gluons, ... Every possible configuration will appear. No matter what the mechanism responsible for a given flavor state, the next vibrational, rotational or pulsed excitation corresponds to the "addition" of one such collective color-singlet multigluon quantum superposition. In the fully relativistic QCD theory, these contributions have to come from summations of appropriate Feynman diagrams, in which dressed n-gluon configurations are exchanged. We rearrange the sum by lumping together contributions from n-gluon irreducible parts, $n = 2, 3, ..., \infty$ and with the same Lorentz quantum numbers. The simplest such system will have the quantum numbers of di-gluon, i.e. $n = 2$. The color singlet external field can thus be constructed from the QCD gluon field as a sum (g_{ab} is the color-$SU(3)$ metric, d_{abc} are the totally symmetric $8 \otimes 8 \otimes 8 \to 1$ coefficients)

$$g_{ab} A^a_\mu A^b_\nu \oplus d_{abc} A^a_\mu A^b_\nu A^c_\sigma \oplus \cdots . \tag{1}$$

In the above, A^a_μ is the dressed gluon field.

2 Chromometric $G_{\mu\nu}$

We suggest that the main feature of hadron excitations is due to a component of QCD representing the exchange of a two-gluon effective gravity-like "chromo-metric" field ($A^a_\mu(x)$ the properly normalized gluon) [1]:

$$G_{\mu\nu}(x) = g_{ab} A^a_\mu A^b_\nu. \tag{2}$$

It will be useful for the applications to separate the "flat connection" N^a_μ, i.e. the zero-mode of the field. Writing for the curvature or field strength

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - i f^a{}_{bc} A^b_\mu A^c_\nu, \tag{3}$$

we define

$$A^a_\mu = N^a_\mu + B^a_\mu, \quad \partial_\mu N^a_\nu - \partial_\nu N^a_\mu = i f^a_{bc} N^b_\mu N^c_\nu, \tag{4}$$

where N^a_μ is the constant component, yielding a vanishing field strength.

Such a vacuum solution might be of the instanton type, for instance. Consider, e.g. the first nontrivial class, with Pontryagin index $n = 0$. Expand around this classical configuration, working, as always for instantons, in a Euclidean metric (i.e. a tunneling solution in Minkowski spacetime). At large distances the instanton field is required to approach a constant value

$$g_{ab}\ N^a_\mu\ \partial_\nu \epsilon^b = \partial_\nu (g_{ab}\ N^a_\mu\ \epsilon^b) \tag{5}$$

with the $B^a_\mu(x)$ field representing a fluctuation around the constant value, vanishing at large distances. One can construct the constant vacuum solution by mapping $SU(3) \to S^4$, namely directly onto the complete Euclidean manifold, compactified by the addition of a point at infinity.

$G_{\mu\nu}$ acts as a "pseudo-metric" field, (passively) gauging effective "pseudo-diffeomorphisms", just as is done by the physical Einstein metric field for the "true" diffeomorphisms of the covariance group.

The variation of the chromo-metric under color-$SU(3)$, due to

$$\delta_\epsilon A^a_\mu = \partial_\mu \epsilon^a + A^b_\mu (\lambda_b)^a_c \epsilon^c, \tag{6}$$

(we use the adjoint representation $\{\lambda_b\}^a_c = -if^a_{b\ c} = if^a_{bc}$) reads

$$\begin{aligned}
\delta_\epsilon G_{\mu\nu} &= \delta_\epsilon \{ g_{ab} (N^a_\mu + B^a_\mu)(N^b_\nu + B^b_\nu) \} \\
&= g_{ab} (\partial_\mu \epsilon^a N^b_\nu + N^a_\mu \partial_\nu \epsilon^b + \partial_\mu \epsilon^a B^b_\nu + B^a_\mu \partial_\nu \epsilon^b) \\
&\quad + i g_{ab} \left\{ f^a_{cd} A^c_\mu \epsilon^d A^b_\nu + f^b_{cd} A^a_\mu A^c_\nu \epsilon^d \right\}.
\end{aligned}$$

The last bracket vanishes, since it represents the homogeneous $SU(3)$ transformation of the $SU(3)$ scalar expression, i.e.

$$i f_{bcd}\ (A^b_\mu\ A^c_\nu + A^c_\mu\ A^b_\nu)\ \epsilon^d$$

(or, more technically, due to the total antisymmetry of f_{abc} in a compact group).

We note that at the IR region distances, any Gauss theorem field-fluxes will only involve the N^a_μ constant component, whereas the $B^a_\mu(x)$ "fluctuation" will not contribute. As a result, when integrating by parts the terms in $B^a_\mu,\ B^b_\nu$ we get

$$g_{ab}\ (\epsilon^a\ \partial_\mu B^b_\nu + \partial_\nu B^a_\mu\ \epsilon^b)$$

an expression whose Fourier transform vanishes for $k \to 0$, i.e. in the infrared sector. A generalized definition of this "IR limit" will be addressed below.

The terms involving the constant N_μ^a, N_ν^b can be rewritten in terms of effective pseudo-diffeomorphisms, defined by

$$\xi_\mu \equiv g_{ab}\epsilon^a N_\mu^b, \quad \delta_\epsilon G_{\mu\nu} = \partial_\mu \xi_\nu + \partial_\nu \xi_\mu. \tag{7}$$

Thus, the local $SU(3)$ color gauge variations contain a subsystem ensuring that the $G_{\mu\nu}$ di-gluon indeed act as a "pseudo-metric" field, precisely emulating gravity.

The definition of our "IR limit" based on the vanishing of the 4-momenta of the 'fluctuating fields' B_μ^a – after an integration by parts in which only the constant fields N_μ^a contribute to the surface terms – will be extended so as to include similar terms with vanishing momenta in all many-gluon zero-color exchanges. This can be taken as an operational definition, sufficient for our general purpose. To gain some additional insight, however, we remind the reader that such an IR approximation of QCD can also be thought of as the first step, the "zeroth approximation", of a *strong coupling* regime – in terms of a "small parameter" representing the number of "hard", or nonsoft, virtual quanta held in the evaluation of any physical quantity. We can write a generic IR state, carrying 4-momentum k, as follows:

$$|\phi_{IR},\ k\rangle = \sum_{m=1}^{\infty} f_m(k_1, k_2, \ldots, k_m)\delta_{k,k_1+k_2+\cdots+k_m}|k_1 k_2 \ldots k_m\rangle \tag{8}$$

where $|k_1 k_2 \ldots k_m\rangle$ represents a state of m soft gluons ($k_i \approx 0,\ i = 1, 2, \ldots m$). Integrating by parts (with surface terms again appearing only for the constant parts), the matrix elements of the terms in B_μ^a, B_ν^b become in this IR approximation

$$\langle \phi'_{IR}, k'|g_{ab}(\epsilon^a \partial_\mu B_\nu^b + \partial_\nu B_\mu^a \epsilon^b)|\phi_{IR},\ k\rangle,$$

an expression that is proportional to the soft 1-gluon momentum, and that vanishes for $k \to 0$, i.e., in the infrared sector. As a result, when changing over to the ξ_μ variable of and reidentifying δ_ξ as a variation under a formal R^4 diffeomorphism, we get $\delta_\epsilon G_{\mu\nu} = \partial_\mu \xi_\nu + \partial_\nu \xi_\mu$. For the sake of completeness, we note that in general one has to consider expressions of the following form

$$\langle \phi'_{IR}, k' | O(B^a_\mu, \partial_\nu B^a_\mu) \delta_\epsilon G_{\mu\nu} | \phi_{IR},\ k \rangle.$$

We evaluate such expressions, in this IR approximation, by inserting a complete set of states, and retaining only the *soft* virtual quanta. It is explained in Ref. [2] that by making use of the Fradkin representation [3] for relevant Green's functions one has a continuous family of "soft", or IR approximations, which maintain gauge invariance. Thus, one finds a consistent gauge-invariant (strong coupling) IR approximation with dressed gluon propagators which incorporate the iteration of all relevant quark bubbles, each carrying all possible internal, soft-gluon lines.

