(Super) String Field Theories

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1 Introduction.

One of the main motivations to construct string field theory (SFT) — an off-shell formulation of a string theory — was a hope to study non-perturbative phenomena in string theory [1]-[8]. The Witten SFT action[1, 2] for open bosonic string is

$$S = \frac{1}{2}\int \Phi \star Q\Phi + \frac{1}{3}\int \Phi \star \Phi \star \Phi. \tag{1.1}$$

Open bosonic string has a tachyon [3] that leads to a classical instability of the perturbative vacuum. In the early works by Kostelecky and Samuel [9] it was proposed to use SFT to describe condensation of the tachyon to a stable vacuum. By using the level truncation scheme they have shown that the tachyon potential in open bosonic string has a nontrivial minimum. Further [10], the level truncation method has been applied to study an effective potential of auxiliary fields in the cubic superstring field theory (SSFT) [11, 12, 13]. It was found that some of the low-lying auxiliary scalar fields acquire non-zero vacuum expectation values that was considered as an indication of supersymmetry breaking in this vacuum.

Later on, Sen has proposed [14, 15, 16] to interpret the tachyon condensation as a decay of unstable D-brane. In the framework of this interpretation the vacuum energy of the open bosonic string in the Kostelecky and Samuel vacuum cancels the tension of the unstable space-filling 25-brane. This cancellation has been checked with high accuracy [17]-[29].

The cubic open string field theory around the tachyon vacuum, or the vacuum string field theory (VSFT) was proposed in [30] and it is investigated now very intensively [31]-[74]. This VSFT has the same form as (1.1) but with a new differential operator, which after restriction to the space of string fields can be written in the following form

$$Q_{new} = Q + \{\Phi_{vac}, \ \cdot \ \}.$$

The triviality of the cohomology of the new BRST operator Q_{new} around the tachyon vacuum is expected [14]. This means that this theory should not contain physical open string excitations. The evidence in favor of this conjecture was also found by demonstrating the absence of kinetic term for the tachyon field in nonperturbative vacuum [23, 24]. VSFT is conjectured to describe the appearance of closed strings after the open string tachyon condensation. Moreover, it was suggested [30] that after some (maybe singular) field redefinition Q_{new} can be written as a pure ghost operator $\mathcal{Q}$. If this conjecture is true it is instructive to search solutions of VSFT equation of motion

$$\mathcal{Q}\Phi + \Phi \star \Phi = 0 \tag{1.2}$$

in the factorized form $\Phi = \Phi_{matter} \otimes \Phi_{ghost}$, thus obtaining a projector-like equation for matter sector:

$$\Phi_{matter} = \Phi_{matter} \star \ \Phi_{matter}. \tag{1.3}$$

An equation similar to (1.3) has appeared in construction of solitonic solutions in noncommutative field theories in the large coupling limit [75].

As an example of the projector-like solution, there is a state called sliver, $|\Xi\rangle$. This projector was constructed in the oscillator formalism by Kostelecky and Potting [33]. In fact a wedge state as a candidate to solve (1.3) was appeared first in the CFT framework [25]. Then it was at first identified with the sliver numerically [34] and later on by direct calculations [42]. The sliver state is special in a sense that it can be defined in an arbitrary boundary CFT [25].

The projector-like form of eq. (1.3) gave a new impulse for the development of the half-string formalism [1],[86]-[90], which drastically simplifies Witten's $\star$-product. To construct projections more systematically, it is useful to find a matrix representation of open string fields. To see matrix representation in a more transparent way it is fruitful to use the so-called "comma" form of the overlap vertex [87, 88],[39],[42]. The Bogoliubov transformation that makes the sliver to be a new vacuum requires the 3-string vertex to take a comma form [33]. The Bogoliubov transformation relating the sliver and initial Fock vacuum cannot be defined as an operator in the Fock space [49]. This is not surprising since we deal with infinite number of degrees of freedom. One can use the dressing transformation technique [76] to give to the sliver a meaning of a state in the Hilbert space. Just due to the infinite dimensional nature of the star product appear associativity anomaly [93, 94, 111] and twist anomaly [48].

Several tools are used to study VSFT:

1. the functional approach, where string fields are discussed in terms of left-right modes functionals, is developed in [36],[37],
2. the operator approach, where explicit Fock-space operators for left-right parts of the string are explored, is presented in [89].

Using these two descriptions the sliver states and their generalizations are studied in [35, 41, 43, 47].

In [30] a simple linear form for $\mathcal{Q} = c_o + \sum f_n(c_n + (-1)^n c_n^\dagger)$ was proposed. Under this assumption the ghost part of the equation admits an analytical investigation [45, 48, 49, 50, 32, 52]. Since an analytic tachyon vacuum solution is unknown, it is impossible to derive this form of VSFT action [30] from the initial Witten action (1.1). However, one can check if the proposed VSFT action passes some tests to be acceptable. In particular, in [44] a classical solutions of VSFT that reproduces the perturbative open string vacuum is constructed (see [48, 52]). Several attempts of getting closed strings from VSFT have been recently performed [51, 32].

This consideration concerns only bosonic cubic SFT. To study the same questions in the superstring case it is worth to work with a formalism which deals with equations of motions in the simple form (1.2) without any modifications. String fields in -1 picture [11], [12],[13] (see also [5]) give us a suitable formalism. We will discuss this formalism in Section 5.

Let us mention that there are several covariant string field theories. We collect them in the Table below.

Bosonic strings	Superstrings
1. Witten's Open Cubic SFT*	1. Modified Cubic SFT* a) 10D SUSY, GSO+ sector b) no SUSY, GSO+ and GSO$-$ sectors
2. Nonpolynomial Closed SFT	2. Nonpolynomial Super SFT
3. Background independent OSFT	3. Background independent OSSFT.

By * we mark the theories which we are going to discuss in details in these lectures (see also a review paper [6] and refs. therein about the others). [1]

In this lecture notes we would like to explain and discuss some ideas concerning string field theories. The material of the lectures is organized as follows. In Section 2, we explain basic mathematical notions of noncommutative differential geometry (NCDG). In Sections 3,4, we expose cubic string field theories focusing on the recent developments in this area. In Section 5, we consider the cubic superstring field theory.

2 Axioms of Noncommutative Differential Geometry.

In this section we sketch the basic ideas of the noncommutative geometry [95]. Noncommutative geometry uses a generalization of the known duality between

[1] Let us note relations between p-adic strings [137, 138, 139, 140, 141] and lump solutions in SFT [151, 152].

a space and its algebra of functions, see [95, 96, 97]. If one knows the associative commutative algebra $\mathcal{A}(M)$ of complex-valued functions on topological space M then one can restore the space M. Therefore, all topological notions can be expressed in terms of algebraic properties of $\mathcal{A}(M)$. For example, the space of continuous sections of a vector bundle over M can be regarded as a projective $\mathcal{A}(M)$-module. (We are speaking about left modules. A projective module is a module that can be embedded into a free module as a direct summand). So, the vector bundle over a compact space M can be identified with projective modules over $\mathcal{A}(M)$.

Let $\mathcal{A}$ be an abstract associative algebra with the multiplication $\star$

$$\star : \ \mathcal{A} \otimes \mathcal{A} \rightarrow \mathcal{A} \tag{2.1}$$

One can interpret $\mathcal{A}$ as an "algebra of functions" on a (nonexisting) noncommutative space.

Let us introduce some basic geometrical notions on this algebra $\mathcal{A}$: external derivative Q (or an algebra $\mathcal{G}$ of derivatives)

$$Q : \mathcal{A} \rightarrow \mathcal{A} \tag{2.2a}$$

and integral (linear map)

$$\int : \mathcal{A} \rightarrow \mathbb{C}. \tag{2.2b}$$

Noncommutative space is the set

$$\left(\mathcal{A}, Q, \int\right), \tag{2.2c}$$

which satisfies the following axioms:

$$\text{associativity} \quad A \star (B \star C) = (A \star B) \star C, \quad A, B, C \in \mathcal{A}; \tag{2.2d}$$

$$\text{nilpotency} \quad Q^2 = 0; \tag{2.2e}$$

$$\text{Leibnitz rule} \quad Q(A \star B) = (QA) \star B) + A \star (QB), \quad A, B \in \mathcal{A}; \tag{2.2f}$$

$$\text{cyclicity} \quad \int (A \star B) = \int (B \star A); \tag{2.2g}$$

$$\text{“integration by part”} \quad \int (QA) = 0. \tag{2.2h}$$

In the most applications the algebra $\mathcal{A}$ is also equipped with a natural $\mathbb{Z}_2$-grading: $|A \star B| = |A| + |B|$. One has to modify Leibnitz rule (2.2f) and cyclicity property (2.2g) to be in agreement with $\mathbb{Z}_2$-grading:

$$Q(A \star B) = (QA) \star B + (-1)^{|A|} A \star (QB), \quad A, B \in \mathcal{A}; \tag{2.2f$'$}$$

$$\int (A \star B) = (-1)^{|A||B|} \int (B \star A), \quad A, B \in \mathcal{A} \tag{2.2g$'$}$$

where $(-1)^{|A||B|} = -1$ if both A and B are odd elements and equal 1 otherwise.

A profit from such kinds of constructions is that we can immediately write an action that is invariant under gauge transformations. If we deal with the set of axioms (2.2e)-(2.2f$'$) we can define the gauge transformation as

$$\delta A = Q\Lambda + A \star \Lambda - \Lambda \star A, \quad A, \Lambda \in \mathcal{A}. \tag{2.3}$$

The gauge invariant action is [1]

$$S[A] = \frac{1}{2} \int A \star QA + \frac{1}{3} \int A \star A \star A. \tag{2.4}$$

This is a generalization of the Chern-Simons action. String field theory (SFT) presents an example of axioms (2.2). There is also a realization of axioms (2.2) on matrices [7].

3 Cubic String Field Theory on Conformal Language.

3.1 Vertex Operators.

By using the language of the conformal field theory (CFT) it is possible to represent each term in the String Field Theory action as some correlation function in CFT on a special two dimensional surface.

