AdS/CFT Correspondence and Supersymmetry

V.K. Dobrev

Institute of Nuclear Research and Nuclear Energy
Bulgarian Academy of Sciences
72 Tsarigradsko Chaussee, 1784 Sofia, Bulgaria

Abstract

We use the group-theoretic interpretation of the AdS/CFT correspondence which we proposed earlier in order to lift intertwining operators acting between boundary conformal representations to intertwining operators acting between bulk conformal representations. Further, we present the classification of the positive energy (lowest weight) unitary irreducible representations of the $D = 6$ superconformal algebras $osp(8^*/2N)$.

1 Introduction

Recently there was renewed interest in (super)conformal field theories in arbitrary dimensions. This happened after the remarkable proposal in [1], according to which the large N limit of a conformally invariant theory in d dimensions is governed by supergravity (and string theory) on $d+1$-dimensional AdS space (often called AdS_{d+1}) times a compact manifold.

Actually the possible relation of field theory on AdS_{d+1} to field theory on $\mathcal{M}_d$ has been a subject of long interest, cf., e.g., [2, 3, 4], and also [5] for discussions motivated by recent developments, and additional references. The proposal of [1] was elaborated in [6] and [7] where was proposed a precise correspondence between conformal field theory observables and those of supergravity: correlation functions in conformal field theory are given by the dependence of the supergravity action on the asymptotic behavior at infinity. More explicitly, a conformal field $\mathcal{O}$ corresponds to an AdS field ϕ when there exists a conformal invariant coupling $\int \phi_0 \mathcal{O}$ where ϕ_0 is the value of ϕ at the boundary of AdS_{d+1} ([7]). Furthermore, the dimension Δ of the operator $\mathcal{O}$ is given by the mass of the particle described by ϕ in supergravity [7]. After these initial papers there was an explosion of related research of which of interest to us are two aspects: 1) calculation of conformal correlators from AdS (super)gravity and various questions of holography, cf., e.g., [8-78]; 2) matching of supergravity and superstring spectra with superconformal theories, cf., e.g., [79-140].

For the *first* aspect of the AdS/CFT correspondence one of the main features furnishing it is that the boundary $\mathcal{M}_d$ of AdS_{d+1} is in fact a copy of d-dimensional Minkowski space (with a cone added at infinity); the symmetry group $SO(d,2)$ of AdS_{d+1} acts on $\mathcal{M}_d$ as the conformal group. The fact that $SO(d,2)$ acts on AdS_{d+1} as a group of ordinary symmetries and on $\mathcal{M}_d$ as a group of conformal symmetries means that there are two ways to get a physical theory with $SO(d,2)$ symmetry: in a relativistic field theory (with or without gravity) on AdS_{d+1}, or in a conformal field theory on $\mathcal{M}_d$.

In an earlier paper [19] we gave a group-theoretic interpretation of the above correspondence. [In fact such an interpretation is partially present in [138] for the $d = 3$ Euclidean version of the AdS/CFT correspondence in the context of the construction of discrete series representations of the group

$SO(4,1)$ involving symmetric traceless tensors of arbitrary nonzero spin.] In short the essence of our interpretation is that the above correspondence is a relation of *representation equivalence* between the representations describing the fields ϕ, ϕ_0 and $\mathcal{O}$. There are actually two kinds of equivalences. The first kind is new (besides the example from [138] mentioned above) and was proved in [19] - it is between the representations describing the bulk fields and the boundary fields. The second kind is well known - it is the equivalence between boundary conformal representations which are related by restricted Weyl reflections, the representations here being the coupled fields ϕ_0 and $\mathcal{O}$. Our interpretation means that the operators relating these fields are *intertwining operators* between (partially) equivalent representations. Operators giving the first kind of equivalence for special cases were actually given in, e.g., [7, 8, 9, 12, 16] - in [19] they were constructed in a more general setting from the requirement that they are intertwining operators. The operators giving the second kind of equivalence are provided by the standard conformal two-point functions. Using both equivalences we have found that the bulk field has naturally *two* boundary fields, namely, the coupled fields ϕ_0 and $\mathcal{O}$, the limits being governed by the corresponding conjugated conformal weights $d-\Delta$ and Δ. Thus, from the point of view of the bulk-to-boundary correspondence the coupled fields ϕ_0 and $\mathcal{O}$ are generically *on an equal footing.* [The appearance of two boundary fields was used later in [22] in a slightly different context, namely, that the theory with the same classical AdS Lagrangian can be interpreted in terms of two different CFT's with the conjugated dimensions. This is possible only for sufficiently negative AdS-mass-squared, so that both dimensions would not be lower than the unitarity bound.]

In the present paper we review also the results of [29] in order to lift intertwining operators acting between boundary conformal representations to intertwining operators acting between bulk conformal representations.

For the *second* aspect of the AdS/CFT correspondence one of the most important tasks is the classification of the UIRs of the corresponding superalgebras. Particularly important are those for $D \leq 6$ since in these cases the relevant superconformal algebras satisfy [139] the Haag-Lopuszanski-Sohnius theorem [140]. Until recently such classification was known only for the $D = 4$ superconformal algebras $su(2,2/N)$ [141] (for $N = 1$), [142, 143, 144, 145]. Recently, the classification for $D = 3,5,6$ was given [146] but the results were conjectural and there was not enough detail in order to check these conjectures. In view of the interesting applications [110, 126, 132] of $D = 6$ unitary irreps to the analysis of OPEs and 1/2 BPS operators we decided to reexamine the list of UIRs of the $D = 6$ superconformal algebras $osp(8^*/2N)$ in detail [135]. We confirm all but one of the conjectures of [146] and thus, we give the final list of UIRs for $D = 6$. Our main tool is the explicit construction of the norms. This, on the one hand, enables us to prove the unitarity list, and, on the other hand, enables us to give explicitly the states of the irreps. We give a brief summary of [135] in the last section here.

2 Conformal field theory representations

As in [19] we consider the Euclidean version of the AdS/CFT correspondence. For definiteness we use the following defining relation of the Sitter group G :

$$\begin{aligned} G \ &= \ \{g \in GL(d+2, I\!R) \mid {}^t g \eta g = \eta \doteq \mathrm{diag}(-1, \ldots, -1, 1), \\ &\qquad \det g = 1, \quad g_{d+2,d+2} \geq 1\} \end{aligned} \tag{1}$$

Thus, $G = SO_e(d+1,1)$, i.e., it is the identity component of $O(d+1,1)$, (${}^t g$ is the transposed of g). Note that for d *even* some expressions are simpler if we work with the extended de Sitter group:

$$G' \doteq \{g \in GL(d+2, I\!R) \mid {}^t g \eta g = \eta, \quad g_{d+2,d+2} \geq 1\} \tag{2}$$

which includes reflections of the first $d+1$ axes. The representations of G used in conformal field theory are called (in the representation theory of semisimple Lie groups) generalized principal series representations (cf. [147]). In [138, 148, 149] they were called *elementary representations* (ERs). They are obtained by induction from the subgroup $P = MAN$, where $M = SO(d)$ is the Euclidean Lorentz group, A is the one-dimensional dilatation group, N is the group of special conformal transformations (isomorphic to $I\!R^d$), P is called a parabolic subgroup of G. The induction is from unitary irreps of $M = SO(d)$, from arbitrary (non-unitary) characters of A, and trivially from N. There are several realizations of these representations. We give now the so-called *noncompact picture* of the ERs - it is the one actually used in physics.