The consistency of this IR approximation requires one to consider only those QCD variations that connect IR gluon configurations *mutually.* Let us consider the expression for the $B = A - N$ variation, i.e. $\delta_\epsilon A^a_\mu = \partial_\mu \epsilon^a + i f^a_{\ bc} A^b_\mu \epsilon^c$ The left hand side of this expression is a difference between two soft gluons, implying that the IR matrix elements of its partial derivative are soft. Thus, we find the following "IR constraint" on the QCD gauge parameters:

$$\langle \phi'_{IR}, k' | \partial_\rho \partial_\mu \epsilon^a + i f^a_{\ bc} B^b_\mu \partial_\rho \epsilon^c | \phi_{IR},\ k \rangle \approx 0. \tag{9}$$

3 $Diff(4,R)$ Structure – n-gluon fields

Let us now consider the multi-gluon colorless configurations [4]. The color-singlet n-gluon field operator has the following form

$$G^{(n)}_{\mu_1 \mu_2 \cdots \mu_n} = d^{(n)}_{a_1 a_2 \cdots a_n} A^{a_1}_{\mu_1} A^{a_2}_{\mu_2} \cdots A^{a_n}_{\mu_n} \tag{10}$$

where

$$
\begin{aligned}
d^{(2)}_{a_1a_2} &= g_{a_1a_2},\\
d^{(3)}_{a_1a_2a_3} &= d_{a_1a_2a_3},\\
d^{(n)}_{a_1a_2\cdots a_n} &= d_{a_1a_2b_1}g^{b_1c_1}d_{c_1b_2a_3}\cdots\\
&\quad\times g^{b_{n-4}c_{n-4}}d_{c_{n-4}b_{n-3}a_{n-2}}g^{b_{n-3}c_{n-3}}d_{c_{n-3}a_{n-1}a_n}, \quad n>3,
\end{aligned}
$$

A^a_μ is the dressed gluon field, $g_{a_1a_2}$ is the $SU(3)$ Cartan metric, and $d_{a_1a_2a_3}$ is the $SU(3)$ totally symmetric $8\times8\times8\to1$ tensor.

But taking Fourier transforms – i.e. the matrix elements for these gluon fluctuations – we find that *these terms are precisely those that vanish in our definition of an IR region.* The terms involving the constant connections $N^{a_i}_{\mu_i}$, $i=1,2,\ldots n$ can be rewritten in terms of effective pseudo-diffeomorphisms,

The QCD variation, in the IR region can be rewritten in terms of effective pseudo-diffeomorphisms,

$$\delta_\epsilon G^{(n)}_{\mu_1\mu_2\cdots\mu_n} = \partial_{\{\mu_1}\xi^{(n-1)}_{\mu_2\mu_3\cdots\mu_n\}} \equiv \delta_\xi G^{(n)}_{\mu_1\mu_2\cdots\mu_n}, \tag{11}$$

where $\{\mu_1\mu_2\cdots\mu_n\}$ denotes symmetrization of indices,

$$\xi^{(n-1)}_{\mu_1\mu_2\cdots\mu_{n-1}} \equiv d^{(n)}_{a_1a_2\cdots a_n}N^{a_1}_{\mu_1}N^{a_2}_{\mu_2}\cdots N^{a_{n-1}}_{\mu_{n-1}}\epsilon^{a_n} \tag{12}$$

while $N^{a_i}_{\mu_i}$, $i=1,2,\ldots n$, being the constant connections.

A subsequent application of two $SU(3)$-induced variations, i.e. the commutator of two such chromo-diffeomorphic variations

$$[\delta_{\epsilon_1},\ \delta_{\epsilon_2}]G^{(n)}_{\mu_1\mu_2\cdots\mu_n} = \delta_{\epsilon_3}G^{(n)}_{\mu_1\mu_2\cdots\mu_n}, \tag{13}$$

i.e.

$$[\delta_{\xi_1},\ \delta_{\xi_2}]G^{(n)}_{\mu_1\mu_2\cdots\mu_n} = \delta_{\xi_3}G^{(n)}_{\mu_1\mu_2\cdots\mu_n}, \tag{14}$$

where

$$\xi_{3\mu} = (\partial_\nu\xi_{1\mu})\ \xi^\nu_2 + (\partial_\mu\xi_{1\nu})\ \xi^\nu_2 - (\partial_\nu\xi_{2\mu})\ \xi^\nu_1 - (\partial_\mu\xi_{2\nu})\ \xi^\nu_1 \tag{15}$$

indeed closes on the covariance group's commutation relations. Thus, one has an infinitesimal nonlinear realization of the $Diff(4,R)$ group in the space of fields $\left\{G^{(n)}_{\mu_1\mu_2\cdots\mu_n}\;\middle|\; n=2,3,\dots\right\}$.

4 $Diff(4,R)$ Structure – $L^{(m)}$ Operators Algebra

Let us consider an ∞-dimensional vector space over the field operators $\left\{G^{(n)}\;\middle|\; n=2,3,\dots\right\}$, i.e.,

$$V(G^{(2)},G^{(3)},\dots)\;=\;V(G^{(2)}_{\mu_1\mu_2},\;G^{(3)}_{\mu_1\mu_2\mu_3},\dots). \tag{16}$$

We can now define an infinite set of field-dependent operators $\left\{L^{(m)}\;\middle|\; m=0,1,2,\dots\right\}$ as follows

$$\begin{aligned}
L^{(0)\rho}_{\nu_1} &= d^{(2)}_{a_1a_2}A^{a_1}_{\nu_1}\frac{\delta}{\delta(g_{a_2b}A^b_\rho)} \;\equiv\; g_{a_1a_2}A^{a_1}_{\nu_1}\frac{\delta}{\delta(g_{a_2b}A^b_\rho)},\\
L^{(1)\rho}_{\nu_1\nu_2} &= d^{(3)}_{a_1a_2a_3}A^{a_1}_{\nu_1}A^{a_2}_{\nu_2}\frac{\delta}{\delta(g_{a_3b}A^b_\rho)} \;\equiv\; d_{a_1a_2a_3}A^{a_1}_{\nu_1}A^{a_2}_{\nu_2}\frac{\delta}{\delta(g_{a_3b}A^b_\rho)},\\
&\cdots\cdots\\
L^{(m)\rho}_{\nu_1\nu_2\cdots\nu_{m+1}} &= d^{(m+2)}_{a_1a_2\cdots a_{m+2}}A^{a_1}_{\nu_1}A^{a_2}_{\nu_2}\cdots A^{a_{m+1}}_{\nu_{m+1}}\frac{\delta}{\delta(g_{a_{m+2}b}A^b_\rho)}.\\
&\cdots\cdots.
\end{aligned}$$