To specify our notations, let us remind some basic facts concerning two dimensional CFT. A general solution of the equation $(\partial_\tau^2 - \partial_\sigma^2) X^\mu = 0$ with Neumann boundary conditions is

$$X^{\mu}(\sigma,\tau) = x^{\mu} + 2\alpha' p^{\mu}\tau + i\left[\frac{\alpha'}{2}\right]^{1/2}\sum_{m\neq 0}\frac{\alpha_m^{\mu}}{m}e^{-im\tau}\left(e^{-im\sigma}+e^{im\sigma}\right), \qquad \text{where} \quad -\infty<\tau<\infty \quad 0\leqslant\sigma\leqslant\pi. \tag{3.1}$$

After Wick rotation we can introduce new complex variables $z = e^{\tau_E + i\sigma}$, $\overline{z} = e^{\tau_E - i\sigma}$. Note that this variables are coordinates on the upper-half complex plane. The solution (3.1) in this coordinates gets the following form

$$X^{\mu}(z,\bar{z}) = X_L^{\mu}(z) + X_R^{\mu}(\bar{z}), \tag{3.2}$$

where

$$X_L^{\mu}(z) = \frac{1}{2}x^{\mu} - \frac{i}{2}\alpha' p^{\mu}\log z^2 + i\left[\frac{\alpha'}{2}\right]^{1/2}\sum_{m\neq 0}\frac{\alpha_m^{\mu}}{mz^m} \quad \text{and} \quad X_R(\bar{z}) = X_L(\bar{z}). \tag{3.3}$$

The similar expansion reads for the ghost-antighost pair $c(z)$, $b(z)$ as

$$c(z) = \sum_n \frac{c_n}{z^{n-1}}, \quad b(z) = \sum_n \frac{b_n}{z^{n+2}}. \tag{3.4}$$

The Neumann correlation function for X's on the upper half-complex plane is of the from

$$\langle X^{\mu}(z,\overline{z})X^{\nu}(w,\overline{w})\rangle = -\frac{\alpha'}{2}\eta^{\mu\nu}\left(\log|z-w|^2 + \log|\overline{z}-w|^2\right). \tag{3.5}$$

But for computations it is fruitful to use the traditional shorthand

$$\langle X_L^{\mu}(z)X_L^{\nu}(w)\rangle = -\frac{\alpha'}{2}\eta^{\mu\nu}\,\log(z-w). \tag{3.6}$$

Note that this expression is really very formal. For example, when one wants to compute a correlation function involving $:e^{ik\cdot X(z,\bar{z})}:$ he encouraged to use the expression (3.5). But if the point $(z,\bar{z})$ is on the real axis (i.e. on the boundary of the UHP), then one can write the equality

$$:e^{ik\cdot X(z,\bar{z})}:\Big|_{z=\bar{z}} =: e^{2ik\cdot X_L(z)}:$$

and use the expression (3.6) to compute correlation functions. Indeed, one can check that these two differently computed correlators are the same while one assumes $z=\bar{z}$. In our calculations we will use this trick.

The ground state is given in terms of conformal fields as the normal ordered exponent:

$$|0,k\rangle =: e^{ik\cdot X(0,0)} : |0\rangle \equiv: e^{2ik\cdot X_L(0)} : |0\rangle, \tag{3.7}$$

and any term in the string field expansion corresponds to the conformal field via

$$\left[\prod_i \alpha^{\mu_i}_{m_i}\right] |0,k\rangle \Longleftrightarrow: \left[\prod_i \partial^{m_i} X^{\mu_i}(0)\right] e^{2ik\cdot X_L(0)} : |0\rangle. \tag{3.8}$$

If it does not lead to misunderstanding we also drop out subscript "L" and use a traditional shorthand $X(z)$

$$\langle X^{\mu}(z) X^{\nu}(w)\rangle = -\frac{\alpha'}{2}\eta^{\mu\nu}\,\log(z-w). \tag{3.9}$$

Another useful form of relation (3.6) is

$$X^{\mu}(z) X^{\nu}(w) \sim -\frac{\alpha'}{2}\eta^{\mu\nu}\,\log(z-w). \tag{3.10}$$

The symbol "$\sim$" means that l.h.s and r.h.s are equal up to regular in $z-w$ terms. For ghosts we have the following OPE

$$c(z)b(w) \sim \frac{1}{z-w}. \tag{3.11}$$

The stress tensor for the matter fields

$$T_X = -\frac{1}{\alpha'}\partial X \cdot \partial X \tag{3.12}$$

has the following OPE

$$T_X(z)T_X(w) \sim \frac{26}{2(z-w)^4} + \frac{2}{(z-w)^2}T_X(w) + \frac{1}{z-w}\partial T_X(w). \tag{3.13}$$

The stress tensor for the ghost fields and the reparametrization algebra are

$$T(z) = -2b\partial c(z) - 1\partial b c(z), \tag{3.14}$$

$$T(z)b(w) \sim \frac{2}{(z-w)^2}b(w) + \frac{1}{z-w}\partial b(w), \tag{3.15}$$

$$T(z)c(w) \sim \frac{-1}{(z-w)^2}c(w) + \frac{1}{z-w}\partial c(w), \tag{3.16}$$

$$T(z)T(w) \sim \frac{-13}{(z-w)^4} + \dots . \tag{3.17}$$

The system has a $U(1)$ number current

$$j(z) = -bc(z) = \sum_n \frac{j_n}{z^{n+1}}, \qquad j(z)j(w) \sim \frac{1}{(z-w)^2}, \tag{3.18}$$

$$j(z)b(w) \sim -\frac{1}{z-w}b(w), \qquad j(z)c(w) \sim \frac{1}{z-w}c(w), \tag{3.19}$$

which charge operator j_0 counts $c = +1$, $b = -1$ charge.

The conformal properties of the current depend on the vacuum being chosen. In the SL_2 invariant vacuum sector $\langle c(z)b(w)\rangle = 1/(z-w)$, then the proper relation is

$$T(z)j(w) \sim \frac{-3}{(z-w)^3} + \frac{1}{(z-w)^2}j(z), \tag{3.20}$$

$$[L_m, j_n] = -nj_{m+n} + \frac{-3}{2}m(m+1)\delta_{m+n,0}, \tag{3.21}$$

so that $j(z)$ is scale and translation covariant ($m = 0, -1$) but not conformal covariant ($m = 1$). Next commutation relations

$$[L_1, j_{-1}] = j_0 - 3, \qquad [L_1, j_{-1}]^\dagger = [L_{-1}, j_1] = -j_0 \tag{3.22}$$

show the charge asymmetry of the system: $j_0^\dagger = -j_0 + 3$. Consequently, operator expectation values of charge neutral operators will vanish since

$$[j_0, \mathcal{O}] = q\mathcal{O}, \qquad j_0|0\rangle = 0, \qquad \langle 0|j_0^\dagger = -Q\langle 0|, \tag{3.23}$$

$$\langle 0|j_0\mathcal{O}|0\rangle = -Q\langle 0|\mathcal{O}|0\rangle = q\langle 0|\mathcal{O}|0\rangle. \tag{3.24}$$

Only operators which cancel the background charge -3 survive. For example,

$$\langle c(z_1)c(z_2)c(z_3)\rangle = -(z_1-z_2)(z_2-z_3)(z_3-z_1). \tag{3.25}$$

3.2 Gluing and Conformal Maps.

There is a method that allows one to represent n-string interactions via correlation functions on special Riemann surfaces R_n (the so-called string configuration).

$$\int \Phi_1 \star \cdots \star \Phi_n = \langle V_n|\Phi_1\rangle_1 \otimes \cdots \otimes |\Phi_n\rangle_n = \langle \Phi_1, \ldots, \Phi_n\rangle_{R_n}$$

Efficiency of this method is a possibility to reduce calculations of correlation functions on n-string configuration to calculations of correlation functions on the upper

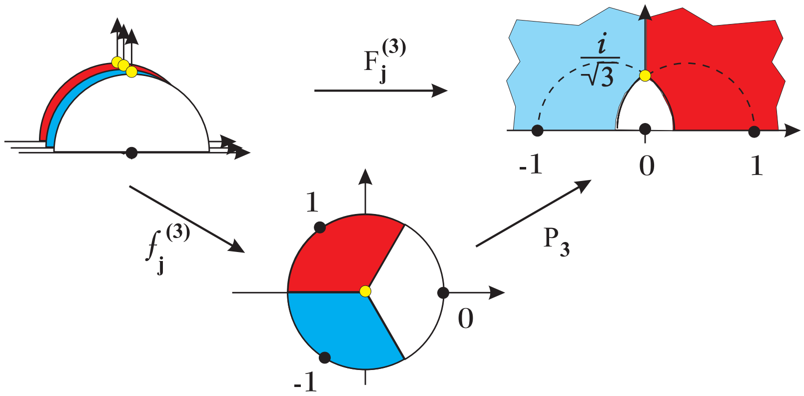

Figure 1: The maps for Witten's vertex, R_3.

half-disk, or the upper half-plane using the equality

$$\langle \Phi_1, \ldots, \Phi_n \rangle_{R_n} = \langle F_1^{(n)} \circ \Phi_1 \ \ldots \ F_n^{(n)} \circ \Phi_n \rangle. \tag{3.26}$$

The maps $F_k^{(n)}$ $(k = 1 \ldots n)$ are defined as follows:

$$F_k^{(n)}(w) = (P_n \circ f_k^{(n)})(w), \tag{3.27a}$$

$$f_k^{(n)}(w) = e^{\frac{2\pi i}{n}(\mu(n)-k)} \left(\frac{1+iw}{1-iw} \right)^{\frac{2}{n}}, \ P_2(z) = i\frac{1-z}{1+z}, \ P_3(z) = \frac{i}{\sqrt{3}} \frac{1-z}{1+z} \tag{3.27b}$$

$$\text{where} \quad \mu(n) = \begin{cases} \frac{n+1}{2} & n = \text{odd}, \\ \frac{n+2}{2} & n = \text{even}. \end{cases} \tag{3.27c}$$

Further we consider only the cases $n = 2$ and $n = 3$. The maps $f_k^{(3)}$ can be rewritten in the form $f_k^{(3)} = T^{k-1} \circ f_1^{(3)}$, where T is the operator of rotation over $-\frac{2\pi}{3}$. This property of the maps as it was shown in [106] is very important to have threefold cyclically symmetric vertex. One can be puzzled why we choose different maps P_2 and P_3 and may think that it gives extra relative factor between the brackets with $n = 2$ and $n = 3$. But fortunately this never happens, because Witten's vertex is invariant under the conformal group [107], so one can choose arbitrary functions P_2 and P_3.