The representation space of these induced representations consists of smooth functions on $I\!R^d$ with values in the corresponding finite-dimensional representation space of M, i.e.:

$$C_\chi = \{f \in C^\infty(I\!R^d, V_\mu)\} \tag{3}$$

where $\chi = [\mu, \Delta]$, Δ is the conformal weight, μ is a unitary irrep of M, V_μ is the finite-dimensional representation space of μ. In addition, these functions have special asymptotic expansion as $x \to \infty$. The leading term of this expansion is $f(x) \sim \frac{1}{(x^2)^\Delta} f_0$, (for more details we refer to [138, 148, 149]). The representation T^χ acts in C_χ by:

$$(T^\chi(g)f)(x) = |a|^{-\Delta} \cdot D^\mu(m)\, f(x') \tag{4}$$

where the nonglobal Bruhat decomposition $g = \tilde{n}man$ is used:

$$g^{-1}\tilde{n}_x = \tilde{n}_{x'} m^{-1} a^{-1} n^{-1}\ , \quad g \in G,\, \tilde{n}_x, \tilde{n}_{x'} \in \tilde{N},\, m \in M,\, a \in A,\, n \in N \tag{5}$$

where $\tilde{N}$ is the abelian group of Euclidean translations (isomorphic to $I\!R^d$), $D^\mu(m)$ is the representation matrix of μ in V_μ .

Note that the representation data given by $\chi = [\mu, \Delta]$ fixes also the value of the Casimir operators $\mathcal{C}_i$ in the ER C_χ , independently of the latter reducibility. For later use we write:

$$\mathcal{C}_i \, f(x) \;=\; \lambda_i(\mu, \Delta) \, f(x) \ , \qquad i = 1, \ldots, \mathrm{rank}\, G \;=\; [\tfrac{d}{2}] + 1, \tag{6}$$

Next, we would like to recall the general expression of the conformal two-point function $G_\chi(x_1 - x_2)$ (for special cases cf. [150, 151, 152, 153], for the general formula with special stress on the role of the conformal inversion, cf. [154], also [138]):

$$G_\chi(x) \;=\; \frac{\gamma_\chi}{(x^2)^\Delta} \, D^\mu(r(x)) \tag{7}$$

$$r(x) \;=\; \begin{pmatrix} \tilde{r}(x) & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \in M \ , \quad \tilde{r}(x) \;=\; \left(\frac{2}{x^2} \, x_i x_j - \delta_{ij} \right)$$

where γ_χ is an arbitrary constant for the moment. (Note that for d *even* $r(x)$ $\in O(d)$, so we work with G', cf. (2).)

Finally, we note the intertwining property of $G_\chi(x)$. Namely, let $\tilde{\chi}$ be the representation conjugated to χ by a restricted Weyl reflection, i.e., by the nontrivial element of the restricted Weyl group $W(G, A)$ [138]. Then we have:

$$\tilde{\chi} \;\doteq\; [\, \tilde{\mu}, \, d - \Delta \,] \ , \quad \text{for } \chi = [\mu, \Delta], \tag{8}$$

where $\tilde{\mu}$ is the *mirror image* representation of μ. (For d odd $\tilde{\mu} \cong \mu$, while for d even $\tilde{\mu}$ may be obtained from μ by exchanging the representation labels of the two distinguished Dynkin nodes of $SO(d)$.) Then there is the following intertwining operator [154, 138]:

$$G_\chi \ : \ C_{\tilde{\chi}} \longrightarrow C_\chi \ , \qquad T^\chi(g) \circ G_\chi \;=\; G_\chi \circ T^{\tilde{\chi}}(g) \ , \quad \forall g \ , \tag{9}$$

$$(G_\chi f)(x_1) \doteq \int G_\chi(x_1 - x_2)\, f(x_2)\, dx_2 \ , \quad dx \equiv d^d x \ ,$$

$$C_\chi \simeq C_{\tilde\chi} \tag{10}$$

the last line uses our symbolic notation for partial equivalence between χ and $\tilde\chi$. Note that because of this equivalence the values of all Casimirs coincide:

$$\lambda_i(\tilde\mu, d - \Delta) = \lambda_i(\mu, \Delta) \ , \qquad \forall i \ . \tag{11}$$

3 Representations on the bulk space

In the previous section we discussed representations on $I\!R^d \cong \tilde N$ induced from the parabolic subgroup MAN which is natural since the abelian subgroup $\tilde N$ is locally isomorphic to the factor space G/MAN (via the Bruhat decomposition). Similarly, it is natural to discuss representations on the bulk space $\mathcal{S} \cong \tilde N A$ which are induced from the maximal compact subgroup $K = SO(d+1)$ since the solvable group $\tilde N A$ is isomorphic to the factor space G/K via the global *Iwasawa* decomposition $G = \tilde N A K$ (cf. the details in [19][1]). Namely, we consider the representation spaces:

$$\hat C_\tau = \{\phi \in C^\infty(I\!R^d \times I\!R_{>0}\, , U_\tau)\} \tag{12}$$

where τ is an arbitrary unitary irrep of K, U_τ is the finite-dimensional representation space of τ, with representation action:

$$(\hat T^\tau(g)\phi)(x, |a|) = \hat D^\tau(k)\, \phi(x', |a'|) \tag{13}$$

where the Iwasawa decomposition is used:

$$g^{-1}\tilde n_x a = \tilde n_{x'} a' k^{-1} \ , \quad g \in G,\, k \in K,\, \tilde n_x, \tilde n_{x'} \in \tilde N,\, a, a' \in A \tag{14}$$

[1] Note that in [19] the bulk space $\mathcal{S} \cong \tilde N A \cong SO(d+1,1)/SO(d+1)$ was called de Sitter space, though in the literature the latter name is used for the space $SO(d+1,1)/SO(d,1)$. The latter space was used recently for extensive study of the so-called *dS/CFT* correspondence, cf., e.g., [155, 156, 157, 158].

and $\hat{D}^\tau(k)$ is the representation matrix of τ in U_τ . However, unlike the ERs, these representations are reducible, and to single out an irrep equivalent, say, a subrepresentation of an ER, one has to look for solutions of the eigenvalue problem related to the Casimir operators.

In the actual implementation of (13) and (14) we use the following parametrization of k:

$$k = \begin{pmatrix} k_{ij} & k_{i,d+1} & 0 \\ k_{d+1,j} & k_{d+1,d+1} & 0 \\ 0 & 0 & 1 \end{pmatrix} \in K \ , \quad (k_{\alpha\beta}) \in SO(d+1) \ . \tag{15}$$

Further we shall need also the unique decomposition:

$$k = m(k)k_f \ , \qquad m(k) = \begin{pmatrix} \tilde{m}(k) & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \in M \ , \qquad k_f = \begin{pmatrix} \tilde{k}_f & 0 \\ 0 & 1 \end{pmatrix} \in K \tag{16}$$

representing the decomposition of K into its subgroup M and the coset K/M : $\quad K \cong M \ K/M$. Explicitly, we have (for $k_{d+1,d+1} \neq -1$):

$$\begin{aligned} \tilde{m}(k) &= \left(k_{ij} - \frac{1}{1 + k_{d+1,d+1}} \, k_{i,d+1} \, k_{d+1,j} \right) \\ \tilde{k}_f &= \begin{pmatrix} \delta_{ij} - \frac{2}{1+x^2} \, x_i x_j & -\frac{2}{1+x^2} \, x_i \\ \frac{2}{1+x^2} \, x_j & \frac{1-x^2}{1+x^2} \end{pmatrix} \doteq \tilde{k}_x \ , \quad x \in I\!R^d \ , \end{aligned} \tag{17}$$

$x_i = k_{d+1,i}/(1 + k_{d+1,d+1})$. Note that $k_x \doteq \begin{pmatrix} \tilde{k}_x & 0 \\ 0 & 1 \end{pmatrix}$ appeared in (1.30a) of [138].