In the general case, $L^{(m)\rho}_{\nu_1\nu_2\cdots\nu_{m+1}}$, $m=0,1,2,\dots$ action on the field operators
$\left\{G^{(n)}\;\middle|\; n=2,3,\dots\right\}$ reads

$$
\begin{aligned}
L^{(m)\rho}_{\nu_1\nu_2\cdots\nu_{m+1}} G^{(2)}_{\mu_1\mu_2} &= \delta^\rho_{\mu_1} G^{(2+m)}_{\nu_1\nu_2\cdots\nu_{m+1}\mu_2} + \delta^\rho_{\mu_2} G^{(2+m)}_{\mu_1\nu_1\nu_2\cdots\nu_{m+1}}, \\
L^{(m)\rho}_{\nu_1\nu_2\cdots\nu_{m+1}} G^{(3)}_{\mu_1\mu_2\mu_3} &= \delta^\rho_{\mu_1} G^{(3+m)}_{\nu_1\nu_2\cdots\nu_{m+1}\mu_2\mu_3} + \delta^\rho_{\mu_2} G^{(3+m)}_{\mu_1\nu_1\nu_2\cdots\nu_{m+1}\mu_3} \\
&\quad + \delta^\rho_{\mu_3} G^{(3+m)}_{\mu_1\mu_2\nu_1\nu_2\cdots\nu_{m+1}}, \\
&\cdots\cdots \\
L^{(m)\rho}_{\nu_1\nu_2\cdots\nu_{m+1}} G^{(n)}_{\mu_1\mu_2\cdots\mu_n} &= \delta^\rho_{\mu_1} G^{(n+m)}_{\nu_1\nu_2\cdots\nu_{m+1}\mu_2\cdots\mu_n} + \delta^\rho_{\mu_2} G^{(n+m)}_{\mu_1\nu_1\nu_2\cdots\nu_{m+1}\mu_3\cdots\mu_n} \\
&\quad + \cdots + \delta^\rho_{\mu_n} G^{(n+m)}_{\mu_1\mu_2\cdots\mu_{n-1}\nu_1\nu_2\cdots\nu_{m+1}}, \\
&\cdots\cdots .
\end{aligned}
$$

Let us now consider the algebraic structure defined by the $\left\{L^{(m)} \;\middle|\; m = 0,1,2,\dots\right\}$ operators Lie brackets. For the $L^{(0)}$ operators themselves we find

$$[L^{(0)}, L^{(0)}] \subset L^{(0)}, \tag{17}$$

i.e.

$$[L^{(0)\rho_1}_{\nu_1}, L^{(0)\rho_2}_{\sigma_1}] = \delta^{\rho_1}_{\sigma_1} L^{(0)\rho_2}_{\nu_1} - \delta^{\rho_2}_{\nu_1} L^{(0)\rho_1}_{\sigma_1}, \tag{18}$$

In the most general case, for the brackets of $L^{(l)}$ and $L^{(m)}$ we find

$$[L^{(l)}, L^{(m)}] \subset L^{(l+m)}, \tag{19}$$

and more specifically,

$$
\begin{aligned}
[L^{(l)\rho_1}_{\nu_1\nu_2\cdots\nu_{l+1}}, L^{(m)\rho_2}_{\sigma_1\sigma_2\cdots\sigma_{m+1}}] = & \sum_{i=1}^{m+1} \delta^{\rho_1}_{\sigma_i} L^{(l+m)\rho_2}_{\sigma_1\sigma_2\cdots\sigma_{i-1}\nu_1\nu_2\cdots\nu_{l+1}\sigma_{i+1}\cdots\sigma_{m+1}} \\
& - \sum_{j=1}^{l+1} \delta^{\rho_2}_{\nu_j} L^{(l+m)\rho_1}_{\nu_1\nu_2\cdots\nu_{j-1}\sigma_1\sigma_2\cdots\sigma_{m+1}\nu_{j+1}\cdots\nu_{l+1}}.
\end{aligned}
$$

We have constructed an ∞-component vector space, $V = V(G^{(2)}_{\mu_1\mu_2},\ G^{(3)}_{\mu_1\mu_2\mu_3}, \dots)$, over the n-gluon field operators, as well as the corresponding algebra of homogeneous diffeomorphisms,

$$diff_0(4,R) \;=\; \left\{L^{(m)\rho}_{\nu_1\nu_2\cdots\nu_{m+1}} \middle| m = 0,1,2,\dots\right\}; \tag{20}$$

the vector space V is invariant under the action of the $diff_0(4,R)$ algebra.

Let us point out that there exists a subalgebra of the entire algebra when m-values are even, i.e. one has the following structure

$$[L^{(even)}, L^{(even)}] \subset L^{(even)},$$
$$[L^{(even)}, L^{(odd)}] \subset L^{(odd)},$$
$$[L^{(odd)}, L^{(odd)}] \subset L^{(even)}.$$

Let us define the dilation-like operator (chromo-dilation) D as a trace of $L^{(0)\rho}_{\nu}$, i.e.,

$$D = L^{(0)\rho}_{\rho}. \tag{21}$$

This operator commutes with the $L^{(0)\rho}_{\nu}$ operators,

$$[D, L^{(0)\rho}_{\nu}] = 0, \tag{22}$$

and belongs to the center of the $gl(4,R)$ chromo-gravity subalgebra generated by the $L^{(0)\rho}_{\nu}$ operators. On account of the chromo-dilation operator one can make the following decomposition

$$gl(4,R) = r \oplus sl(4,R), \tag{23}$$

where D corresponds to the subalgebra r, while the basis of the $sl(4,R)$ subalgebra is given by

$$T^{(0)\rho}_{\nu} = L^{(0)\rho}_{\nu} - \tfrac{1}{4}\delta^{\rho}_{\nu} D. \tag{24}$$

The commutation relation of D with a generic $diff_0(4,R)$ operator $L^{(m)\rho}_{\nu_1\nu_2\cdots\nu_{m+1}}$ reads

$$[D, L^{(m)\rho}_{\nu_1\nu_2\cdots\nu_{m+1}}] = m L^{(m)\rho}_{\nu_1\nu_2\cdots\nu_{m+1}} \tag{25}$$

and thus, the chromo-dilation operator D provides us with a Z_+ grading. This grading justifies and/or explains the m-label used for the $L^{(m)\rho}_{\nu_1\nu_2\cdots\nu_{m+1}}$ operators.

The chromo-dilation operator D counts the number of single gluon fields in a multi-gluon configuration, as seen from the following commutation relation

$$[D, G^{(n)}_{\mu_1\mu_2\cdots\mu_n}] = nG^{(n)}_{\mu_1\mu_2\cdots\mu_n}. \tag{26}$$

Clearly, the $J = 1$ Yang-Mills gauge of QCD contains (in the IR limit) local diffeomorphisms, gauged à la Einstein. As a matter of fact, this is not surprising, as the truncated massless sector of the open string reduces to a $J = 1$ Yang-Mills field theory and that the same truncation for the closed string reduces to a $J = 2$ gravitational field theory; but the closed string is nothing but the contraction of two open strings!

5 Chromogravity Matter Fields

The effective QCD interaction fields $G_{\mu\nu}$ couple to the hadron systems themselves, and thus, in order to complete the Chromogravity approximation of the QCD IR region, we have to address the question of the effective hadron fields as well. It is well known that the constructions of hadrons, i.e. the composite objects made of quarks and gluons, is due to the strong coupling regime one of the most challenging issues in QCD. Thus, in order to define the effective hadron fields of Chromogravity we rely as much as possible on the symmetry considerations.