We illustrate the maps (3.27) on the Figure 1. Here we have three half discs with two marked points. Into the black point we usually insert the vertex operators corresponding to the states of the string. The grey point is the center of the string (we insert picture changing operators into this point in the cubic superstring field theory). Also on the Figure 1 we illustrate where half discs and marked points maps under the transformations (3.27).

Note that the choice (3.27b) of the map $f_k^{(n)}$ from the upper half disc to a sector of disc differs from usual one. But this choice is very convenient for computer calculations, since it should not evaluate powers of negative numbers. The maps for $n=2$ have very simple form:

$$f_1^{(2)}(w) = -\frac{1}{w}, \qquad f_2^{(2)}(w) = w. \tag{3.28}$$

$(f \circ \Phi)$ in (3.26) means the conformal transform of Φ by f. Let us explain this in more details. If Φ is a primary field of conformal weight h, then $f \circ \Phi(z)$ is given by

$$(f \circ \Phi)(w) = (f'(w))^h \Phi(f(w)). \tag{3.29}$$

For a derivative of a primary field of conformal weight h one has

$$(f \circ \partial\Phi)(w) = (f'(w))^{h+1} \partial\Phi(f(w)) + h f''(w)(f'(w))^{h-1}\Phi(f(w)). \tag{3.30}$$

Since we work with free conformal fields, all the operators in the theory can be constructed as normal product of these free fields. Therefore, we have to know how to find the conformal transformations of the composite operators. Let us illustrate this technique on the example of bc-current. The operators b and c are primary ones with weights 2 and -1 respectively, so (3.29) yields

$$(f \circ b)(w) = [f'(w)]^2\, b(f(w)) \quad \text{and} \quad (f \circ c)(w) = [f'(w)]^{-1}\, c(f(w)). \tag{3.31}$$

The bc-current j_{bc} is defined by the following expressions

$$j_{bc}(w) = -:b(w)c(w) := -\lim_{z\to w}\left(b(z)c(w) - \frac{1}{z-w}\right), \tag{3.32}$$

where in the latter equality we have used the OPE of b with c. Now we apply a conformal map f to both sides of the equality (3.32):

$$(f \circ j_{bc})(w) = f'(w)\, j_{bc}(f(w)) + \frac{3}{2}\frac{f''(w)}{f'(w)}. \tag{3.33}$$

So we get the conformal transformation of the composite operator j_{bc} using only OPE and conformal transforms of the operators from which it is built. This scheme can be applied to find a conformal transform of any composite operator.

EXERCISE: Show that the stress energy tensor of the bosonic string $T_X = -\frac{1}{\alpha'}\partial X\cdot\partial X$ changes under conformal transformations as

$$(f\circ T_X)(w) = f'^2 T_X(f) + \frac{26}{12}\left(\frac{f'''}{f'} - \frac{3}{2}\frac{f''^2}{f'^2}\right). \tag{3.34}$$

EXERCISE: Check that

$$(f\circ\partial(bc))(w) = f'^2\partial(bc)(f) + f''b(f)c(f) + \frac{3}{2}\left(\frac{f'''}{f'} - \frac{f''^2}{f'^2}\right),$$

$$(f\circ\partial bc)(w) = f'^2\partial b(f)c(f) + 2f''b(f)c(f) + \frac{5}{6}\frac{f'''}{f'} + \frac{1}{4}\frac{f''^2}{f'^2},$$

$$(f\circ b\partial c)(w) = f'^2 b(f)\partial c(f) - f''b(f)c(f) + \frac{2}{3}\frac{f'''}{f'} - \frac{7}{4}\frac{f''^2}{f'^2}.$$

For future use we list Taylor's series of maps (3.27) in the origin:

$$f_1^{(3)}(w) = 1 + 2\gamma w + 3\gamma^2 w^2 + \frac{31}{8}\gamma^3 w^3 + \frac{39}{8}\gamma^4 w^4 + \frac{813}{128}\gamma^5 w^5 + O(w^6), \tag{3.35a}$$

$$f_2^{(3)}(w) = \frac{1}{2}\gamma w - \frac{5}{32}\gamma^3 w^3 + \frac{57}{512}\gamma^5 w^5 + O(w^7), \tag{3.35b}$$

$$f_3^{(3)}(w) = -1 + 2\gamma w - 3\gamma^2 w^2 + \frac{31}{8}\gamma^3 w^3 - \frac{39}{8}\gamma^4 w^4 + \frac{813}{128}\gamma^5 w^5 + O(w^6). \tag{3.35c}$$

Here $\gamma = \frac{4}{3\sqrt{3}}$.

3.3 Calculations of the Action for Tachyon and Vector Fields.

We want to compute Witten's action

$$S[\Phi] = -\frac{1}{g_o^2}\left[\frac{1}{2}\langle\langle\Phi,\, Q_B\Phi\rangle\rangle + \frac{1}{3}\langle\langle\Phi,\,\Phi,\,\Phi\rangle\rangle\right] \tag{3.36}$$

for tachyon and vector string fields

$$\Phi(w) = \int\frac{d^{26}k}{(2\pi)^{26}}\left[t(k)\mathrm{V}_t(k,w) + A_\mu(k)\mathrm{V}_v^\mu(k,w) + B(k)\mathrm{V}_B(k,w)\right], \tag{3.37}$$

where vertex operators V are of the form

$$\begin{aligned}
\mathrm{V}_t(w,k) &= :c(z)e^{2ik\cdot X(w)}:, \\
\mathrm{V}_v^\mu(w,k) &= i\left[\frac{2}{\alpha'}\right]^{1/2} :c(w)\partial X^\mu(w)e^{2ik\cdot X(w)}:, \\
\mathrm{V}_B(w,k) &= :\partial c(w)e^{2ik\cdot X(w)}: .
\end{aligned} \tag{3.38}$$

All the coefficients in (3.37) are chosen in such a way that the reality condition $\Phi^\dagger = \Phi$ for string field produces reality conditions for component fields.

3.3.1 Action of the BRST charge.

First of all we need to calculate the action of the BRST charge on the vertex operators. The action of the BRST charge on the vertex of the auxiliary field is the following:

$$Q_B \mathrm{V}_B(w,k) = \operatorname*{Res}_{\zeta=w}(cT_B + bc\partial c)(\zeta)(\partial c e^{2ik\cdot X})(w) = \partial_w(c\partial c e^{2ik\cdot X})(w). \tag{3.39}$$

The action of the BRST charge on the tachyon vertex is the following:

$$Q_B \mathrm{V}_t(w,k) = [-\alpha' k^2 + 1](c\partial c e^{2ik\cdot X})(w). \tag{3.40}$$

And the action of the BRST charge on the massless vector vertex is the following:

$$Q_B \mathrm{V}_v^\mu(w,k) = i\left[\frac{2}{\alpha'}\right]^{\frac{1}{2}}\left[\frac{i\alpha' k^\mu}{2}c\partial^2 c + (-\alpha' k^2)c\partial c\partial X^\mu\right]e^{2ik\cdot X(w)}. \tag{3.41}$$

It is convenient to rewrite $Q_B\Phi$ in the following way

$$\begin{aligned}
(Q_B\Phi)(0) = \int \frac{d^{26}k}{(2\pi)^{26}}\Big[t(k)(-\alpha' k^2+1)\,c\partial c e^{2ik\cdot X(0)} + B'\partial(c\partial c e^{2ik\cdot X(0)}) \\
+\sqrt{2\alpha'}A_\mu(k)(\eta^{\mu\nu}k^2 - k^\mu k^\nu)c\partial c\partial X^\nu e^{2ik\cdot X(0)}\Big],
\end{aligned} \tag{3.42}$$

where

$$B'(k) = B(k) - \left[\frac{\alpha'}{2}\right]^{\frac{1}{2}} ik^\mu A_\mu(k). \tag{3.43}$$

3.3.2 Conformal transformations.

The operator for the tachyon field is primary, therefore, its conformal transformation is the following

$$f\circ \mathrm{V}_t(w,k) = [f'(w)]^{\alpha' k^2 - 1}\,\mathrm{V}_t(f(w),k). \tag{3.44}$$

The operator for the auxiliary field is transformed as follows

$$f \circ \mathrm{V}_B(w,k) = [f'(w)]^{\alpha' k^2} \left(\mathrm{V}_B(f(w),k) - \frac{f''}{f'^2} c(f(w)) e^{2ik\cdot X(f(w))} \right). \quad (3.45)$$

The operator for the vector field is transformed in the following way

$$f \circ \mathrm{V}_v^\mu(w,k) = [f'(w)]^{\alpha' k^2} \left(\mathrm{V}_v^\mu(f(w),k) + \left[\frac{\alpha'}{2}\right]^{\frac{1}{2}} k^\mu \frac{f''}{f'^2} c e^{2ik\cdot X}(f(w)) \right) \quad (3.46)$$

From (3.46) and (3.45) follows that the operator

$$\mathrm{V}_V^\mu(w,k) = \mathrm{V}_v^\mu(w,k) + \left[\frac{\alpha'}{2}\right]^{\frac{1}{2}} k^\mu \mathrm{V}_B(w,k) \quad (3.47)$$

is the primary operator of weight $\alpha' k^2$.