Further, we would like to extract from $\hat{C}_\tau$ a representation that may be equivalent to C_χ , $\chi = [\mu, \Delta]$. The first condition for this is that the M-representation μ is contained in the restriction of the K-representation τ to M. Another condition is that the two representations would have the same Casimir values $\lambda_i(\mu, \Delta)$. Having in mind the degeneracy of Casimir values for partially equivalent representations (e.g., (11)) we add also the appropriate asymptotic condition. Furthermore, from now on we shall suppose that Δ is real. Thus, we shall use the representations:

$$\hat{C}^\tau_\chi \;=\; \{\, \phi \in \hat{C}_\tau \;:\; \mathcal{C}_i\, \phi(x,|a|) \;=\; \lambda_i(\mu,\Delta)\, \phi(x,|a|) \ , \quad \forall i \ , \quad \mu \in \tau|_M \ ,$$

$$\phi(x,|a|) \;\sim\; |a|^\Delta\, \varphi(x) \text{ for } |a| \to 0 \,\} \tag{18}$$

Unlike their action on the ERs C_χ the Casimirs $\mathcal{C}_i$ here are differential operators and the elements of $\hat{C}^\tau_\chi$ are solutions of the equations in (18).

In [19] it was shown that generically the functions in (18) have also a second limit with $\Delta \to d - \Delta$:

$$\tilde{\varphi}(x) \;=\; \lim_{|a|\to 0} |a|^{\Delta - d}\, \phi(x,|a|) \tag{19}$$

For generic representations this establishes the following important relation:

$$\hat{C}^\tau_\chi \;=\; \hat{C}^\tau_{\tilde{\chi}} \ , \qquad \chi = [\mu,\Delta],\ \tilde{\chi} = [\tilde{\mu}, d-\Delta] \ . \tag{20}$$

For non-generic representations the second asymptotic expansion of ϕ contains logarithms (cf., e.g., (7.45) of [138] and [10, 15]), and then the representations χ and $\tilde{\chi}$ are only partially equivalent:

$$\hat{C}^\tau_\chi \;\simeq\; \hat{C}^\tau_{\tilde{\chi}} \ . \tag{21}$$

4 Intertwining relations between conformal and bulk representations

This Section contains the main result of [19], explicating the relations between CFT and bulk representations as intertwining relations. We first give the intertwining operator from the bulk to the CFT realization. The operator is mapping a function on the bulk space to its boundary value and was used in a restricted sense (explained below) in many papers, starting from [7].

Theorem: *Let us define the operator:*

$$L_\chi^\tau \ : \ \hat{C}_\chi^\tau \ \longrightarrow \ C_\chi \,, \tag{22}$$

with the following action:

$$(L_\chi^\tau \phi)(x) \ = \ \lim_{|a|\to 0} \ |a|^{-\Delta} \ \Pi_\mu^\tau \ \phi(x,|a|) \tag{23}$$

where Π_μ^τ *is the standard projection operator from the* K*-representation space* U_τ *to the* M*-representation space* V_μ, *which acts in the following way on the* K*-representation matrices:*

$$\Pi_\mu^\tau \ \hat{D}^\tau(k) \ = \ D^\mu(m(k)) \ \Pi_\mu^\tau \ \hat{D}^\tau(k_f) \tag{24}$$

where we have used (16). Then L_χ^τ *is an intertwining operator, i.e.:*

$$L_\chi^\tau \circ \hat{T}^\tau(g) \ = \ T^\chi(g) \circ L_\chi^\tau \ , \quad \forall g \in G \ . \tag{25}$$

In addition, in (23) the operator Π_μ^τ *acts in the following truncated way:*

$$\Pi_\mu^\tau \ \hat{D}^\tau(k) \ = \ D^\mu(m(k)) \ \Pi_\mu^\tau \tag{26}$$

The proof is given in [19].

Next we consider the operator inverse to L_χ^τ which would restore a function on the bulk space from its boundary value, as discussed in [7-18]. Again what was new in [19] was that the operator was defined as intertwining operator between exactly defined spaces in a more general setting. Moreover, the operator was constructed just from the condition that it is an intertwining integral operator. Namely, we started with the operator:

$$\tilde{L}_\chi^\tau \ : \ C_\chi \ \longrightarrow \ \hat{C}_\chi^\tau \ , \tag{27}$$

using the following Ansatz:

$$\left(\tilde{L}_\chi^\tau \ f\right)(x,|a|) \ = \ \int K_\chi^\tau(x,|a|;x') \, f(x') \, dx' \tag{28}$$

where $K^\tau_\chi(x,|a|;x')$ is a linear operator acting from the space V_μ to the space U_τ, and supposed that $\tilde{L}^\tau_\chi$ is an intertwining operator, i.e.:

$$\hat{T}^\tau(g) \circ \tilde{L}^\tau_\chi \ = \ \tilde{L}^\tau_\chi \circ T^\chi(g) \ , \quad \forall g \in G \ . \tag{29}$$

From this we obtained (cf. the details in [19]) that K^τ_χ is fixed up to an overall multiplicative constant N^τ_χ and explicitly is:

$$\begin{aligned} K^\tau_\chi(x,|a|;x') \ &= \ K^\tau_\chi(x-x',|a|) \\ K^\tau_\chi(x,|a|) \ &= \ N^\tau_\chi \left(\frac{|a|}{x^2+|a|^2}\right)^{d-\Delta} \ \hat{D}^\tau(k_{-\frac{x}{|a|}})\, \Pi^\mu_\tau \end{aligned} \tag{30}$$

where Π^μ_τ is the canonical embedding operator from V_μ to U_τ, such that $\Pi^\tau_\mu \circ \Pi^\mu_\tau \ = \ \mathbf{1}_\mu$, and k_x is given in (17).

The above operator exists for arbitrary representations τ of $K = SO(d+1)$ which contain the representation μ of $M = SO(d)$. We use the standard $SO(p)$ representation parametrization: $[\ell_1,\ldots,\ell_{\tilde{p}}]$, ($\tilde{p} \equiv [\frac{p}{2}]$), where all ℓ_j are simultaneously integer or half-integer, all are positive except for p *even* when ℓ_1 can also be negative, and they are ordered: $|\ell_1| \leq \ell_2 \leq \ldots \leq \ell_{\tilde{p}}$. The condition that $\tau = [\ell'_1,\ldots,\ell'_{\hat{d}}]$, ($\hat{d} \equiv [\frac{d+1}{2}]$), contains $\mu = [\ell_1,\ldots,\ell_{\tilde{d}}]$, ($\tilde{d} \equiv [\frac{d}{2}]$), explicitly is:

$$\begin{aligned} &|\ell'_1| \leq \ell_1 \leq \ldots \leq \ell_{\tilde{d}} \leq \ell'_{\hat{d}} \ , \qquad d \ \ odd, \ \ \hat{d} = \tilde{d}+1 \\ &-\ell'_1 \leq \ell_1 \leq \ell'_1 \ldots \leq \ell_{\tilde{d}} \leq \ell'_{\hat{d}} \ , \qquad d \ \ even, \ \ \hat{d} = \tilde{d} \end{aligned} \tag{31}$$