The $G_{\mu\nu}$ fields, that transform w.r.t. the second-rank symmetric representation of the $Diff(4, R)$ group, are naturally coupled to the bosonic and fermionic hadron fields that transform themselves w.r.t. representations of the $Diff(4, R)$ group as well [5].

The construction of the fermionic fields requires the study of the quantum-mechanical $Diff(4, R)$ group, i.e. $Diff(4, R)_{QM}$. Note that the topological properties of the $Diff(4, R)$ group that determine nontrivial minimal group-extensions of the $Diff(4, R)$ group by the $U(1)$ group of the quantum-mechanical Hilbert space phase factors

$$1 \to U(1) \to Diff(4,R)_{QM} \to Diff(4,R) \to 1 \tag{27}$$

are given by the corresponding properties of the group chain:

$$Diff(4,R) \supset GL(4,R) \supset SL(4,R) \supset SO(3,1) \supset SO(3). \tag{28}$$

It is well known that, in contradistinction to $SO(3,1)$ and $SO(3)$ cases, the $SL(4,R)$ group cannot be embedded into any group of finite complex matrices, and that the universal covering of the $SL(4,R)$ group, i.e. $\overline{SL}(4,R)$, is a group of infinite matrices – likewise for the $\overline{Diff}(4,R)$. The universal covering is actually given by the double covering, and the corresponding relations among relevant symmetry groups are as presented in the following diagram:

$$\begin{array}{ccccccccc}
1 & \to & Z_2 & \to & \overline{Diff}(4,R) & \to & Diff(4,R) & \to & 1 \\
 & & & & \cup & & \cup & & \\
1 & \to & Z_2 & \to & \overline{SL}(4,R) & \to & SL(4,R) & \to & 1 \\
 & & & & \cup & & \cup & & \\
1 & \to & Z_2 & \to & \overline{SO}(3,1) & \to & SO(3,1) & \to & 1 \\
 & & & & \cup & & \cup & & \\
1 & \to & Z_2 & \to & \overline{SO}(3) & \to & SO(3) & \to & 1
\end{array}$$

An immediate consequence is that there are no finite-dimensional spinorial representations of the $\overline{SL}(4,R)$, i.e. $\overline{Diff}(4,R)$ group – *all unitary and non-unitary spinorial representations of these groups are necessarily infinite-dimensional*. In practice, the $\overline{SL}(4,R)$ representations are constructed by making use of the "standard" linear representations techniques, while the $\overline{Diff}(4,R)$ representations are induced from these $\overline{SL}(4,R)$ representations. This fact fits very well with our Chromogravity picture of hadrons, where the entire set of presumably infinitely many excitations of given flavor are to be described by a single infinite-component effective field – "manifield".

In order to set up all basic quantum mechanical objects, that are necessary for particle physics applications, we have to consider:

(i) The fermionic and bosonic (infinite-component) representations of the $\overline{SL}(4,R)$ group that characterize respectively the baryonic and mesonic quantum manifields,

(ii) The fermionic and bosonic (infinite-component) representations of the inhomogenious $SA(4,R) = T_4 \wedge \overline{SL}(4,R)$ group (affine generalization of the Poincaré group) that characterize the quantum states of the manifields quanta,

(iii) The wave equation type relations that insure consistency between manifields and the corresponding quantum states, and

(iv) The physical requirements that are primarily related to the unitarity properties of various observable facts.

The affine group $\overline{SA}(4,R) = T_4 \wedge \overline{SL}(4,R)$, is a semidirect product of translations generated by $P_\mu,\ \mu = 0,1,2,3$ and $\overline{SL}(4,R)$ generated by $Q_{\mu\nu}$ $(\mu,\nu = 0,1,2,3)$. The antisymmetric operators $M_{\mu\nu} = \frac{1}{2}(Q_{\mu\nu} - Q_{\nu\mu})$ generate the Lorentz subgroup $\overline{SO}(3,1)$, while the symmetric traceless operators (shears) $T_{\mu\nu} = \frac{1}{2}(Q_{\mu\nu} + Q_{\mu\nu}) - \frac{1}{4}\eta_{\mu\nu}Q_\sigma{}^\sigma$ generate the proper 4-volume-preserving deformations.

As in the Poincaré case, the $\overline{SA}(4,R)$ unitary irreducible representations are induced from the representations of the corresponding little group $T_3' \wedge \overline{SL}(3,R),\ m \neq 0$. In the physically most interesting case T_3' is represented trivially. The corresponding particle states have to be described by the unitary representations of the remaining part of the little group, i.e. $\overline{SL}(3,R)$. All these representations, both spinorial and tensorial, are infinite-dimensional owing to the $\overline{SL}(3,R)$ noncompactness. Therefore, the corresponding $\overline{SL}(4,R)$ matter fields $\Psi(x),\ \Phi(x)$ are necessarily infinite-dimensional and when reduced with respect to the $\overline{SL}(3,R)$ subgroup should transform with respect to its unitary irreducible quantum-states representations .

If the whole $\overline{SL}(4,R)$ group would be represented unitarily, the Lorentz boost generators intrinsic part would be hermitian and as a result, when boosting a particle, one would obtain a particle with a different spin, i.e. another particle - contrary to experience. There exists however a remarkable inner *deunitarizing* automorphism $\mathcal{A}$ [8] of the $SL(4,R)$ group, which leaves its $R_+ \otimes \overline{SL}(3,R)$ subgroup intact, and which maps the T_{0k}, M_{0k} generators into iM_{0k}, iT_{0k} respectively ($k = 1,2,3$). In other words it exchanges the $\overline{SO}(4)$ and $\overline{SO}(3,1)$ subgroups of the $\overline{SL}(4,R)$ group mutually. The deunitarizing automorphism allows us to start with the unitary (irreducible) representations of the $\overline{SL}(4,R)$ group, and upon its application, to identify the finite (unitary) representations of the "abstract" $\overline{SO}(4)$ compact subgroup with nonunitary representations of the physical Lorentz group – $\overline{SO}(3,1) = \overline{SO}(4)^{\mathcal{A}}$. In this way, we avoid a disease common to most of infinite-component wave equations, in particular those based on groups containing the $\overline{SL}(4,R)$ group.

6 Hadron Spectroscopy

The catalogue of the $\overline{SL}(4,R)$ multiplicity-free unitary irreducible representations is presented in [8] , and by making use of the deunitarizing automorphism $\mathcal{A}$, we arrive at the infinite-dimensional $\overline{SL}(4,R)$ representations for which the Lorentz subgroup is represented nonunitarily. Moreover, for the relevant cases the Lorentz-covariant (flat-space) infinite-component wave equations which determine the physical (propagating) degrees of freedom are given in [6], and thus we can proceed with the actual applications to hadron classification.