It is also convenient to rewrite $f \circ \Phi$ in the following form

$$(f\circ\Phi)(w) \overset{3.47}{=} \int \frac{d^{26}p}{(2\pi)^{26}} [f'(w)]^{\alpha' p^2} \left[t(p)[f'(w)]^{-1} c(f(w)) - \left[\frac{2}{\alpha'}\right]^{\frac{1}{2}} A_\mu(p)[c\partial X^\mu \right.$$
$$\left. -i\frac{\alpha' p^\mu}{2}\partial c](f(w)) + B'\left(\partial c(f(w)) - \frac{f''}{f'^2} c(f(w))\right)\right] e^{2ip\cdot X(f(w))}. \quad (3.48)$$

3.3.3 Calculations of kinetic terms.

Substituting (3.48) and (3.42) in the quadratic part of the action (3.36), expanding the obtained expression and using the conservation of momenta and the fact that $\eta^{\mu\nu}k^2 - k^\mu k^\nu$ is orthogonal to k_μ we obtain

$$\langle f \circ \Phi\, Q_B \Phi(0)\rangle = \int \frac{d^{26}k}{(2\pi)^{26}\alpha'^{13}} \frac{f'^{\alpha' p^2}}{f^{2\alpha' p^2}} \left[t(-k)t(k)\frac{f^2}{f'}[-\alpha' k^2 + 1] \right.$$
$$+ t(-k)B'(k)(\alpha' k^2 - 1)(\frac{2f}{f'} - 2f + f^2\frac{f''}{f'^2}) + 2B'(-k)B'(k)\times$$
$$\left. \times \left[1 + (2 - \frac{ff''}{f'^2})(\alpha' k^2 - 1)\right] + \alpha'(\eta^{\mu\nu}k^2 - k^\mu k^\nu)A_\nu(-k)A_\mu(k)\right]. \quad (3.49)$$

Recall, that the map $f(w)$ (see also (3.28)) is defined as follows

$$f(w) = -\frac{1}{w}, \qquad f'(w) = \frac{1}{w^2}, \qquad f''(w) = -\frac{2}{w^3}. \quad (3.50)$$

Substituting this map in (3.49) and putting $w = 0$ we get the following expression for the quadratic action

$$S[t, A_\mu, B] = \frac{1}{g_o^2 \alpha'^{13}} \int \frac{d^{26}k}{(2\pi)^{26}} \left[\frac{1}{2} t(-k)t(k)[-\alpha' k^2 + 1] + B'(-k)B'(k) \right.$$
$$\left. -\frac{\alpha'}{2}(\eta^{\mu\nu}k^2 - k^\mu k^\nu)A_\nu(-k)A_\mu(k) \right]. \quad (3.51)$$

3.3.4 Calculation of interaction.

Here we compute the cubic term of the action (3.36). First, we compute the interaction term for three tachyons. To this end we substitute only the part of the string field Φ which is proportional to $t(k)$:

$$\frac{1}{3}\langle\langle \Phi,\, \Phi,\, \Phi \rangle\rangle = \frac{1}{3}\langle f_1 \circ \Phi\, f_2 \circ \Phi\, f_3 \circ \Phi \rangle = -\frac{1}{3} \int \frac{d^{26}k_1}{(2\pi)^{26}} \frac{d^{26}k_2}{(2\pi)^{26}} \frac{d^{26}k_3}{(2\pi)^{26}} \times$$
$$\times\, t(k_1)t(k_2)t(k_3)(2\pi)^{26}\alpha'^{-13}\delta(p+k+q) f_1'^{\,\alpha' k_1^2 - 1} f_2'^{\,\alpha' k_2^2 - 1} f_3'^{\,\alpha' k_3^2 - 1} \times$$
$$\times\, (f_1 - f_2)^{2\alpha' k_1 \cdot k_2 + 1}(f_1 - f_3)^{2\alpha' k_1 \cdot k_3 + 1}(f_2 - f_3)^{2\alpha' k_2 \cdot k_3 + 1}. \quad (3.52)$$

Substituting the maps (3.35) into this expression and performing simplification one gets

$$S_{int}[t] = -\frac{1}{g_o^2 \alpha'^{13}} \int \frac{d^{26}k_1 d^{26}k_2 d^{26}k_3}{(2\pi)^{26}(2\pi)^{26}} \delta(k_1 + k_2 + k_3)\, \frac{1}{3} t(k_1)t(k_2)t(k_3)\, \gamma^{\alpha' k_1^2 + \alpha' k_2^2 + \alpha' k_3^2 - 3}. \quad (3.53)$$

Next we compute the term which consists of two vector fields and one tachyon. So, we compute the following odd bracket

$$\langle\langle \mathrm{V}_t(k_1),\, \mathrm{V}_V^\mu(k_2),\, \mathrm{V}_V^\nu(k_3) \rangle\rangle = (2\pi)^{26}\alpha'^{-13}\delta(k_1 + k_2 + k_3)[\eta^{\mu\nu} + \alpha' k_2^\nu k_1^\mu + \alpha' k_3^\mu k_1^\nu]$$
$$\times\, f_1'^{\,\alpha' k_1^2 - 1} f_2'^{\,\alpha' k_2^2} f_3'^{\,\alpha' k_3^2} (f_1 - f_2)^{2\alpha' k_1 k_2 + 1}(f_1 - f_3)^{2\alpha' k_1 k_3 + 1}(f_2 - f_3)^{2\alpha' k_2 k_3 - 1}. \quad (3.54)$$

To compute the whole interaction term for the tachyon and two vector fields we need to subtract from (3.54) the same terms with permutation $(f_1 \leftrightarrow f_2)$ and $(f_1 \leftrightarrow f_3)$. After substitution of the maps (3.35) we get the following action

$$S_{int}[t, A] = -\frac{1}{g_o^2 \alpha'^{13}} \int \frac{d^{26}k_1 d^{26}k_2 d^{26}k_3}{(2\pi)^{26}(2\pi)^{26}} \delta(k_1 + k_2 + k_3)$$
$$\times\, t(k_1)A_\mu(k_2)A_\nu(k_3)\, [\eta^{\mu\nu} + \alpha' k_1^\mu k_2^\nu + \alpha' k_3^\mu k_1^\nu]\gamma^{\alpha' k_1^2 + \alpha' k_2^2 + \alpha' k_3^2}. \quad (3.55)$$

The cubic term of the action (3.36) for whole string field Φ (3.48) is more complicate. It was computed using the program on Maple, and the answer is the following

$$S_{int}[t, A_\mu, B'] = \frac{1}{g_o^2 \alpha'^{13}} \int \frac{d^{26}k_1 d^{26}k_2 d^{26}k_3}{(2\pi)^{26}(2\pi)^{26}} \gamma^{\alpha'(k_1^2+k_2^2+k_3^2)} \Big[-\gamma^{-3}\, t(k_1)t(k_2)t(k_3)$$
$$-\gamma^{-1}\, A_\beta(k_1)A_\mu(k_2)t(k_3)\,(\eta^{\mu\beta} + \alpha' k_2^\beta k_3^\mu + \alpha' k_1^\mu k_3^\beta)$$
$$+\gamma^{-1}\, B'(k_1)B'(k_2)t(k_3) + 2\gamma^{-1} \left[\frac{\alpha'}{2}\right]^{\frac{1}{2}} B'(k_1) i k_1^\mu A_\mu(k_2)t(k_3)\Big]. \quad (3.56)$$

This expression can also be rewritten in x-representation:

$$S_{int}[t, A_\mu, B'] = \frac{1}{g_o^2 \alpha'^{13}} \int d^{26}x \Big[-\frac{1}{3}\gamma^{-3}\, \tilde{t}\tilde{t}\tilde{t} - \gamma^{-1}\, \tilde{t}\tilde{A}_\mu \tilde{A}^\mu + 2\gamma^{-1}\alpha'\, \partial^\mu \tilde{t} \partial^\beta \tilde{A}_\mu \tilde{A}_\beta$$
$$+\gamma^{-1}\, \tilde{t}\tilde{B}'\tilde{B}' + 2\gamma^{-1} \left[\frac{\alpha'}{2}\right]^{\frac{1}{2}} \tilde{t}\tilde{B}'\partial^\mu \tilde{A}_\mu\Big]. \quad (3.56')$$

At all levels the particle fields ϕ in the interaction action are smeared over the distance $\sqrt{\alpha'}$:

$$\tilde{\phi}(x) = \exp\left[-\alpha' \log\gamma\, \partial_\mu \partial^\mu\right]\ \phi(x). \quad (3.57)$$

3.4 Sliver.

Rastelli and Zwiebach [25] have found a family of string fields generating a commutative subalgebra of the open string star algebra:

$$|n\rangle \star |m\rangle = |n+m-1\rangle. \quad (3.58)$$

The state $|n\rangle$ is defined by the following requirement for any state $|\phi\rangle$

$$\langle n|\phi\rangle = \langle f_n \circ \phi(0)\rangle, \quad (3.59)$$

where [34]

$$f_n(z) = \frac{n}{2} \tan\left(\frac{2}{n}\tan^{-1}(z)\right). \quad (3.60)$$

This family of states is called wedge states, because the half-disks representing their local coordinates could be considered as wedges of the full unit disks.

For $n = 1$ the wedge corresponds to the identity state $|I\rangle$ and for $n = 2$ the wedge corresponds to the vacuum $|0\rangle$. In the $n \to \infty$ limit the map (3.60) is

$$f(z) \equiv f_\infty(z) = \tan^{-1}(z) \quad (3.61)$$

and this wedge state has the property

$$|\Xi\rangle \star |\Xi\rangle = |\Xi\rangle. \tag{3.62}$$

This wedge state is called sliver. It was found [34] that the sliver $|\Xi\rangle$ can be written in the factorized form

$$|\Xi\rangle = |\Xi_m\rangle \otimes |\Xi_g\rangle\,. \tag{3.63}$$

The matter part of (3.63) can be defined using the CFT technique [106]

$$|\Xi_m\rangle \propto \exp\Big(-\frac{1}{2}\eta_{\mu\nu}a^{\mu\dagger}\cdot S\cdot a^{\nu\dagger}\Big)|0\rangle\,, \tag{3.64}$$

where

$$S_{mn} = -\frac{1}{\sqrt{mn}}\oint\frac{dw}{2\pi i}\frac{dz}{2\pi i}\frac{1}{z^n w^m(1+z^2)(1+w^2)(\tan^{-1}(z)-\tan^{-1}(w))^2}. \tag{3.65}$$

First Rastelli, Sen and Zwiebach [34] have compared numerically the sliver (3.64) with the matter part of the solution found by Kostelecky and Potting [33] and have got an evidence that these two solutions are the same. Later on these two states have been identified by direct calculations [42].