If one is primarily concerned with the ERs $\chi = [\mu,\Delta]$ it is convenient to chose a 'minimal' representation $\tau(\mu)$ of $K = SO(d+1)$ containing μ. This depends on the parity of d. Thus, for μ as above, when d is *odd* we would choose:

$$\tau(\mu) \ = \ [\ell_1,\ell_1,\ldots,\ell_{\tilde{d}}] \qquad \text{or} \qquad \tilde{\tau}(\mu) \ = \ [-\ell_1,\ell_1,\ldots,\ell_{\tilde{d}}] \ , \qquad \mu \cong \tilde{\mu} \ , \tag{32}$$

while for *even* d we would choose:

$$\tau(\mu) \;=\; [|\ell_1|, \ell_2 \ldots, \ell_{\tilde{d}}] \;=\; \tau(\tilde{\mu}) \;\cong\; \tilde{\tau}(\mu) \;=\; \tilde{\tau}(\tilde{\mu})\;, \quad \tilde{\mu} \;=\; [-\ell_1, \ell_2 \ldots, \ell_{\tilde{d}}]\;. \tag{33}$$

Thus, in the odd d case for each μ we would choose between two K-irreps which are mirror images of one another, while in the even d case to each two mirror-image irreps of M we choose one and the same irrep of K.

The explicit examples which appeared in the literature are actually in the cases in which $\tau = \tau(\mu)$, e.g., [7, 8, 9, 12, 16], though there is no such interpretation as we have here.

5 Intertwining operators on the bulk space

In this section we review [29]. We show how to lift intertwining operators acting between boundary conformal representations to intertwining operators acting between bulk conformal representations. Of course, for generic representations there is nothing to do, since the pairs of equivalent boundary representations χ and $\tilde{\chi}$ are equivalent to the same bulk representation. However, for nongeneric boundary representations when χ and $\tilde{\chi}$ are partially equivalent but not equivalent, the situation is much more interesting. In this case besides the pair $\chi_0 \equiv \chi$ and $\tilde{\chi}_0 \equiv \tilde{\chi}$ there exist $\tilde{d}$ more such pairs χ_i and $\tilde{\chi}_i$ $(i = 1, ..., \tilde{d})$ so that these $2\tilde{d}+2$ ERs are partially equivalent between themselves, (for the explicit parametrization of these ERs we refer to [138, 148] for early partial cases, and [159] for the general case). These partial equivalences are realized by $2\tilde{d}$ intertwining differential operators. The latter come in pairs, i.e., if there exists an intertwining differential operator $\mathcal{D}$ acting from C_{χ_j} to $C_{\chi_{j'}}$, $(j, j' = 0, 1, ..., \tilde{d};\ j \neq j')$, then there exists an intertwining differential operator $\mathcal{D}'$ acting from $C_{\tilde{\chi}_{j'}}$ to $C_{\tilde{\chi}_j}$.

Now we shall use these operators and the operators bulk$\leftrightarrow$boundary operators of the previous section to build operators acting between bulk representations. For notational simplicity we write χ, χ' instead of $\chi_j\,, \chi_{j'}$. We start with the operator $\mathcal{D}$ acting from C_χ to $C_{\chi'}$, where $\chi' = [\mu', \Delta']$, and lift it to an operator $\tilde{\mathcal{D}}$ acting from $\hat{C}^\tau_\chi$ to $\hat{C}^{\tau'}_{\chi'}$, where τ' is a UIR of K containing μ'. Explicitly, we have:

$$\tilde{\mathcal{D}} \ : \ \hat{C}^\tau_\chi \ \longrightarrow \ \hat{C}^{\tau'}_{\chi'} \ , \qquad \tilde{\mathcal{D}} \ = \ \tilde{L}^{\tau'}_{\chi'} \circ \mathcal{D} \circ L^\tau_\chi \tag{34}$$

Analogously, we have for the operator $\tilde{\mathcal{D}}'$ acting from $\hat{C}^{\tau'}_{\tilde{\chi}'}$ to $\hat{C}^\tau_{\tilde{\chi}}$:

$$\tilde{\mathcal{D}}' \ : \ \hat{C}^{\tau'}_{\tilde{\chi}'} \ \longrightarrow \ \hat{C}^\tau_{\tilde{\chi}} \ , \qquad \tilde{\mathcal{D}}' \ = \ \tilde{L}^\tau_{\tilde{\chi}} \circ \mathcal{D}' \circ L^{\tau'}_{\tilde{\chi}'} \tag{35}$$

The explicit parametrization and expressions of all possible operators $\tilde{\mathcal{D}}, \tilde{\mathcal{D}}'$ will be given elsewhere.

Finally, we notice that all operators that we have used may be found on the following diagram:

$$
\begin{array}{ccc}
 & \hat{C}^{\tau}_{\tilde{\chi}} \;\simeq\; \hat{C}^{\tau}_{\chi} & \\
L^{\tau}_{\tilde{\chi}} \;\swarrow\!\nearrow \tilde{L}^{\tau}_{\tilde{\chi}} & & \tilde{L}^{\tau}_{\chi} \;\nwarrow\!\searrow L^{\tau}_{\chi} \\
C_{\tilde{\chi}} & \begin{array}{c} G_{\tilde{\chi}} \\ \longleftarrow \\ \longrightarrow \\ G_{\chi} \end{array} & C_{\chi} \\
\mathcal{D}' \uparrow & & \downarrow \mathcal{D} \\
C_{\tilde{\chi}'} & \begin{array}{c} G_{\tilde{\chi}'} \\ \longleftarrow \\ \longrightarrow \\ G_{\chi'} \end{array} & C_{\chi'} \\
L^{\tau'}_{\tilde{\chi}'} \;\nwarrow\!\searrow \tilde{L}^{\tau'}_{\tilde{\chi}'} & & \tilde{L}^{\tau'}_{\chi'} \;\swarrow\!\nearrow L^{\tau'}_{\chi'} \\
 & \hat{C}^{\tau'}_{\tilde{\chi}'} \;\simeq\; \hat{C}^{\tau'}_{\chi'} &
\end{array}
\tag{36}
$$

6 Positive energy UIRs of D=6 conformal supersymmetry

In this section we review [135]. The superconformal algebras in $D = 6$ are $\mathcal{G} = osp(8^*/2N)$ (real forms of $osp(8/2N) \cong D(4,N)$, [160]). We label their physically relevant representations by the signature:

$$\chi = [\, d\, ;\, n_1\, ,\, n_2\, ,\, n_3\, ;\, a_1\, , ..., a_N\,] \tag{37}$$

where d is the conformal weight, n_1, n_2, n_3 are non-negative integers which are Dynkin labels of the finite-dimensional irreps of the $D = 6$ Lorentz algebra $so(5,1)$, and $a_1, ..., a_N$ are non-negative integers which are Dynkin labels of the finite-dimensional irreps of the internal (or R) symmetry algebra $usp(2N)$. The even subalgebra of $osp(8^*/2N)$ is the algebra $so^*(8) \oplus usp(2N)$, and $so^*(8) \cong so(6,2)$ is the $D = 6$ conformal algebra.