In the case of mesons there are two $\overline{SL}(4,R)$ representations at our disposal: $D^{ladd}_{SL(4,R)}(0)^{\mathcal{A}}$ and $D^{ladd}_{SL(4,R)}(\frac{1}{2})^{\mathcal{A}}$. Having in mind the quark model, it is most natural to classify the $\bar{q}q$ meson states according to the $D^{ladd}_{SL(4,R)}(\frac{1}{2})^{\mathcal{A}}$ representation, i.e., to have as the lowest level the $J = 0, 1$ (${}^1S_0$ and ${}^3S_1$) states. The $D^{ladd}_{SL(4,R)}(0)^{\mathcal{A}}$ representation would be an appropriate choice for the possible glueballs. In the case of baryons, for the flavor $SU(3)$ octet states we have a unique choice of the system based on the $[D^{disc}_{SL(4,R)}(\frac{1}{2},0) \oplus D^{disc}_{SL(4,R)}(0,\frac{1}{2})]^{\mathcal{A}}$ system, while for the decuplet states we have to make use of the symmetrized product of this reducible representation and the finite-dimensional $\overline{SL}(4,R)$ representation $(\frac{1}{2},\frac{1}{2})$ (generalizing the Rarita - Schwinger approach). The $\overline{SL}(4,R)$ generators have definite space-time properties, and in particular a constrained behavior under the parity operation: The $J_i = \epsilon_{ijk}M_{jk}$, T_{ij}, and T_{00} operators are parity even, while the $K_i = M_{0i}$ and $N_i = T_{0i}$ are parity odd. All states of the same $\overline{SL}(3,R)$ subgroup unitary irreducible representation (Regge trajectory) thus have the same parity; the states of an $SL(2,C) \simeq \overline{SO}(3,1)$ or an $\overline{SO}(4)$ subgroup representation have alternating parities. For a given $SL(2,C) = \overline{SO}(4)^{\mathcal{A}}$ representation (j_1, j_2), the total (spin) angular momentum is

$$J = J^{(1)} + J^{(2)}, \tag{29}$$

while the boost operator is given by

$$K = J^{(1)} - J^{(2)}. \tag{30}$$

We find the following J^P content of a (j_1, j_2) $\overline{SO}(4)^{\mathcal{A}}$ representation:

$$J^P = (j_1 + j_2)^P,\ (j_1 + j_2 - 1)^{-P},\ (j_1 + j_2 - 2)^P,\ \cdots,\ (|j_1 - j_2|)^{\pm P}. \tag{31}$$

Thus, by assigning a given parity to any state of an $\overline{SL}(4,R)$ representation, say the lowest state, the parities of all other states are determined.

The $\overline{SL}(3,R)$ subgroup unitary irreducible representations [7] determine the Regge trajectory states of a given $\overline{SL}(4,R)^{\mathcal{A}}$ representation. In decomposing an $\overline{SL}(4,R)^{\mathcal{A}}$ representation with respect to the $\overline{SL}(3,R)$ unitary irreducible representations, it is convenient to use an integer quantum number n that is in one-to-one correspondence with the T_{00} operator eigen values.

The $\overline{SL}(4,R)$ ladder unitary irreducible representations contain an infinite sum of $\overline{SL}(3,R)$ ladder unitary irreducible representations, i.e.,

$$D^{ladd}_{SL(4,R)}(0;e_2) \rightarrow \sum_{n\ even}{}^{\oplus} D^{ladd}_{SL(3,R)}(0;\sigma_2) \oplus \sum_{n\ odd}{}^{\oplus} D^{ladd}_{SL(3,R)}(1;\sigma_2),$$

$$D^{ladd}_{SL(4,R)}(\frac{1}{2};e_2) \rightarrow \sum_{n\ even}{}^{\oplus} D^{ladd}_{SL(3,R)}(1;\sigma_2) \oplus \sum_{n\ odd}{}^{\oplus} D^{ladd}_{SL(3,R)}(0;\sigma_2).$$

An analysis shows that the reduction of the $D^{disc}_{SL(4,R)}(\frac{1}{2},0)$ and $D^{disc}_{SL(4,R)}(0,\frac{1}{2})$ representations with respect to the unitary irreducible representations of $\overline{SL}(3,R)$ is given by the symbolic expression

$$D^{disc}_{SL(4,R)}(\frac{1}{2},0) \oplus D^{disc}_{SL(4,R)}(0,\frac{1}{2}) \rightarrow \sum_{n\ even}{}^{\oplus} D^{ladd}_{SL(3,R)}(\frac{1}{2}) \oplus \sum_{n\ odd}{}^{\oplus} D^{disc}_{SL(3,R)}(\frac{3}{2};\sigma_2);$$

each $\overline{SL}(3,R)$ unitary irreducible representation appears infinitely many times.

We find it necessary, from a comparison with the experimental situation, to use parity doubling, the actual spectrum displaying approximate exchange-degeneracy features. The parity of states within an $\overline{SL}(4,R)^{\mathcal{A}}$ representation is determined by the parity of the lowest - J state.

Thus, *we assign all hadron states of a given flavor to the wave-equation-projected states corresponding to parity-doubled* $\overline{SL}(4,R)^{\mathcal{A}}$ *irreducible representations* (their lowest - J states have opposite parities) [9].

TABLE I Assignment of N, Λ and Σ $SU(3)$ octet states

$D(0,\frac{1}{2})$					$D(0,\frac{1}{2})$				
(j_1,j_2)	J^P	$\{N\}$	$\{\Lambda\}$	$\{\Sigma\}$	(j_1,j_2)	J^P	$\{N\}$	$\{\Lambda\}$	$\{\Sigma\}$
$(\frac{1}{2},0)$	$\frac{1}{2}^+$	$N(940)$	$\Lambda(1116)$	$\Sigma(1193)$	$(0,\frac{1}{2})$	$\frac{1}{2}^-$	$N(1535)$	$\Lambda(1670)$	$\underline{\Sigma}(\sim 1500)$
$(\frac{3}{2},1)$	$\frac{1}{2}^+$	$N(1440)$	$\Lambda(1600)$	$\Sigma(1660)$	$(1,\frac{3}{2})$	$\frac{1}{2}^-$	$N(1650)$	$\Lambda(1800)$	$\Sigma(1620)$
	$\frac{3}{2}^-$	$N(1520)$	$\Lambda(1690)$	$\Sigma(1670)$		$\frac{3}{2}^+$	$\underline{N}(1540)$	$\Lambda(1890)$	$\Sigma(1670)$
	$\frac{5}{2}^+$	$N(1680)$	$\Lambda(1820)$	$\Sigma(1915)$		$\frac{5}{2}^-$	$N(1675)$	$\Lambda(1830)$	$\Sigma(1775)$
$(\frac{5}{2},2)$	$\frac{1}{2}^+$	$N(1710)$	$\Lambda(1800)$	$\Sigma(1880)$	$(2,\frac{5}{2})$	$\frac{1}{2}^-$	$N(2090)$		$\Sigma(1750)$
	$\frac{3}{2}^-$	$N(1700)$	$\Lambda(2000)$			$\frac{3}{2}^+$	$N(1720)$		$\Sigma(1840)$
	$\frac{5}{2}^+$	$N(2000)$	$\Lambda(2110)$			$\frac{5}{2}^-$	$N(2200)$		
	$\frac{7}{2}^-$	$N(2190)$				$\frac{7}{2}^+$	$N(1990)$	$\underline{\Lambda}(2020)$	
	$\frac{9}{2}^+$	$N(2220)$	$\Lambda(2350)$	$\Sigma(2455)$		$\frac{9}{2}^-$	$N(2250)$		
$(\frac{7}{2},3)$	$\frac{1}{2}^+$	$\underline{N}(2100)$		$\Sigma(2250)$	$(3,\frac{7}{2})$	$\frac{1}{2}^-$			
	$\frac{3}{2}^-$	$N(2080)$	$\Lambda(2325)$			$\frac{3}{2}^+$			
	$\frac{5}{2}^+$					$\frac{5}{2}^-$			
	$\frac{7}{2}^-$					$\frac{7}{2}^+$			
	$\frac{9}{2}^+$					$\frac{9}{2}^-$			
	$\frac{11}{2}^-$	$N(2600)$				$\frac{11}{2}^+$			
	$\frac{13}{2}^+$	$N(2700)$				$\frac{13}{2}^-$			