4 Cubic Super String Field Theory.

4.1 Fermions.

The action for the open superstring in the conformal gauge has the form

$$S = \frac{1}{2\pi\alpha'}\int_{-\infty}^{\infty}d\tau\int_0^{\pi}d\sigma\left[\frac{1}{2}\partial_a X^\mu\partial^a X_\mu - \frac{i}{2}\bar{\Psi}^\mu\gamma^a\partial_a\Psi_\mu\right]. \tag{4.1}$$

Here Ψ is a Majorana spinor and $\bar{\Psi} = \Psi^T\gamma^0$. We introduce the explicit representation of Euclidean Dirac γ-matrices:

$$\gamma^0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \gamma^1 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}. \tag{4.2}$$

The spinor $\Psi^\mu(\sigma,\tau)$ has two components

$$\Psi^\mu(\sigma,\tau) = \begin{pmatrix} \psi^\mu(\sigma,\tau) \\ \tilde{\psi}^\mu(\sigma,\tau) \end{pmatrix}.$$

Performing matrix multiplications in (4.1) and changing the variables we get the following action for fermions

$$S = -\frac{1}{2\pi\alpha'}\int d^2z\, [\psi^\mu\overline{\partial}\psi_\mu + \tilde{\psi}^\mu\partial\tilde{\psi}_\mu]. \tag{4.3}$$

Since $\Psi^\mu(\sigma,\tau)$ is spinor, we can impose the following boundary conditions (the first one can always be reached by redefinition of the fields)

$$\psi^\mu(\pi,\tau) = \tilde{\psi}^\mu(\pi,\tau), \qquad \psi^\mu(0,\tau) = \pm\tilde{\psi}^\mu(0,\tau).$$

For "+" the sector is called Ramond (R), and for "−" the sector is called Neveu-Schwartz (NS). Therefore, the solution of the equations of motion (merging ψ and $\tilde{\psi}$) is of the form

$$\psi^\mu(z) = \sum_{n\in\mathbb{Z}+r} \frac{\psi^\mu_n}{z^{n+1/2}}, \tag{4.4}$$

where $r = 1/2$ for R-sector, and $r = 0$ for NS-sector (z is the same as in Sect. (3.1)).

Following the ordinary quantization procedure we get the following commutation relations

$$\{\psi^\mu_m, \psi^\nu_n\} = \eta^{\mu\nu}\delta_{m+n,0}. \tag{4.5}$$

The correlation function is

$$\langle\psi^\mu(z)\psi^\nu(w)\rangle = -\frac{\alpha'}{2}\eta^{\mu\nu}\frac{1}{z-w}. \tag{4.6}$$

4.2 Superghosts, Bosonization.

Consider a general first order action [99]

$$S = \frac{1}{2\pi}\int d^2z\, \mathbf{b}\overline{\partial}\mathbf{c}, \tag{4.7}$$

where $\mathbf{b}$ and $\mathbf{c}$ denote general conjugate fields of dimension λ and $1-\lambda$ respectively; they can be either Bose or Fermi fields[2].

The $\mathbf{b},\mathbf{c}$ operator product expansion is

$$\mathbf{c}(z)\mathbf{b}(w) \sim \frac{1}{z-w}, \qquad \mathbf{b}(z)\mathbf{c}(w) \sim \frac{\varepsilon}{z-w}. \tag{4.8}$$

[2]The formulae in the end of Sec 5.1 are a special case of the general formulae presented in this section.

Here and in the sequel, $\varepsilon = +1$ for Fermi statistics and $\varepsilon = -1$ for Bose statistics. The fields have the following mode expansions and hermitian properties

$$\mathbf{c}(z) = \sum_n \frac{\mathbf{c}_n}{z^{n+1-\lambda}}, \quad \mathbf{c}_n^\dagger = \mathbf{c}_{-n}, \mathbf{b}(z) = \sum_n \frac{\mathbf{b}_n}{z^{n+\lambda}}, \mathbf{b}_n^\dagger = \varepsilon \mathbf{b}_{-n}. \tag{4.9}$$

The OPE (4.8) determines the (anti)commutation relations

$$\mathbf{c}_m \mathbf{b}_n + \varepsilon \mathbf{b}_n \mathbf{c}_m = \delta_{m+n,0}. \tag{4.10}$$

There are NS and R sectors of the theory, specified by

NS	R
$\mathbf{b}_n,\ n \in \mathbb{Z} - \lambda$	$\mathbf{b}_n,\ n \in \frac{1}{2} + \mathbb{Z} - \lambda$
$\mathbf{c}_n,\ n \in \mathbb{Z} + \lambda$	$\mathbf{c}_n,\ n \in \frac{1}{2} + \mathbb{Z} + \lambda$

The stress tensor and reparametrization algebra are

$$T(z) = -\lambda \mathbf{b}\partial\mathbf{c}(z) + (1-\lambda)\partial\mathbf{b}\mathbf{c}(z), \tag{4.11}$$

$$T(z)\mathbf{b}(w) \sim \frac{\lambda}{(z-w)^2}\mathbf{b}(w) + \frac{1}{z-w}\partial\mathbf{b}(w), \tag{4.12}$$

$$T(z)\mathbf{c}(w) \sim \frac{1-\lambda}{(z-w)^2}\mathbf{c}(w) + \frac{1}{z-w}\partial\mathbf{c}(w), \tag{4.13}$$

$$T(z)T(w) \sim \frac{c/2}{(z-w)^4} + \ldots, \qquad c = \varepsilon(1-3Q^2), \qquad Q = \varepsilon(1-2\lambda). \tag{4.14}$$

Special cases are the reparametrization ghost algebra with $\varepsilon = 1$, $\lambda = 2$, $Q = -3$ and $c = -26$; and the local superconformal ghost algebra with $\varepsilon = -1$, $\lambda = 3/2$, $Q = 2$ and $c = 11$.

The system has a global $U(1)$ current

$$j(z) = -\mathbf{b}\mathbf{c}(z) = \sum_n \frac{j_n}{z^{n+1}}, \qquad j(z)j(w) \sim \frac{\varepsilon}{(z-w)^2}, \tag{4.15}$$

$$j(z)\mathbf{b}(w) \sim -\frac{1}{z-w}\mathbf{b}(w), \qquad j(z)\mathbf{c}(w) \sim \frac{1}{z-w}\mathbf{c}(w), \tag{4.16}$$

which charge operator j_0 counts $\mathbf{c} = +1$, $\mathbf{b} = -1$ charge. The conformal properties of the current depend on the vacuum being chosen. In the SL_2 invariant vacuum

$\langle \mathbf{c}(z)\mathbf{b}(w)\rangle = 1/(z-w)$, the proper relation is

$$T(z)j(w) \sim \frac{Q}{(z-w)^3} + \frac{1}{(z-w)^2}j(z), \tag{4.17a}$$

$$[L_m, j_n] = -nj_{m+n} + \frac{Q}{2}m(m+1)\delta_{m+n,0}. \tag{4.17b}$$

So $j(z)$ is scale and translation covariant ($m = 0, -1$), but not conformal covariant ($m = 1$). Next commutation relations

$$[L_1, j_{-1}] = j_0 + Q, \qquad [L_1, j_{-1}]^\dagger = [L_{-1}, j_1] = -j_0, \tag{4.18}$$

show the charge asymmetry of the system: $j_0^\dagger = -j_0 - Q$. Consequently, operator expectation values of charge neutral operators will vanish since

$$[j_0, \mathcal{O}] = q\mathcal{O}, \qquad j_0|0\rangle = 0, \qquad \langle 0|j_0^\dagger = -Q\langle 0|,$$
$$\langle 0|j_0\mathcal{O}|0\rangle = -Q\langle 0|\mathcal{O}|0\rangle = q\langle 0|\mathcal{O}|0\rangle.$$

Only operators which cancel the background charge Q survive.

The $U(1)$ number current may be used to build a stress energy tensor that reproduces the relation (4.17a)

$$T_j(z) = -\frac{\varepsilon}{2}(-j^2(z) + Q\partial j(z)), \tag{4.19a}$$

$$T_j(z)j(w) \sim \frac{Q}{(z-w)^3} + \frac{j(z)}{(z-w)^2}, \tag{4.19b}$$

$$T_j(z)T_j(w) \sim \frac{1-3\varepsilon Q^2}{2(z-w)^4} + \dots \tag{4.19c}$$

Thus the new stress tensor has $c_j = 1 - 2\varepsilon Q^2$; the original $\mathbf{b}$, $\mathbf{c}$ stress tensor has

$$\begin{aligned} c &= c_j, & \varepsilon &= 1, & &\text{(Fermi)} \\ c &= c_j - 2, & \varepsilon &= -1, & &\text{(Bose)} \end{aligned}$$

To reproduce correct central charge in the Bose case one has to add to (4.19a) a proper Fermi system with central charge -2 such that

$$T = T_j + T_{-2}. \tag{4.20}$$

More precisely, this has to be Fermi system with $\lambda_{-2} = 1$ and $Q_{-2} = -1$ composed of a dimension 1 field $\eta(z)$ and dimension 0 field $\xi(z)$.