In [135] we gave a constructive proof for the UIRs of $osp(8^*/2N)$ following the methods used for the $D = 4$ superconformal algebras $su(2,2/N)$, cf. [143, 144, 145]. The main tool is an adaptation of the Shapovalov form on the Verma modules V^χ over the complexification $\mathcal{G}^{\mathbb{C}} = osp(8/2N)$ of $\mathcal{G}$. The UIRs are realized as irreducible factor-modules of the Verma modules V^χ. (The reducibility conditions of V^χ are derived according to [161].) The main result is:

Theorem: All positive energy unitary irreducible representations of the conformal superalgebra $osp(8^*/2N)$ characterized by the signature χ in (37) are obtained for real d and are given in the following list:

$$d \ \geq\ d^-_{11} \ =\ \tfrac{1}{2}(3n_1 + 2n_2 + n_3) + 2r_1 + 6\ , \quad \text{no restrictions on } n_j \tag{38}$$

$$d \ =\ d^-_{21} \ =\ \tfrac{1}{2}(n_3 + 2n_2) + 2r_1 + 4\ , \quad n_1 = 0 \tag{39}$$

$$d \ =\ d^-_{31} \ =\ \tfrac{1}{2}n_3 + 2r_1 + 2\ , \quad n_1 = n_2 = 0 \tag{40}$$

$$d \ = \ d_{41}^- \ = \ 2r_1 \ , \quad n_1 = n_2 = n_3 = 0 \tag{41}$$

where d_{j1}^- are the four distinguished reducibility points of the Verma modules:

$$d_{j1}^- \ \doteq \ \tfrac{1}{2}(3n_1 + 2n_2 + n_3) \ - \ 2\sum_{s=1}^{j-1} n_j \ + \ 2r_1 \ + \ 8 - 2j \tag{42}$$

Remark: For $N = 1, 2$ the Theorem was conjectured by Minwalla [146], except that he conjectured unitarity also for the open interval (d_{31}^-, d_{21}^-) with conditions on n_j as in (40). We should note that this conjecture could be reproduced neither by methods of conformal field theory [110], nor by the oscillator method [162] (cf. [146]), and thus was in doubt. To compare with the notations of [146] one should use the following substitutions: $n_1 = h_2 - h_3$, $n_2 = h_1 - h_2$, $n_3 = h_2 + h_3$, $r_1 = k$, and h_j are all integer or all half-integer. The fact that $n_j \geq 0$ for $j = 1, 2, 3$ translates into: $h_1 \geq h_2 \geq |h_3|$, i.e., the parameters h_j are of the type often used for representations of $so(2N)$ (though usually for $N \geq 4$). Note also that the statement of the Theorem is arranged in [146] according to the possible values of n_i first and then the possible values of d. To compare with the notation of [110] we use the substitution $(n_1, n_2, n_3) \to (J_3, J_2, J_1)$. Some UIRs at the four exceptional points d_{i1}^- were constructed in [4] by the oscillator method (some of these were identified with Cartan-type signatures like (37) in, e.g., [146], [126]).

Acknowledgement. The author would like to thank the organizers and especially B. Dragovich for the invitation to give lectures at the School and for the warm hospitality at Sokobanja.

References

[1] J. Maldacena, Adv. Theor. Math. Phys. **2** (1998) 231 (hep-th/971120).

[2] M. Flato and C. Fronsdal, J. Math. Phys. **22** (1981) 1100; E. Angelopoulos, M. Flato, C. Fronsdal and D. Sternheimer, Phys. Rev. **D23** (1981) 1278; C. Fronsdal, Phys. Rev. **D26** (1982) 1988.

[3] H. Nicolai and E. Sezgin, Phys. Let. **143B** (1984) 103.

[4] M. Gunaydin, P. van Nieuwenhuizen and N.P. Warner, Nucl. Phys. **B255** (1985) 63.

[5] S. Ferrara and C. Fronsdal, Class. Quant. Grav. **15** (1998) 2153 (hep-th/9712239); Phys. Lett. **433B** (1998) 19 (hep-th/9802126).

[6] S.S. Gubser, I.R. Klebanov and A.M. Polyakov, Phys. Lett. **428B** (1998) 105 (hep-th/9802109).

[7] E. Witten, Adv. Theor. Math. Phys. **2** (1998) 253 (hep-th/9802150).

[8] M. Henningson and K. Sfetsos, Phys. Lett. **431B** (1998) 63 (hep-th/9803251).

[9] W. Mück and K.S. Viswanathan, Phys. Rev. **D58** (1998) 041901 (hep-th/9804035), Phys. Rev. **D58** (1998) 106006 (hep-th/9805145), Phys. Rev. **D60** (1999) 081901 (hep-th/9906155).

[10] D.Z. Freedman, S.D. Mathur, A. Matusis and L. Rastelli, Nucl. Phys. **B546** (1999) 96 (hep-th/9804058); Phys. Lett. **452B** (1999) 61 (hep-th/9808006).

[11] A.M. Ghezelbash, K. Kaviani, S. Parvizi and A.H. Fatollahi, Phys. Lett. **435B** (1998) 291 (hep-th/9805162); K. Kaviani and A. M. Ghezelbash, Phys. Lett. **469B** (1999) 81 (hep-th/9902104).

[12] H. Liu and A.A. Tseytlin, Phys. Rev. **D59** (1999) 086002 (hep-th/9807097).

[13] E. D'Hoker, D.Z. Freedman and W. Skiba, Phys. Rev. **D59** (1999) 045008 (hep-th/9807098); E. D'Hoker and D.Z. Freedman, Nucl. Phys. **B544** (1999) 612 (hep-th/9809179), Nucl. Phys. **B550** (1999) 261 (hep-th/9811257); E. D'Hoker, D. Z. Freedman, S.D. Mathur, A. Matusis and L. Rastelli, Nucl. Phys. **B562** (1999) 353 (hep-th/9903196); in: Shifman, M.A. (ed.) 'The Many Faces of the Superworld', p. 332

(hep-th/9908160); E. D'Hoker, D.Z. Freedman and L. Rastelli, Nucl. Phys. **B562** (1999) 395 (hep-th/9905049).

[14] S. Lee, S. Minwalla, M. Rangamani and N. Seiberg, Adv. Theor. Math. Phys. **2** (1998) 697 (hep-th/9806074).

[15] A.M. Ghezelbash, M. Khorrami and A. Aghamohammadi, Int. J. Mod. Phys. **A14** (1999) 2581 (hep-th/9807034).

[16] S. Corley, hep-th/9808184; A. Volovich, hep-th/9809009; W.S. l'Yi, hep-th/9809132, hep-th/9811097.

[17] D. Berenstein, R. Corrado, W. Fischler and J. Maldacena, Phys. Rev. **D59** (1999) 105023 (hep-th/9809188); J. Maldacena and H. Ooguri, J. Math. Phys. **42**, 2929 (2001) [hep-th/0001053], hep-th/0111180; J. Maldacena, hep-th/0106112.

[18] M. Bianchi and S. Kovacs, ROM2F-98-37, hep-th/9811060; Phys. Lett. **468B** (1999) 102 (hep-th/9910016).

[19] V.K. Dobrev, Nucl. Phys. **B553** (1999) 559-582 (hep-th/9812194).

[20] P. Minces and V.O. Rivelles, Phys. Lett. **455B** (1999) 147 (hep-th/9902123), hep-th/9907079, J. High En. Phys. **0112**, 010 (2001) [arXiv:hep-th/0110189].

[21] R.C. Rashkov, Mod. Phys. Lett. **A14** (1999) 1783 (hep-th/9904098); hep-th/9907093.

[22] I.R. Klebanov and E. Witten, Nucl. Phys. **B556** (1999) 89 (hep-th/9905104).

[23] I.R. Klebanov, hep-th/9908165, hep-th/0009139.

[24] H.O. Girotti and V.O. Rivelles, hep-th/9910017; V.O. Rivelles, hep-th/9912139.