Mesons $(q\bar{q})$: $D^{ladd}_{SL(4,R)}(\frac{1}{2};e_2)^{\mathcal{A}},\ \Phi,$

$$\{(j_1,j_2)\} = \{(\frac{1}{2},\frac{1}{2}),\ (\frac{3}{2},\frac{3}{2}),\ (\frac{5}{2},\frac{5}{2}),\ \cdots\} \tag{32}$$

Baryons $(qqq)_{mixedsymmetry}$: $[D^{disc}_{SL(4,R)}(\frac{1}{2},0) \oplus D^{disc}_{SL(4,R)}(0,\frac{1}{2})]^{\mathcal{A}},\ \Psi,$

$$\{(j_1,j_2)\} = \{(\frac{1}{2},0),\ (\frac{3}{2},1),\ (\frac{5}{2},0),\ \cdots\}\oplus\{(0,\frac{1}{2}),\ (1,\frac{3}{2}),\ (2,\frac{5}{2}),\ \cdots\}. \tag{33}$$

Baryons $(qqq)_{symmetric}$: $\{[D^{disc}_{SL(4,R)}(\frac{1}{2},0)\oplus D^{disc}_{SL(4,R)}(0,\frac{1}{2})]^{\mathcal{A}}\otimes D^{(\frac{1}{2},\frac{1}{2})}\}_{sym},\ \Psi_\rho,$

$$\{(j_1,j_2)\} = \{(1,\frac{1}{2}),(2,\frac{3}{2}),(3,\frac{5}{2}),\cdots\} \oplus \{(\frac{1}{2},1),(\frac{3}{2},2),(\frac{5}{2},3),\cdots\}. \tag{34}$$

The $\overline{SO}(4)^{\mathcal{A}}$ states, when reorganized with respect to the $\overline{SL}(3,R)$ subgroup, form an infinite sum of Regge-type $\Delta J = 2$ recurrences with the J content

$$\{J\} = \{\frac{1}{2},\ \frac{5}{2},\ \frac{9}{2},\ \cdots\},\{J\} = \{\frac{3}{2},\ \frac{7}{2},\ \frac{11}{2},\ \cdots\}. \tag{35}$$

The former states belong to $D^{ladd}_{SL(3,R)}(\frac{1}{2})$, while the latter ones are projected out of $D^{ladd}_{SL(3,R)}(\frac{3}{2}, \sigma_2)$ by the field equations. Note that we have thus achieved the goal of a fully relativistic algebraic model in terms of the total angular momentum J.

TABLE II Assignment of Δ and Σ $SU(3)$ decuplet states.

$D(\frac{1}{2},0)_\mu$				$D(0,\frac{1}{2})_\mu$			
(j_1,j_2)	J^P	$\{\Delta\}$	$\{\Sigma\}$	(j_1,j_2)	J^P	$\{\Delta\}$	$\{\Sigma\}$
$(1,\frac{1}{2})$	$\frac{1}{2}^-$	$\Delta(1620)$		$(\frac{1}{2},1)$	$\frac{1}{2}^+$	$\underline{\Delta}(1550)$	$\Sigma(1770)$
	$\frac{3}{2}^+$	$\Delta(1232)$	$\Sigma(1385)$		$\frac{3}{2}^-$	$\Delta(1700)$	$\Sigma(1580)$
$(2,\frac{3}{2})$	$\frac{1}{2}^-$	$\Delta(1900)$	$\underline{\Sigma}(2000)$	$(\frac{3}{2},2)$	$\frac{1}{2}^+$	$\Delta(1910)$	
	$\frac{3}{2}^+$	$\Delta(1600)$	$\Sigma(1690)$		$\frac{3}{2}^-$	$\underline{\Delta}(1940)$	$\Sigma(1940)$
	$\frac{5}{2}^-$				$\frac{5}{2}^+$	$\Delta(1905)$	
	$\frac{7}{2}^+$	$\Delta(1950)$	$\Sigma(2030)$		$\frac{7}{2}^-$		
$(3,\frac{5}{2})$	$\frac{1}{2}^-$	$\underline{\Delta}(2150)$		$(\frac{5}{2},3)$	$\frac{1}{2}^+$		
	$\frac{3}{2}^+$	$\Delta(1920)$	$\Sigma(2080)$		$\frac{3}{2}^-$		
	$\frac{5}{2}^-$	$\Delta(1930)$			$\frac{5}{2}^+$	$\Delta(2000)$	$\underline{\Sigma}(2070)$
	$\frac{7}{2}^+$				$\frac{7}{2}^-$	$\underline{\Delta}(2200)$	$\underline{\Sigma}(2150)$
	$\frac{9}{2}^-$	$\Delta(2400)$			$\frac{9}{2}^+$	$\Delta(2300)$	
	$\frac{11}{2}^+$	$\Delta(2420)$	$\Sigma(2620)$		$\frac{11}{2}^-$		
$(4,\frac{7}{2})$	$\frac{1}{2}^-$			$(\frac{7}{2},4)$	$\frac{1}{2}^+$		
	$\frac{3}{2}^+$				$\frac{3}{2}^-$		
	$\frac{5}{2}^-$	$\underline{\Delta}(2350)$			$\frac{5}{2}^-$		
	$\frac{7}{2}^+$	$\Delta(2390)$			$\frac{7}{2}^-$		
	$\frac{9}{2}^-$				$\frac{9}{2}^+$		
	$\frac{11}{2}^+$				$\frac{11}{2}^-$		
	$\frac{13}{2}^-$	$\Delta(2750)$			$\frac{13}{2}^+$		
	$\frac{15}{2}^+$	$\Delta(2950)$			$\frac{15}{2}^-$		

As an example we present in the Table I the N, Λ, and Σ octet states $\{\mathbf{8}\}$, while the Δ and Σ decuplet states $\{\mathbf{10}\}$ are presented in Table II. The $SU(3)$ Σ assignment is not known completely. Note that the $J = \frac{1}{2}$ $\{\mathbf{10}\}$ states come from the $J = 0$ part of the $(\frac{1}{2}, \frac{1}{2})$ explicit index in Ψ_μ of (C9), while the other $\{\mathbf{10}\}$ states come from the $J = 1$ part - thus a discrepancy in mass.

We find a striking match between the (J^P, mass) values and the wave - equation - projected $\overline{SL}(4, R)^{\mathcal{A}}$ representation states. Moreover, a remarkably simple mass formula (straightforward generalization of the mass-spin - Regge relation) fits these infinite systems of hadronic states. For the $\{\mathbf{8}\}$ and the higher-spin $\{\mathbf{10}\}$ baryon resonances we write:

$$m^2 = m_0^2 + (\alpha'_f)^{-1}(j_1 + j_2 - \frac{1}{2} - \frac{1}{2}n), \tag{36}$$

where m_0 is the mass of the lowest-lying state, α'_f is the slope of the Regge trajectory for that flavor. The linear $J \simeq m^2$ relation is taken here phenomenologically. However, it will be demonstrated below that this relation can indeed be derived from Chromogravity dynamics with a natural choice of a Lagrangian.