Now let us discuss bosonization of $\mathbf{b}, \mathbf{c}$-system. To this end, we introduce bosonic field ϕ such that

$$j(z) = \varepsilon \partial\phi(z), \qquad \phi(z)\phi(w) \sim \varepsilon \log(z-w). \tag{4.21}$$

This means that the stress energy tensor of the field ϕ is of the form

$$T_\phi = -\frac{\varepsilon}{2}\partial\phi\partial\phi - \frac{Q}{2}\partial^2\phi.$$

The main object in bosonization construction is a normal ordered exponent $e^{q\phi(z)}$. It has the following OPEs with stress-energy tensor T_ϕ and $U(1)$-current j

$$T_\phi(z)e^{q\phi(w)} = \left[\frac{\frac{\varepsilon}{2}q(q+Q)}{(z-w)^2} + \frac{1}{z-w}\partial_w\right]e^{q\phi(w)}, \tag{4.22a}$$

$$j(z)e^{q\phi(w)} \sim \frac{q}{z-w}e^{q\phi(w)} \quad \Rightarrow \quad [j_0, e^{q\phi(w)}] = qe^{q\phi(w)}, \tag{4.22b}$$

$$e^{q\phi(z)}e^{q'\phi(w)} \sim (z-w)^{\varepsilon qq'}e^{q\phi(z)+q'\phi(w)}. \tag{4.22c}$$

So $e^{q\phi(w)}$ has j_0-charge equal to q and conformal weight $\frac{\varepsilon}{2}q(q+Q)$. From (4.22c) it follows that $e^{q\phi(z)}$ for odd q behaves like anticommuting operator. So the main bosonization formula are[3]

Fermi	Bose
$\mathbf{c}(z) = e^{\phi(z)} \equiv e^{\sigma(z)}$	$\mathbf{c}(z) \equiv \gamma = \eta(z)e^{\phi(z)}$
$\mathbf{b}(z) = e^{-\phi(z)} \equiv e^{-\sigma(z)}$	$\mathbf{b}(z) \equiv \beta = e^{-\phi(z)}\partial\xi(z)$

The (η, ξ)-system has its own conserved current $\xi\eta$. Using this current we can construct a new one, which is called the picture current and is of the form

$$j_p(z) = -\partial\phi - \xi\eta.$$

In Table 1 we collect the assignments of the ghost number n_g and the picture number n_p for the conformal operators described above. Let us note that $SL(2,\mathbb{R})$ invariant vacuum has zero ghost and picture numbers.

Correlation functions are to be computed in the following normalization

$$^*\langle\xi(z)c\partial c\partial^2 c(w)e^{-2\phi(y)}\rangle = -2.$$

[3]There are more general formula [85]

field	n_g	n_p	h	field	n_g	n_p	h
b	-1	0	2	$e^{q\phi}$	0	q	$-\frac{1}{2}q(q+2)$
c	1	0	-1	ξ	-1	1	0
γ	1	0	-1/2	η	1	-1	1
β	-1	0	3/2				

Table 1: Ghost number n_g, picture number n_p and conformal weight h assignments

It is a good point to note here, that the above expression is written in the so-called large Hilbert space. The problem is that in the bosonization formula we have used only $\partial\xi$ rather then ξ, therefore, the zero mode ξ_0 is not important for bosonization. Moreover the Hilbert space, which can be associated in a standard way with the bosonized superghosts, does not form an irreducible representation of the superghost algebra. One can make it irreducible by excluding the mode ξ_0. In the succeeding sections we will assume this restriction and correlation functions will be normalized as follows

$$\langle c\partial c\partial^2 c(w) e^{-2\phi(y)}\rangle = -2, \quad \text{where} \quad \langle 0| = {}^*\langle 0|\xi_0.$$

4.3 Problems with Action in -1 Picture.

The original Witten's proposal [2] for NSR superstring field theory action reads:

$$S_W \cong \int \mathcal{A} \star Q_{NS}\mathcal{A} + \frac{2}{3}\int \mathrm{X}\mathcal{A}\star\mathcal{A}\star\mathcal{A} + \int Y(i)\Psi\star Q_R\Psi + 2\int \Psi\star\mathcal{A}\star\Psi. \quad (4.23)$$

Here Q_{NS} and Q_R are the BRST charges in NS and R sectors, $\int$ and $\star$ are Witten's string integral and star product to be specified below. States in the extended Fock space $\mathcal{H}$ are created by the modes of the matter fields X^μ and ψ^μ, conformal ghosts b, c and superghosts β, γ:

$$\mathcal{A} = \sum_{m,j,i\in\mathbb{Z}+\frac{1}{2}} A_{i\ldots}(x)\,\beta_i\ldots\gamma_j\ldots b_k\ldots c_l\ldots\alpha_n^\mu\ldots\psi_m^\nu|0\rangle_{-1}, \quad (4.24)$$

$$\Psi = \sum_{m,j,i\in\mathbb{Z}} \Psi^A{}_{i\ldots}(x)\,\beta_i\ldots\gamma_j\ldots b_k\ldots c_l\ldots\alpha_n^\mu\ldots\psi_m^\nu|A\rangle_{-\frac{1}{2}}. \quad (4.25)$$

The characteristic feature of the action (4.23) is the choice of the -1 picture for the string field $\mathcal{A}$. The vacuum $|0\rangle_{-1}$ in the NS sector is defined as

$$|0\rangle_{-1} = c(0)e^{-\phi(0)}|0\rangle, \tag{4.26}$$

where $|0\rangle$ stands for $SL(2,\mathbb{R})$-invariant vacuum. In the description of the open NSR superstring the string field $\mathcal{A}$ is subjected to be GSO+.

The vacuum $|A\rangle_{-\frac{1}{2}}$ in the R sector is defined as

$$|A\rangle_{-\frac{1}{2}} = c(0)e^{-\frac{1}{2}\phi(0)}S_A(0)|0\rangle, \tag{4.27}$$

where $S_A(z)$ is a spin operator of weight $\frac{5}{8}$.

The insertion of the picture-changing operator [98]

$$\mathrm{X} = \frac{1}{\alpha'}e^{\phi}\psi\cdot\partial X + c\partial\xi + \frac{1}{4}b\partial\eta e^{2\phi} + \frac{1}{4}\partial(b\eta e^{2\phi}) \tag{4.28}$$

in the cubic term is aimed to absorb the unwanted unit of the ϕ charge as only $\langle 0|e^{-2\phi}|0\rangle \neq 0$.

The action (4.23) suffers from the contact term divergencies [109, 110, 111, 112] which arise when a pair of X-s collides in a point. This sort of singularities appears already at the tree level.

Really, the tree level graphs are generated by solving the classical equation of motion by perturbation theory. For Witten's action (4.23) the equation reads

$$Q_B\mathcal{A} + \mathrm{X}\mathcal{A}\star\mathcal{A} = 0.$$

Here $Q_B = Q_{NS}$. The first nontrivial iteration (4-point function) involving the pair of $\mathrm{X}A\star A$ vertices produces the contact term singularity when two of X-s collide in a point.

4.4 Action in 0 Picture and Double Step Inverse Picture Changing Operator.

To overcome the troubles that we have discussed in the previous subsection it was proposed to change the picture of NS string fields from -1 to 0, i.e. to replace $|0\rangle_{-1}$ in (4.24) by $|0\rangle$ [11, 12]. States in the -1 picture can be obtained from the states in the 0 picture by the action of the inverse picture-changing operator Y [98]

$$Y = 4c\partial\xi e^{-2\phi}(w)$$

with $\mathrm{X}Y = Y\mathrm{X} = 1$. This identity holds outside the ranges $\ker \mathrm{X}(w)$ and $\ker Y(w)$ [117]. These kernels are:

$$\ker X(w) = \{\mathcal{O}(w) \mid \lim_{z\to w} X(w)\mathcal{O}(z) = 0\}, \quad \ker Y(w) = \{\mathcal{O}(w) \mid \lim_{z\to w} Y(w)\mathcal{O}(z) = 0\}.$$

This means that $\ker X(w)$ and $\ker Y(w)$ depend on a point w. In particular, the following statement is true: for all operators $\mathcal{O}(z)$, $z \neq w$ the product $X(w)\mathcal{O}(z) \neq 0$.

The existence of these local kernels leads to the fact that at the 0 picture there are states that can not be obtained by applying the picture changing operators X and Y to the states at the -1 picture.

The action for the NS string field in the 0 picture has the cubic form with the insertion of a double-step inverse picture-changing operator Y_{-2} [11, 12][4]:

$$S \cong \int Y_{-2}\mathcal{A} \star Q_B\mathcal{A} + \frac{2}{3}\int Y_{-2}\mathcal{A} \star \mathcal{A} \star \mathcal{A}. \tag{4.29}$$

We discuss Y_{-2} in the next subsection.

In the 0 picture there is a variety of auxiliary fields as compared with the -1 picture. These fields become zero due to the free equation of motion: $Q_B\mathcal{A} = 0$, but they play a significant role in the off-shell calculations. For instance, a low level off-shell NS string field expands as

$$\begin{aligned}\mathcal{A} \cong \int dk\ \{&u(k)c_1 - \frac{1}{2}A_\mu(k)ic_1\alpha^\mu_{-1} - \frac{1}{4}B_\mu(k)\gamma_{\frac{1}{2}}\psi^\mu_{-\frac{1}{2}} \\ &+ \frac{1}{2}F_{\mu\nu}(k)c_1\psi^\mu_{-\frac{1}{2}}\psi^\nu_{-\frac{1}{2}} + B(k)c_0 + r(k)c_1\gamma_{\frac{1}{2}}\beta_{-\frac{3}{2}} + \dots\}e^{ik\cdot X(0,0)}|0\rangle.\end{aligned} \tag{4.30}$$

Here u and r are just the auxiliary fields mentioned above.

The SSFT based on the action (4.29) is free from the drawbacks of the Witten's action (4.23). The absence of contact singularities can be explained shortly. The action (4.29) yields the following equation

$$Y_{-2}(Q_B\mathcal{A} + \mathcal{A} \star \mathcal{A}) = 0. \tag{4.31}$$

[4]One can cast the action into the same form as the action (2.4) for the bosonic string if one modifies the NS string integral accounting the "measure" Y_{-2}: $\int' = \int Y_{-2}$.

Since operator Y_{-2} is inserted in the mid point in the bulk and $\mathcal{A}$ is an operator in a point on the boundary we can drop out Y_{-2} and write the following equation

$$Q_B\mathcal{A} + \mathcal{A} \star \mathcal{A} = 0. \tag{4.32}$$

So the interaction vertex does not contain any insertion leading to singularity. The complete proofs of this fact can be found in [11, 12, 13].