[25] K.H. Rehren, hep-th/9905179, hep-th/9910074, Phys. Lett. **493B**, 383 (2000) [hep-th/0003120].

[26] R.R. Metsaev, Nucl. Phys. **B563** (1999) 295 (hep-th/9906217), Phys. Lett. **468B** (1999) 65 (hep-th/9908114), hep-th/0011112, Nucl. Phys. Proc. Suppl. **102**, 100 (2001) [hep-th/0103088], hep-th/0201226.

[27] E. Witten and S.T. Yau, hep-th/9910245.

[28] M. Flato, C. Fronsdal and D. Sternheimer, Lett. Math. Phys. **48**, 109 (1999).

[29] V.K. Dobrev, Talk at the VI International Wigner Symposium, Istanbul, 1999, Turk. J. Phys., to appear.

[30] F. Bastianelli, S. Frolov and A.A. Tseytlin, J. High En. Phys. **0002**, 013 (2000) [hep-th/0001041].

[31] D. Berenstein and R.G. Leigh, J. High En. Phys. **0001**, 038 (2000) [hep-th/0001055].

[32] M. Mihailescu and S. Ramgoolam, Nucl. Phys. **B587**, 179 (2000) [hep-th/0002002].

[33] L. Hoffmann, A.C. Petkou and W. Rühl, Phys. Lett. B **478B**, 320 (2000) [hep-th/0002025], hep-th/0002154; L. Hoffmann, L. Mesref and W. Rühl, Nucl. Phys. **B589**, 337 (2000) [hep-th/0006165], Nucl. Phys. **B608**, 177 (2001) [hep-th/0012153]; L. Hoffmann, T. Leonhardt, L. Mesref and W. Rühl, hep-th/0102162; S.V. Ketov, T. Leonhardt and W. Rühl, hep-th/0109081; L. Hoffmann, L. Mesref, A. Meziane and W. Rühl, hep-th/0112191.

[34] G. Arutyunov and S. Frolov, hep-th/0002152.

[35] R. Manvelian and A.C. Petkou, Phys. Lett. B **483B**, 264 (2000) [hep-th/0003017]; D. Klemm, A.C. Petkou, G. Siopsis and D. Zanon, Nucl. Phys. **B620**, 519 (2002) [hep-th/0104141]; A.C. Petkou, hep-th/0201258.

[36] Y. Oz, hep-th/0004009.

[37] J. de Boer, Class. Quant. Grav. **17**, 1027 (2000).

[38] L. Brink, R.R. Metsaev and M.A. Vasiliev, Nucl. Phys. **B586**, 183 (2000) [hep-th/0005136].

[39] A.M. Polyakov and V.S. Rychkov, Nucl. Phys. **B594**, 272 (2001) [hep-th/0005173].

[40] G. Giribet and C. Nunez, J. High En. Phys. **0006**, 033 (2000) [hep-th/0006070]; L. Anchordoqui, C. Nunez and K. Olsen, J. High En. Phys. **0010**, 050 (2000) [hep-th/0007064].

[41] E. D'Hoker and B. Pioline, J. High En. Phys. **0007**, 021 (2000) [hep-th/0006103].

[42] A. Jevicki, M. Mihailescu and S. Ramgoolam, J. High En. Phys. **0010**, 008 (2000) [hep-th/0006239].

[43] C. Bachas, J. Hoppe and B. Pioline, J. High En. Phys. **0107**, 041 (2001) [hep-th/0007067].

[44] I. Benkaddour, A. El Rhalami and E.H. Saidi, Eur. Phys. J. **C21**, 735 (2001) [hep-th/0007142].

[45] M. Niedermaier, Phys. Lett. **498B**, 83 (2001) [hep-th/0007227].

[46] B. Eden, A.C. Petkou, C. Schubert and E. Sokatchev, Nucl. Phys. **B607**, 191 (2001) [hep-th/0009106].

[47] S.S. Gubser and I. Mitra, hep-th/0009126, J. High En. Phys. **0108**, 018 (2001) [hep-th/0011127].

[48] M. Cadoni, P. Carta, D. Klemm and S. Mignemi, Phys. Rev. **D63**, 125021 (2001) [hep-th/0009185]; D. Klemm and W.A. Sabra, Phys. Lett. **503B**, 147 (2001) [hep-th/0010200], J. High En. Phys. **0102**, 031 (2001) [hep-th/0011016]; M.M. Caldarelli, D. Klemm and W.A. Sabra, J. High En. Phys. **0105**, 014 (2001) [hep-th/0103133]; S. Cacciatori, D. Klemm, L. Martucci and D. Zanon, hep-th/0201103; M. Brigante, S. Cacciatori, D. Klemm and D. Zanon, hep-th/0202073.

[49] J. Lee, Phys. Rev. **D64**, 046012 (2001) [hep-th/0010191].

[50] S.E. Konstein, M.A. Vasiliev and V.N. Zaikin, J. High En. Phys. **0012**, 018 (2000) [hep-th/0010239].

[51] R. Argurio, A. Giveon and A. Shomer, J. High En. Phys. **0012**, 025 (2000) [hep-th/0011046], Fortsch. Phys. **49**, 409 (2001) [hep-th/0012117];

[52] A. Giveon and D. Kutasov, Nucl. Phys. **B621**, 303 (2002) [hep-th/0106004]; A. Giveon, D. Kutasov and A. Schwimmer, Nucl. Phys. **B615**, 133 (2001) [hep-th/0106005].
[53] H. Steinacker, Int. J. Mod. Phys. **B14**, 2499 (2000); math.qa/0102110.
[54] M. Porrati, Phys. Lett. **498B**, 92 (2001) [hep-th/0011152].
[55] Y.S. Myung, hep-th/0012082.
[56] M. Bianchi, Nucl. Phys. Proc. Suppl. **102**, 56 (2001) [hep-th/0103112].
[57] A. Hebecker and J. March-Russell, Nucl. Phys. **B608**, 375 (2001) [hep-ph/0103214].
[58] M. Perez-Victoria, J. High En. Phys. **0105**, 064 (2001) [hep-th/0105048].
[59] O. Lunin and S.D. Mathur, Nucl. Phys. **B623**, 342 (2002) [hep-th/0109154].
[60] O. DeWolfe, D.Z. Freedman, S.S. Gubser, G.T. Horowitz and I. Mitra, hep-th/0105047.
[61] H. Ooguri, Int. J. Mod. Phys. A **A16**, 677 (2001); P. Lee, H. Ooguri, J.W. Park and J. Tannenhauser, Nucl. Phys. **B610**, 3 (2001) [hep-th/0106129]; P. Lee, H. Ooguri and J.W. Park, hep-th/0112188.
[62] Y. Hikida and Y. Sugawara, hep-th/0107189.
[63] M.H. Dehghani, A.M. Ghezelbash and R.B. Mann, Phys. Rev. **D65**, 044010 (2002) [hep-th/0107224]; A.M. Ghezelbash, D. Ida, R.B. Mann and T. Shiromizu, hep-th/0201004.
[64] M. Blau, K.S. Narain and G. Thompson, hep-th/0108122.
[65] J.T. Liu and W.Y. Wen, Phys. Lett. **525B**, 157 (2002) [hep-th/0110213].
[66] R. Iengo and R. Ramachandran, hep-th/0111200.
[67] R. Bousso and L. Randall, hep-th/0112080.
[68] S.B. Giddings and A. Nudelman, hep-th/0112099.
[69] G.T. Horowitz and T. Jacobson, J. High En. Phys. **0201**, 013 (2002) [hep-th/0112131].