7 $J \simeq m^2$ relation

In the absence of Chromogravity, the matter Lagrangian would be [10]

$$L_M = \overline{\Psi} i X^\mu \partial_\mu \Psi + \partial^\mu \Phi \partial_\mu \Phi, \tag{37}$$

invariant under global $\overline{SL}(4,R)$. The Hilbert spaces of Ψ and Φ are given by the representations of $\overline{SA}(4,R)$. Chromogravity enters through the replacement $\partial_\mu \rightarrow \hat{D}_A$, where the index "$A$" denotes a local frame: $\hat{D}_\mu = \partial_\mu - \Gamma^A{}_{B\mu} Q_A{}^B$, $\hat{D}_A = e_A{}^\mu \hat{D}_\mu$ with Γ the connection and $e_A{}^\mu \cdot e_\mu{}^B = \delta_A{}^B$, e the chromogravity tetrad; $Q_A{}^B$ is the $sl(4,R)$ algebraic generator in the tangent frame. We use $\hat{D}$ for the full covariant derivative with $sl(4,R)$ connection. $e^A{}_\mu(x)$ and $\Gamma^A{}_{B\mu}(x)$ can be taken as gauge fields for $\overline{SA}(4,R)$:

$$\delta_{(\epsilon,\alpha)} \Psi = [-\epsilon^A(x)\partial_A - \alpha^A{}_B(x) V_A{}^B]\Psi. \tag{38}$$

As in gravity, the corresponding field strengths are the torsion and the (generalized) curvature [11], i.e.

$$\hat{R}^A{}_{\mu\nu} = \partial_\mu e^A{}_\nu + \Gamma^A{}_{B\mu} e^B{}_\nu - (\mu \leftrightarrow \nu) \tag{39}$$

$$\hat{R}^A{}_{B\mu\nu} = \partial_\mu \Gamma^A{}_{B\nu} + \Gamma^C{}_{B\mu}\Gamma^A{}_{C\nu} - (\mu \leftrightarrow \nu) \tag{40}$$

The Noether currents resulting from this $\overline{SA}(4,R)$ invariance are the energy-momentum and hypermomentum,

$$\Theta_A{}^\mu = \frac{1}{\hat{e}}\,\frac{\delta L_M}{\delta e^A{}_\mu}, \qquad \hat{e} \equiv det(e^A{}_\mu), \tag{41}$$

$$\Upsilon^B{}_{A\mu} = \frac{1}{\hat{e}}\,\frac{\delta L_M}{\delta \Gamma^A{}_{B\mu}}, \tag{42}$$

with the symmetric (AB) pairs denoting shear currents and the antisymmetric pairs $[AB]$ representing angular momentum.

The effective action for this IR (zero-color) hadron sector of QCD, written as a Chromogravitational theory, with matter in $\overline{SL}(4,R)$ manifields, then becomes

$$I = \int d^4x\sqrt{-G}\big\{-aR_{\mu\nu}R^{\mu\nu}+bR^2-cl_G^{-2}R+l_S^{-2}\Sigma_{\mu\nu}{}^\sigma\Sigma^{\mu\nu}{}_\sigma+l_Q^{-2}\Delta_{\mu\nu}{}^\sigma\Delta^{\mu\nu}{}_\sigma+L_M\big\}. \tag{43}$$

The first three terms constitute the Lagrangian that yields the p^{-4} propagators. The fourth and fifth terms are spin-spin and shear-shear contact interaction terms.

We linearize the theory in terms of $H_{\mu\nu}(x) = G_{\mu\nu}(x) - \eta_{\mu\nu}$, where $\eta_{\mu\nu}$ is the Minkowski metric. Taking just the homogeneous part, as required for the evaluation of the propagator, we get for the $H_{\mu\nu}$ field the equation of motion

$$\Big(\frac{a}{4}\,\Box^{\,2} - \frac{1}{2}l_S^{-2}\,\Sigma_{\rho\eta}{}^\lambda\Sigma^{\rho\eta}{}_\lambda - \frac{1}{2}l_Q^{-2}\,\Delta_{\rho\eta}{}^\lambda\Delta^{\rho\eta}{}_\lambda\Big)H_{\mu\nu} = 0, \tag{44}$$

which becomes in momentum space

$$\Big(\frac{a}{4}\,(p^2)^2 - \frac{1}{2}l_S^{-2}f_S\;M_\eta{}^\lambda M^\eta{}_\lambda - \frac{1}{2}l_Q^{-2}f_Q\;T_\eta{}^\lambda T^\eta{}_\lambda\Big)H_{\mu\nu}(p) = 0. \tag{45}$$

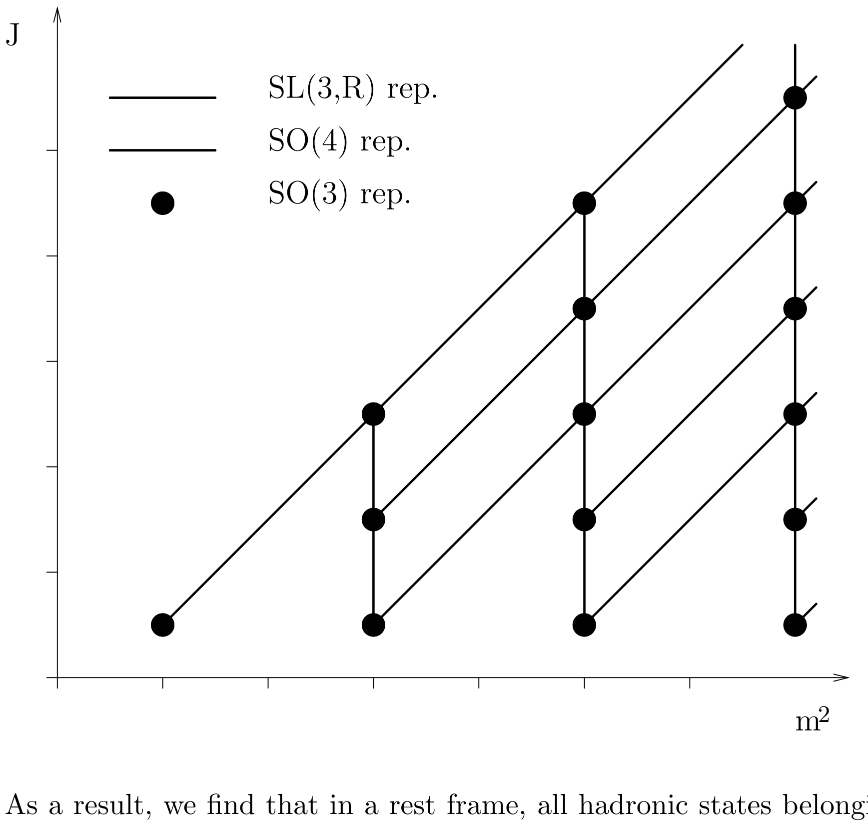
For pseudo-gravity, we may regard these equations as *the dynamical equations above the theory's "vacuum", as represented by hadron matter itself.*

In the rest frame (stability) "little" group is $\overline{SL}(3,R) \subset \overline{SL}(4,R)$. Taking a hadron's rest frame

$$M_\eta{}^\lambda M^\eta{}_\lambda \to M_i{}^j M^i{}_j \to J(J+1), \tag{46}$$

$$T_\eta{}^\lambda T^\eta{}_\lambda \to T_i{}^j T^i{}_j \to M_i{}^j M^i{}_j - A^2_{sl(3,R)} \to J(J+1) - C^2_{sl(3,R)}, \tag{47}$$

where C^2 is the $sl(3,R)$ quadratic invariant.