To have well defined SSFT (4.29) the double step inverse picture changing operator must be restricted to be

a) in accord with the identity[5]:

$$Y_{-2}\mathrm{X} = \mathrm{X}Y_{-2} = Y, \tag{4.33}$$

b) BRST invariant: $[Q_B, Y_{-2}] = 0$,

c) scale invariant conformal field, i.e. conformal weight of Y_{-2} is 0,

d) Lorentz invariant conformal field, i.e. Y_{-2} does not depend on momentum.

The point a) provides the formal equivalence between the improved (4.29) and the original Witten (4.23) actions. The point b) allows us "to move" BRST charge from one string field to another. The point c) is necessary to make the insertion of Y_{-2} compatible with the $\star$-product. And the point d) is necessary to preserve unbroken Lorentz symmetry.

As it was shown in paper [12], there are two (up to BRST equivalence) possible choices for the operator Y_{-2}.

The first operator, the chiral one [11], is built from holomorphic fields in the upper half plane and is given by

$$Y_{-2}(z) = -4e^{-2\phi(z)} - \frac{16}{5\alpha'}e^{-3\phi}c\partial\xi\psi\cdot\partial X(z). \tag{4.34}$$

The identity (4.33) for this operator reads

$$Y_{-2}(z)\mathrm{X}(z) = \mathrm{X}(z)Y_{-2}(z) = Y(z).$$

This Y_{-2} is uniquely defined by the constraints a) – d).

The second operator, the nonchiral one [12], is built from both holomorphic and antiholomorphic fields in the upper half plane and is of the form:

$$Y_{-2}(z,\overline{z}) = Y(z)Y(\overline{z}). \tag{4.35}$$

[5]We assume that this equation is true up to BRST exact operators.

Here by $Y(\overline{z})$ we denote the antiholomorphic field $4c(\overline{z})\overline{\partial}\xi(\overline{z})e^{-2\phi(\overline{z})}$. For this choice of Y_{-2} the identity (4.33) takes the form:

$$Y_{-2}(z,\overline{z})\mathrm{X}(z) = \mathrm{X}(z)Y_{-2}(z,\overline{z}) = Y(\overline{z}).$$

One can find the inverse operators W to $Y_{-2}(z,\overline{z})$ and $Y_{-2}(z)$

$$W(z,\overline{z})Y_{-2}(z,\overline{z}) = 1,\; W(z,\overline{z}) = X(z)X(\overline{z}); \tag{4.36a}$$

$$W(z)Y_{-2}(z) = 1, \qquad W(z) = X^2(z) + \alpha_1 W_1(z) + \alpha_2 W_2(z) + \alpha_3 W_3(z), \tag{4.36b}$$

where $W_i(z) = \{Q, \Omega_i(z)\}$, Ω_i is given by

$$\Omega_1 = (\partial b)\partial e^{2\phi}, \quad \Omega_2 = (\partial^2 b)e^{2\phi}, \quad \Omega_3 = b\partial^2 e^{2\phi} \tag{4.37}$$

and α_i are the following numbers

$$\alpha_1 = -\frac{19}{192},\; \alpha_2 = -\frac{5}{48},\; \alpha_3 = -\frac{1}{96}. \tag{4.38}$$

The issue of equivalence between the theories based on chiral or nonchiral insertions still remains open. The first touch to the problem was performed in [108]. It was shown that the actions for low-level space-time fields are different depending on insertion being chosen.

4.5 SSFT in Conformal Language.

As for the bosonic string (see Sec. 3.2) for SSFT calculations it is convenient to employ the tools of CFT [106]. In the conformal language $\int$ and $\star$-product mentioned above are replaced by the odd bracket $\langle\langle \ldots | \ldots \rangle\rangle$, defined as follows

$$\begin{aligned}\langle\langle Y_{-2}|A_1,\ldots,A_n\rangle\rangle &= \Big\langle P_n \circ Y_{-2}(0,0)\, f_1^{(n)} \circ A_1(0) \ldots f_n^{(n)} \circ A_n(0) \Big\rangle_{R_n} \\ &= \Big\langle f_j^{(n)} \circ Y_{-2}(i,\overline{i})\, f_1^{(n)} \circ A_1(0) \ldots f_n^{(n)} \circ A_n(0) \Big\rangle_{R_n}, \quad n=2,3.\end{aligned} \tag{4.39}$$

Here r.h.s. contains $SL(2,\mathbb{R})$-invariant correlation function in CFT on string configuration surface R_n described in Section 3.2. A_j $(j = 1,\ldots,n)$ are vertex operators and $\{f_j^{(n)}\}$ are given by eq.(3.27b)

Y_{-2} is the double-step inverse picture changing operator (4.35) inserted in the center of the unit disc. This choice of the insertion point is very important, since all the functions $f_j^{(n)}$ maps the points i (the middle points of the individual strings) to the same point that is the origin. In other words, the origin is a unique common point for all strings (see Figure 1). The next important fact is the zero weight of the operator Y_{-2}, so its conformal transformation is very simple $f \circ Y_{-2}(z,\overline{z}) = Y_{-2}(f(z),\overline{f}(\overline{z}))$. Due to this property it can be inserted in any string. This note shows that the definition (4.39) is self-consistent and does not depend on a choice of a string on which we insert Y_{-2}.

Due to the Neumann boundary conditions, there is a relation between holomorphic and antiholomorphic fields. So it is convenient to employ a doubling trick (see details in [4]). Therefore, $Y_{-2}(z,\overline{z})$ can be rewritten in the following form[6]

$$Y_{-2}(z,\overline{z}) = Y(z)Y(z^*).$$

Here $Y(z)$ is the holomorphic field and z^* denotes the conjugated point of z with respect to a boundary, i.e. for the unit disc $z^* = 1/\overline{z}$ and $z^* = \overline{z}$ for the upper half plane. From now on we work only on the whole complex plane. Hence the odd bracket takes the form

$$\langle\langle Y_{-2}|A_1,\dots,A_n\rangle\rangle = \Big\langle\, Y(P_n(0))Y(P_n(\infty))\, f_1^{(n)} \circ A_1(0)\dots f_n^{(n)} \circ A_n(0)\Big\rangle\,. \quad (4.40)$$

To summarize, the action we start with reads

$$S[\mathcal{A}] = \frac{1}{g_o^2}\left[\frac{1}{2}\langle\langle Y_{-2}|\mathcal{A},Q_B\mathcal{A}\rangle\rangle + \frac{1}{3}\langle\langle Y_{-2}|\mathcal{A},\mathcal{A},\mathcal{A}\rangle\rangle\right], \quad (4.41)$$

where g_o is a dimensionless coupling constant.

4.6 Restriction on String Fields.

The aim of this section is to try to reduce the number of excitations appearing in the zero picture string field. Such reductions can be made if we have degenerate quadratic part of the action. But the reduction we describe is not of this sort. In this construction we use only the fact that the expectation value of the operator on the complex plane in super conformal two dimensional field theory is non-vanishing only if it compensates all background charges of the theory.

[6] Because of this formula, the operator $Y_{-2}(z,\overline{z})$ is sometimes called bilocal.

Let us decompose the string field $\mathcal{A}$ according to the ϕ-charge q:

$$\mathcal{A} = \sum_{q\in\mathbb{Z}} \mathcal{A}_q, \qquad \mathcal{A}_q \in V_q,$$

where

$$[j_0,\, \mathcal{A}_q] = q\mathcal{A}_q \quad \text{with} \quad j_0 = \frac{1}{2\pi i}\oint d\zeta\, \partial\phi(\zeta). \tag{4.42}$$

The BRST charge Q_B has also the natural decomposition over ϕ-charge:

$$Q_B = Q_0 + Q_1 + Q_2. \tag{4.43}$$

Since ${Q_B}^2 = 0$ we get the identities:

$${Q_0}^2 = 0,\ \{Q_0,\, Q_1\} = 0,\ \{Q_1,\, Q_2\} = 0,\ {Q_2}^2 = 0,\ \{Q_0,\, Q_2\} + {Q_1}^2 = 0. \tag{4.44}$$

The non-chiral inverse double step picture changing operator Y_{-2} has ϕ-charge equal to -4. Therefore to be non zero the expression in the brackets $\langle\langle Y_{-2}|\ldots\rangle\rangle$ must have ϕ-charge equal to $+2$. Hence the quadratic S_2 and cubic S_3 terms of the action (2.4) read:

$$S_2 = -\frac{1}{2}\sum_{q\in\mathbb{Z}} \left(\langle\langle Y_{-2}|(\mathcal{A}_{2-q},\, Q_0\mathcal{A}_q + \mathcal{A}_{1-q},\, Q_1\mathcal{A}_q + \mathcal{A}_{-q},\, Q_2\mathcal{A}_q)\rangle\rangle\right). \tag{4.45a}$$

$$S_3 = -\frac{1}{3}\sum_{q,\,q'\in\mathbb{Z}} \langle\langle Y_{-2}|\mathcal{A}_{2-q-q'}, \mathcal{A}_{q'}, \mathcal{A}_q\rangle\rangle. \tag{4.45b}$$

We see that all the fields $\mathcal{A}_q$, $q \neq 0, 1$, give only linear contribution to the quadratic action (4.45a). We propose to exclude such fields. We will consider the action (4.45) with fields that belong to the space $V_0 \oplus V_1$ only. To make this prescription meaningful we have to check that the restricted action is gauge invariant.

The action restricted to subspaces V_0 and V_1 takes the form

$$S_{2,\,\mathrm{r}} = -\frac{1}{2}\langle\langle Y_{-2}|\mathcal{A}_0,\, Q_2\mathcal{A}_0\rangle\rangle - \langle\langle Y_{-2}|\mathcal{A}_0,\, Q_1\mathcal{A}_1\rangle\rangle - \frac{1}{2}\langle\langle Y_{-2}|\mathcal{A}_1,\, Q_0\mathcal{A}_1\rangle\rangle, \tag{4.46a}$$

$$S_{3,\,\mathrm{r}} = -\langle\langle Y_{-2}|\mathcal{A}_0, \mathcal{A}_1, \mathcal{A}_1\rangle\rangle. \tag{4.46b}$$

The action (4.46) has a nice structure. One sees that since the charge Q_2 does not contain zero modes of stress energy tensor the fields $\mathcal{A}_0$ play a role of auxiliary fields. On the contrary, all fields $\mathcal{A}_1$ are physical ones, i.e. they have non-zero kinetic terms.