[70] B. Ponsot, V. Schomerus and J. Teschner, hep-th/0112198.

[71] J. Polchinski and L. Susskind, hep-th/0112204.

[72] E. Witten, hep-th/0112258.

[73] Y.I. Manin and M. Marcolli, hep-th/0201036.

[74] W. Mück, hep-th/0201100.

[75] P. Minces, hep-th/0201172.

[76] S. Ferrara, C. Fronsdal and A. Zaffaroni, Nucl. Phys. **B532** (1998) 153 (hep-th/9802203); S. Ferrara and C. Fronsdal, Lett. Math. Phys. **46** (1998) 157 (hep-th/9806072).

[77] M. Flato and C. Fronsdal, Lett. Math. Phys. **44** (1998) 249 (hep-th/9803013).

[78] Y. Oz and J. Terning, Nucl. Phys. **B532** (1998) 163 (hep-th/9803167).

[79] L. Andrianopoli and S. Ferrara, Phys. Lett. **430** (1998) 248 (hep-th/9803171); Lett. Math. Phys. **46** (1998) 265 (hep-th/9807150); Lett. Math. Phys. **48** (1999) 145 (hep-th/9812067); S. Ferrara, M.A. Lledo and A. Zaffaroni, Phys. Rev. **D58** (1998) 105029 (hep-th/9805082); S. Ferrara and A. Zaffaroni, CERN-TH-98-229, hep-th/9807090; to appear in the proceedings of the Moshe Flato Conference, Dijon, 1999, CERN-TH/99-255, hep-th/9908163; S. Ferrara, M. Porrati and A. Zaffaroni, Lett. Math. Phys. **47** (1999) 255 (hep-th/9810063); A. Ceresole, G. Dall'Agata, R. D'Auria and S. Ferrara, Phys. Rev. **D61**, 066001 (2000), hep-th/9905226; Class. Quant. Grav. **17**, 1017 (2000), hep-th/9910066; S. Ferrara, TMR Conference, Paris, 1-7 Sep 1999, hep-th/0002141; R. D'Auria, S. Ferrara and S. Vaula, J. High En. Phys. **0010**, 013 (2000) [hep-th/0006107], Fortsch. Phys. **49**, 459 (2001) [hep-th/0101066].

[80] A. Nelson and M.J. Strassler, Phys. Rev. **D60**, 015004 (1999), hep-ph/9806346; J. High En. Phys. (9) U553 (2000), hep-ph/0006251.

[81] M.J. O'Loughlin and S. Randjbar-Daemi, hep-th/9807208.

[82] H. Osborn, Ann. Phys. (NY) **272**, 243 (1999), hep-th/9808041.

[83] O. Aharony, J. Pawelczyk, S. Theisen and S. Yankielowicz, Phys. Rev. **D60** (1999) 066001 (hep-th/9901134).

[84] M.R. Douglas and S. Randjbar-Daemi, in: *Nonperturbative aspects of strings, branes and supersymmetry*, Proceedings (Trieste 1998) p. 157, hep-th/9902022.

[85] J.-H. Park, Nucl. Phys. **B559**, 455 (1999), hep-th/9903230.

[86] D.Z. Freedman, S.S. Gubser, K. Pilch and N.P. Warner, Adv. Theor. Math. Phys. **3**, 363 (1999) (hep-th/9904017).

[87] M. Gremm and A. Kapustin, J. High En. Phys. 9907 (1999) 005 (hep-th/9904050).

[88] D.P. Jatkar and S. Randjbar-Daemi, Phys. Lett. **460B** 281 (1999) hep-th/9904187.

[89] M. Blau, E. Gava and K.S. Narain, J. High En. Phys. 9909 (1999) 018 (hep-th/9904179); E. Gava, K.S. Narain and M.H. Sarmadi, hep-th/9908125; E. Gava, A.B. Hammou, J.F. Morales and K.S. Narain, J. High En. Phys. **0103**, 035 (2001) [hep-th/0102043], hep-th/0201265.

[90] O. Aharony, S.S. Gubser, J. Maldacena, H. Ooguri and Y. Oz, Phys. Rep. **323**, 184 (2000) (hep-th/9905111).

[91] A. Ceresole, G. Dall'Agata and R. D'Auria, DFTT-40-99, hep-th/9907216.

[92] M. Bianchi, S. Kovacs, G. Rossi and Y.S. Stanev, J. High En. Phys. (8) U397 (1999), hep-th/9906188; Nucl. Phys. **B584** (2000) 216, hep-th/0003203; J. High En. Phys. (5) U960 (2001), hep-th/0104016.

[93] M. Berkooz and H. Verlinde, J. High En. Phys. 9911 (1999) 037 (hep-th/9907100).

[94] M. Alishahiha and Y. Oz, Phys. Lett. **465B** (1999) 136 (hep-th/9907206).

[95] S.S. Gubser, hep-th/9910117, hep-th/9912001.

[96] L. Andrianopoli, S. Ferrara, E. Sokatchev and B. Zupnik, Adv. Theor. Math. Phys. **3**, 1149 (1999), hep-th/9912007; S. Ferrara and E.

Sokatchev, hep-th/9912168; J. High En. Phys. **0005**, 038 (2000), hep-th/0003051; Int. J. Mod. Phys. **B14**, 2315 (2000), hep-th/0007058; J. Math. Phys. **42**, 3015 (2001), hep-th/0010117.

[97] O. Pelc, J. High En. Phys. (3) U216 (2000), (hep-th/0001054).

[98] R.R. Metsaev, hep-th/0002008; R.R. Metsaev, C.B. Thorn and A.A. Tseytlin, Nucl. Phys. **B596**, 151 (2001) [hep-th/0009171].

[99] R. Argurio, A. Giveon and A. Shomer, J. High En. Phys. **0004**, 010 (2000) [hep-th/0002104].

[100] J. Rasmussen, Nucl. Phys. **B582**, 649 (2000) [hep-th/0002188].

[101] N. Alonso-Alberca, P. Meessen and T. Ortin, Class. Quant. Grav. **17**, 2783 (2000) [hep-th/0003071].

[102] L. Castellani and L. Sommovigo, J. High En. Phys. **0007**, 044 (2000) [hep-th/0003102].

[103] E. D'Hoker, J. Erdmenger, D.Z. Freedman and M. Perez-Victoria, Nucl. Phys. **B589**, 3 (2000) [hep-th/0003218]; E. D'Hoker and D.Z. Freedman, hep-th/0201253.

[104] F. Bastianelli and R. Zucchini, J. High En. Phys. U1041 (2000), hep-th/0003230.

[105] E.E. Donets, A. Pashnev, V.O. Rivelles, D. Sorokin and M. Tsulaia, Phys. Lett. **484B**, 337 (2000) [hep-th/0004019].

[106] S. Cacciatori, D. Klemm, W.A. Sabra and D. Zanon, Nucl. Phys. **B587**, 277 (2000) [hep-th/0004077]; D. Klemm and W.A. Sabra, J. High En. Phys. (2) U632 (2001) (hep-th/0011016); D. Klemm, Fortsch. Phys. **49**, 581 (2001) [hep-th/0012214].

[107] S. Randjbar-Daemi, Invited talk at International Symposium on Quantum Theory and Symmetries, Goslar, Germany, 18-22 Jul 1999, Proceedings, *Quantum Theory and Symmetries*, eds. H.-D. Doebner et al, (World Sci, Singapore, 2000) 311; hep-th/0004204.