As a result, we find that in a rest frame, all hadronic states belonging to a single $\overline{SL}(3,R)$ (unitary) irreducible representation (i.e. one value of $C^2_{sl(3,R)}$) lay on a single trajectory in the Chew-Frautschi plane, i.e.

$$(J+\frac{1}{2})^2 = (\alpha' m^2)^2 + \alpha_0^2, \tag{48}$$

$$(\alpha')^2 = \big[\frac{2}{a}(l_S^{-2} f_S + l_Q^{-2} f_Q)\big]^{-1}, \tag{49}$$

$$\alpha_0^2 = \frac{1}{4} + \frac{l_Q^{-2} f_Q}{l_S^{-2} f_S + l_Q^{-2} f_Q} C^2_{sl(3,R)}. \tag{50}$$

α' is the (asymptotic) trajectory slope, S is the Cartan's chromo-torsion tensor, while $Q = D\ G$ is the chromo-non-metricity tensor. Neglecting a slight bending at small m^2,i.e. the α_0^2 term, we finally obtain the linear Regge trajectory

$$J = \alpha' m^2 - \frac{1}{2}. \tag{51}$$

A combined result of the $\overline{SL}(4,R) \supset R_+ \otimes \overline{SL}(3,R)$ representation reduction states and the $J \simeq m^2$ relation is illustrated on the above figure.

8 IBM – Interacting Boson Model Derivation

The Interacting Boson Model has been very successful as a dynamical symmetry in correlating as well as providing an understanding of a large amount of data which manifest the collective behavior of nuclei. The model's point of departure is the observation that the two lowest levels in the great majority of even-even nuclei are the 0^+ and 2^+ levels, with relatively close excitation energies, realized by proton or neutron pairs. The model postulates a corresponding phenomenological $U(6)$ symmetry.

As demonstrated above, the strongly-coupled IR region in QCD is approximated by the exchange of a phenomenological chromometric di-gluon field $G_{\mu\nu}(x)$. The $G_{\mu\nu}(x)$ acts formally as a Riemannian metric, i.e. it obeys the following Riemannian constraint:

$$D_\sigma G_{\mu\nu}(x) = 0. \tag{52}$$

where D_σ is the covariant derivative of the effective gravity, with the connection given by a Christoffel symbol constructed with this effective metric. As a result, the surviving quanta are color neutral and have $J^P = 0^+,\ 2^+$, with symmetric couplings to matter fields.

We now stretch the Chromogravity application from a single hadron case over to the composite hadronic system of nuclear matter [12] in a Van der Waals fashion. As in the hadronic case, the next vibrational, rotational or pulsed excitations will correspond to the "addition" of one such collective color-singlet multigluon quantum superposition, while the basic exchanged quantum is generated by "gluonium" $G_{\mu\nu}(x)$.

An effective Riemannian metric induces the corresponding Einsteinian dynamics. The invariant action in which the Einstein-like Lagrangian R is accompanied by a parametrized combination of the allowed quadratic terms reads

$$I_{inv} = -\int d^4x\sqrt{-G}(\alpha R_{\mu\nu}R^{\mu\nu} - \beta R^2 + \gamma\kappa^{-2}R). \tag{53}$$

The theory is renormalizable, a feature befitting the present application, since QCD is renormalizable, but is not unitary, which also befits this application: a "piece" of QCD should not be unitary, considering that QCD is an irreducible theory. The renormalizability is caused by p^{-4} propagators. *p^{-4} propagators are dynamically equivalent to confinement*!

Moreover, it has been shown that the presence of the quadratic terms in the action induces a potential $\sim \frac{1}{r} + r + r^2$.

Nuclei. Out of the 10 components of $G_{\mu\nu}$ the 6 that survive the 4 Riemannian constraints have spin/parity assignments $J^P = 0^+,\ 2^+$.

The non-relativistic subgroup of $SL(4,R)$ is $SL(3,R)$. Under this group, the 0^+ and 2^+ states span together one irreducible 6-dimensional representation, thus both 0^+ and 2^+ couple with the same strength to nucleons. There is thus full justification for the IBM postulate of a $U(6)$ symmetry between the defining states!

The closed shells assume the role of "vacua", as rigid structures. Gluonium excitations should then be searched for in the valence nucleon systematics.

The corresponding Hamiltonian in terms of $b = \{s, d\}$ and $b^+ = \{s^+, d^+\}$ that represent the destruction and creation of a 6-dimensional gluonium quantum reads:

$$H = \frac{1}{M^3} \int dk \{ C_1 \frac{k^2}{\kappa^2} (b^+ b) + C_2 \frac{k^2}{\kappa^2} (b^+ \bullet b)(b^+ \bullet b) + A_1 k^4 (b^+ \bullet b)(b^+ \bullet b)$$

$$+ A_2 k^4 (b^+ \bullet b)(b^+ \bullet b)(b^+ \bullet b) + A_3 k^4 (b^+ \bullet b)(b^+ \bullet b)(b^+ \bullet b)(b^+ \bullet b)). \quad (54)$$

Symmetries of deformed nuclei

In the quantum case, we can write, $G_{\mu\nu} = T_{\mu\nu} + U_{\mu\nu}$, where

$$T_{\mu\nu} = \eta_{ab} \int d\tilde{k} [\alpha_\mu^{a+}(k) \alpha_\nu^{b+}(k) e^{2ikx} + \alpha_\mu^a(k) \alpha_\nu^b(k) e^{-2ikx}], \quad (55)$$

and

$$U_{\mu\nu} = \eta_{ab} \int d\tilde{k} [\alpha_\mu^{a+}(k) \alpha_\nu^b(k) + \alpha_\mu^a(k) \alpha_\nu^{b+}(k)]. \quad (56)$$

This time we use the creation and annihilation operators $\alpha_\mu^{a+}, \alpha_\nu^b$ of the QCD gluon itself, which can be regarded somewhat like a tetrad field with respect to $G_{\mu\nu}$ as a metric. For this to fit the formalism, we have to separate out the "rigid" piece (analogous to $e_\mu^i = \delta_\mu^i + h_\mu^i$ in the tetrad case). Here this is the "flat connection" N_μ^a, i.e. the zero-mode of the field.

The operators $T_{\mu\nu}$ and $U_{\mu\nu}$, together with the operators

$$S_{\mu\nu} = \eta_{ab} \int d\tilde{k} [\alpha_\mu^{a+}(k) \alpha_\nu^b(k) - \alpha_\mu^a(k) \alpha_\nu^{b+}(k)], \quad (57)$$

close respectively on the algebras of $GL(4, R)$ and $U(1, 3)$. Note that the largest (linearly realized) algebra with generators quadratic in the ${\alpha_\mu}^+$, α_μ operators is the algebra of $Sp(4, R)$. This algebra contains both previous ones:

$$Sp(4, R) \supset \left\{ \begin{array}{ccc} U(1,3) & \supset & SU(1,3) \\ GL(4,R) & \supset & SL(4,R) \\ T_{10} \wedge SO(1,3) & \supset & T_9 \wedge SO(1,3) \end{array} \right\} \supset SO(1,3) \quad (58)$$

The $GL(4,R)$ algebra represents a Spectrum-Generating Algebra for the set of hadron states of a given flavor. In the case of $U(1,3)$, when selecting a time-like vector (for massive states), the stability subgroup is $U(3)$, a compact group with finite representations – as against the non-compact $SL(3,R)$ for $SL(4,R)$. This fits with a situation in nuclei in which the symmetries are physically realized over pairs of "valency" nucleons outside of closed shells, as in the case of IBM: there is a finite number of such pairs, and the excitations thus have to fit within finite representations.

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