[108] J. Lee, Nucl. Phys. **B589**, 119 (2000) [hep-th/0005081].

[109] P. Heslop and P.S. Howe, Class. Quant. Grav. **17**, 3743 (2000), hep-th/0005135; TMR Conference, Paris, September 2000, hep-th/0009217; Phys. Lett. **516B**, 367 (2001), hep-th/0106238; hep-th/0107212.

[110] S. Ferrara and E. Sokatchev, Int. J. Theor. Phys. **40**, 935 (2001), hep-th/0005151.

[111] M. Billo, D. Fabbri, P. Fre, P. Merlatti and A. Zaffaroni, Nucl. Phys. **B591**, 139 (2000) [hep-th/0005220].

[112] M.J. Duff, J.T. Liu and K.S. Stelle, J. Math. Phys. **42**, 3027 (2001) hep-th/0007120.

[113] S. Randjbar-Daemi and M.E. Shaposhnikov, Phys. Lett. **491B** 329 (2000) hep-th/0008087, Phys. Lett. **492B** 361 (2000) hep-th/0008079.

[114] J.T. Liu and H. Sati, Nucl. Phys. **B605**, 116 (2001), hep-th/0009184.

[115] M. Grana and J. Polchinski, Phys. Rev. **D63**, 026001 (2001) [hep-th/0009211].

[116] D.Z. Freedman and P. Henry-Labordere, TMR Conference, Paris, 7-13 Sep 2000, hep-th/0011086.

[117] P.A. Grassi and P. van Nieuwenhuizen, Phys. Lett. **499B**, 174 (2001) [hep-th/0011278].

[118] S.P. Martin and J.D. Wells, Phys Rev **D64**, 036010 (2001), hep-ph/0011382.

[119] S. Deger, A. Kaya, E. Sezgin, P. Sundell and Y. Tanii, hep-th/0012139.

[120] P.C. Argyres, Nucl. Phys. Proc. Sup. **B101**, 175 (2001), hep-th/0102006.

[121] P.S. Howe and P.C. West, Class. Quant. Grav. **18**, 3143 (2001) [hep-th/0105218].

[122] G. Arutyunov, B. Eden, A.C. Petkou and E. Sokatchev, Nucl. Phys. **B620**, 380 (2002), hep-th/0103230; G. Arutyunov, B. Eden and E.

Sokatchev, Nucl. Phys. **B619**, 359 (2001), hep-th/0105254; B. Eden and E. Sokatchev, Nucl. Phys. **B618**, 259 (2001), hep-th/0106249.

[123] H. Nicolai and H. Samtleben, Phys. Lett. **514B**, 165 (2001) [hep-th/0106153].

[124] S. Fernando, M. Gunaydin and S. Takemae, hep-th/0106161.

[125] S. Penati and A. Santambrogio, Nucl. Phys. **B614**, 367 (2001), hep-th/0107071.

[126] B. Eden, S. Ferrara and E. Sokatchev, J. High En. Phys. 0111:020 (2001), hep-th/0107084.

[127] J. Kubo and D. Suematsu, Phys. Rev. **D64**, 115014 (2001), hep-ph/0107133.

[128] P. West, Nucl. Phys. Proc. Sup. **B101**, 112 (2001).

[129] P.J. Heslop, Class. Quant. Grav. **19**, 303 (2002), hep-th/0108235.

[130] A.V. Ryzhov, J. High En. Phys. 0111:046 (2001), hep-th/0109064.

[131] E. D'Hoker and A.V. Ryzhov, hep-th/0109065.

[132] S. Ferrara and E. Sokatchev, to be published in NJP Focus Issue: Supersymmetry in condensed matter and high-energy physics, hep-th/0110174.

[133] V.G. Kac and A. Rudakov, math-ph/0112022.

[134] F.A. Dolan and H. Osborn, hep-th/0112251.

[135] V.K. Dobrev, hep-th/0201076, INRNE-TH-02-01.

[136] M. Bagnoud, L. Carlevaro and A. Bilal, hep-th/0201183.

[137] J. Lukierski, hep-th/0201233.

[138] V.K. Dobrev, G. Mack, V.B. Petkova, S.G. Petrova and I.T. Todorov, *Harmonic Analysis on the n - Dimensional Lorentz Group and Its Applications to Conformal Quantum Field Theory*, Lecture Notes in Physics, Vol. 63 (Springer, 1977).

[139] W. Nahm, Nucl. Phys. **B135**, 149 (1978).

[140] R. Haag, J.T. Lopuszanski and M. Sohnius, Nucl. Phys. **B88**, 257 (1975).

[141] M. Flato and C. Fronsdal, Lett. Math. Phys. **8**, 159 (1984).
[142] V.K. Dobrev and V.B. Petkova, Lett. Math. Phys. **9**, 287 (1985).
[143] V.K. Dobrev and V.B. Petkova, Phys. Lett. **162B**, 127 (1985).
[144] V.K. Dobrev and V.B. Petkova, Fortschr. d. Phys. **35**, 537 (1987).
[145] V.K. Dobrev and V.B. Petkova, Proceedings, eds. A.O. Barut and H.-D. Doebner, Lecture Notes in Physics, Vol. 261 (Springer-Verlag, Berlin, 1986) p. 291 and p. 300.
[146] S. Minwalla, Adv. Theor. Math. Phys. **2**, 781 (1998).
[147] A.W. Knapp, *Representation Theory of Semisimple Groups (An Overview Based on Examples)*, (Princeton Univ. Press, 1986).
[148] V.K. Dobrev and V.B. Petkova, Rep. Math. Phys. **13** (1978) 233.
[149] V.K. Dobrev, Rep. Math. Phys. **25** (1988) 159.
[150] A.M. Polyakov, JETP Lett. **12** (1970) 381.
[151] S. Ferrara, A.F. Grillo, R. Gatto and G. Parisi, Lett. Nuovo Cim. **4** (1972) 115.
[152] A.A. Migdal, 4-dimensional soluble models of conformal field theory, preprint, Landau Inst. Theor. Phys., Chernogolovka (1972).
[153] E.S. Fradkin and M.Ya. Palchik, Nucl. Phys. **B99** (1975) 317.
[154] K. Koller, Comm. Math. Phys. **40** (1975) 15.
[155] A. Strominger, J. High En. Phys. **0110**, 034 (2001) [hep-th/0106113], J. High En. Phys. **0111**, 049 (2001) [hep-th/0110087]; M. Spradlin, A. Strominger and A. Volovich, hep-th/0110007; R. Bousso, A. Maloney and A. Strominger, hep-th/0112218.
[156] D. Klemm, Nucl. Phys. **B625**, 295 (2002) [hep-th/0106247].
[157] A.C. Petkou and G. Siopsis, hep-th/0111085.
[158] Y.S. Myung, hep-th/0112140, hep-th/0201176.
[159] V.K. Dobrev, Acta Appl. Math. **44** (1996) 81.
[160] V.G. Kac, Adv. Math. **26**, 8-96 (1977); Comm. Math. Phys. **53**, 31-64 (1977); the second paper is an adaptation for physicists of the first.
[161] V.G. Kac, Lect. Notes in Math. **676** (Springer-Verlag, Berlin, 1978) 597-626.

[162] M. Gunaydin and C. Saclioglu, Comm. Math. Phys. **87**, 159 (1982), Phys. Lett. **108B**, 180 (1982), (for algebras); I. Bars and M. Gunaydin, Comm. Math. Phys. **91**, 31 (1983); M. Gunaydin, J. Math. Phys. **29**, 1275 (1988) (for superalgebras).