Let us now check that the action (4.46) has gauge invariance. The action (4.41) is gauge invariant under

$$\delta\mathcal{A} = Q_B\Lambda + [\mathcal{A},\, \Lambda]. \tag{4.47}$$

But after the restriction to the space $V_0 \oplus V_1$ it might be lost. Decomposing the gauge parameter Λ over ϕ-charge $\Lambda = \sum\limits_q \Lambda_q$ we rewrite the gauge transformation (4.47) as:

$$\delta\mathcal{A}_q = Q_0\Lambda_q + Q_1\Lambda_{q-1} + Q_2\Lambda_{q-2} + \sum_{q'\in\mathbb{Z}}[A_{q-q'},\, \Lambda_{q'}]. \tag{4.48}$$

Assuming that $\mathcal{A}_q = 0$ for $q \neq 0, 1$ from (4.48) we get

$$\delta\mathcal{A}_{-2} = Q_0\Lambda_{-2}; \tag{4.49a}$$

$$\delta\mathcal{A}_{-1} = Q_0\Lambda_{-1} + [\mathcal{A}_0,\, \Lambda_{-1}] + Q_1\Lambda_{-2} + [\mathcal{A}_1,\, \Lambda_{-2}]; \tag{4.49b}$$

$$\delta\mathcal{A}_0 = Q_0\Lambda_0 + [\mathcal{A}_0,\, \Lambda_0] + Q_1\Lambda_{-1} + [\mathcal{A}_1,\, \Lambda_{-1}] + Q_2\Lambda_{-2}; \tag{4.49c}$$

$$\delta\mathcal{A}_1 = Q_0\Lambda_1 + [\mathcal{A}_0,\, \Lambda_1] + Q_1\Lambda_0 + [\mathcal{A}_1,\, \Lambda_0] + Q_2\Lambda_{-1}; \tag{4.49d}$$

$$\delta\mathcal{A}_2 = Q_1\Lambda_1 + [\mathcal{A}_1,\, \Lambda_1] + Q_2\Lambda_0; \tag{4.49e}$$

$$\delta\mathcal{A}_3 = Q_2\Lambda_1. \tag{4.49f}$$

To make the restriction consistent with transformations (4.49) the variations of the fields $\mathcal{A}_{-2}$, $\mathcal{A}_{-1}$, $\mathcal{A}_2$ and $\mathcal{A}_3$ must be zero. Since the gauge parameters cannot depend on string fields we must put $\Lambda_{-2} = \Lambda_{-1} = \Lambda_1 = 0$. So we are left with the single parameter Λ_0, but to have zero variation of $\mathcal{A}_2$ we must require in addition $Q_2\Lambda_0 = 0$. Therefore the gauge transformations take the form

$$\begin{aligned} \delta\mathcal{A}_0 &= Q_0\Lambda_0 + [\mathcal{A}_0,\, \Lambda_0], \\ \delta\mathcal{A}_1 &= Q_1\Lambda_0 + [\mathcal{A}_1,\, \Lambda_0], \quad \text{with} \quad Q_2\Lambda_0 = 0. \end{aligned} \tag{4.50}$$

It is easy to check that (4.50) form a closed algebra. It is also worth to note that the restriction $Q_2\Lambda_0 = 0$ leaves the gauge transformation of the massless vector field unchanged.

4.7 Table of notations, correlation functions and OPE.

$X_L^\mu(z)X_L^\nu(w) \sim -\frac{\alpha'}{2}\eta^{\mu\nu}\log(z-w)$	$\partial X^\mu(z)\partial X^\nu(w) \sim -\frac{\alpha'}{2}\eta^{\mu\nu}\frac{1}{(z-w)^2}$
$\psi^\mu(z)\psi^\nu(w) \sim -\frac{\alpha'}{2}\eta^{\mu\nu}\frac{1}{z-w}$	

$$c(z)b(w) \sim b(z)c(w) \sim \tfrac{1}{z-w} \qquad \gamma(z)\beta(w) \sim -\beta(z)\gamma(w) \sim \tfrac{1}{z-w}$$
$$\phi(z)\phi(w) \sim -\log(z-w) \qquad \xi(z)\eta(w) \sim \eta(z)\xi(w) \sim \tfrac{1}{z-w}$$
$$\langle c(z_1)c(z_2)c(z_3)\rangle = -(z_1-z_2)(z_2-z_3)(z_3-z_1) \qquad \langle e^{-2\phi(y)}\rangle = 1$$
$$\left\langle e^{i2p_\alpha X_L^\alpha(y)-i2b_iX_L^i(y)}e^{i2k_\beta X_L^\beta(z)+i2b_jX_L^j(z)}\right\rangle = (2\pi)^{p+1}{\alpha'}^{-\frac{p+1}{2}}\delta(p+k)(y-z)^{-2\alpha'(k^2+b^2)}$$

$$T_B = -\tfrac{1}{\alpha'}\partial X\cdot\partial X - \tfrac{1}{\alpha'}\partial\psi\cdot\psi \qquad T_F = -\tfrac{1}{\alpha'}\partial X\cdot\psi$$
$$T_{bc} = -2b\partial c - \partial bc \qquad T_{\beta\gamma} = -\tfrac{3}{2}\beta\partial\gamma - \tfrac{1}{2}\partial\beta\gamma$$
$$T_\phi = -\tfrac{1}{2}\partial\phi\partial\phi - \partial^2\phi \qquad T_{\eta\xi} = \partial\xi\eta$$

$$T_B(z)T_B(w) \sim \tfrac{15}{2(z-w)^4} + \tfrac{2}{(z-w)^2}T_B(w) + \tfrac{1}{z-w}\partial T_B(w)$$
$$T_B(z)T_F(w) \sim \tfrac{3/2}{(z-w)^2}T_F(w) + \tfrac{1}{z-w}\partial T_F(w)$$
$$T_F(z)T_F(w) \sim \tfrac{5/2}{(z-w)^3} + \tfrac{1/2}{z-w}T_B(w)$$

$$\gamma = \eta e^{\phi} \qquad \beta = e^{-\phi}\partial\xi$$
$$T_{\beta\gamma} = T_\phi + T_{\eta\xi} \qquad \gamma^2 = \eta\partial\eta e^{2\phi}$$

$$\phi - \text{charge:} \quad \left[\tfrac{1}{2\pi i}\oint d\zeta\partial\phi(\zeta),\, \mathcal{A}_q\right] = q\mathcal{A}_q$$

$$Q_B = \tfrac{1}{2\pi i}\oint d\zeta\ \left[c(T_B + T_{\beta\gamma} + \tfrac{1}{2}T_{bc}) + \tfrac{1}{\alpha'}\gamma\psi\cdot\partial X - \tfrac{1}{4}b\gamma^2\right]$$
$$Q_B = \tfrac{1}{2\pi i}\oint d\zeta\ \left[c(T_B + T_\phi + T_{\eta\xi} + \tfrac{1}{2}T_{bc}) + \tfrac{1}{\alpha'}\eta e^{\phi}\psi\cdot\partial X + \tfrac{1}{4}b\partial\eta\eta e^{2\phi}\right]$$

$$\mathrm{X} = \tfrac{1}{\alpha'}e^{\phi}\psi\cdot\partial X + c\partial\xi + \tfrac{1}{4}b\partial\eta e^{2\phi} + \tfrac{1}{4}\partial(b\eta e^{2\phi})$$
$$Y = 4c\partial\xi e^{-2\phi}$$

5 Cubic (Super)String Field Theory on Branes and Sen Conjectures.

5.1 Sen's Conjecture.

As we have seen in Section 5 there is a tachyon mode in the spectrum of bosonic open string. There is also a tachyon mode in GSO$-$ sector of fermionic string. Normally we impose GSO$+$ projection on superstring spectrum and obtain tachyon free spectrum. But there are objects in the string theory such as non-BPS branes or brane-anti-brane pairs, which necessarily contain both GSO$+$ and GSO$-$ sector [15]. Therefore, it becomes important to explore the tachyon phenomenon. The existence of the tachyon mode does not necessarily signify a sickness of the theory, but may simply indicate an existence of a ground state with the energy density lower than in the starting configuration. One can compare this situation with Higgs phenomenon. The Higgs field has a potential looking like a Mexican hat. On the top of the potential the Higgs field has negative mass square, while at the bottom it has positive mass square.

To find a new vacuum in the theory of strings one has to be able to calculate the effective tachyon potential, that is a functional on a constant field configuration. Since a non zero constant tachyon field is far away from its on-shell value one has to use an off-shell formulation of string theory to do this. As we know from sections 4-6 string filed theories give such off-shell formulations of the string theory. There are also so-called background independent string filed theories (BSFT) which are also used to calculate the effective tachyon potential [131, 133, 134].

First attempts to find a stable vacuum in the bosonic string using SFT was made by Kostelecky and Samuel [9]. They have computed tachyon potential using level truncation scheme and shown that there is another vacuum, in which there is no tachyonic state.

About two years ago A. Sen [130] suggested to interpret tachyon condensation as a decay of unstable D-branes. Let us consider one non-BPS D9-brane. As it is mentioned above the string attached to this brane has a tachyon in the spectrum. For this case, the Sen's conjecture says [15]:

1. The tachyon potential $\mathcal{V}_c(t)$ looks like Mexican hat with minimum at the point t_c. The difference $\mathcal{V}(0) - \mathcal{V}(t_c)$ is precisely the tension $\tilde{\tau}_9$ of the non-BPS D9-brane.

2. The theory around the new vacuum does not contain any open string excitation. Therefore, this new vacuum can be also considered as a vacuum of a closed string.

3. All D-branes in type IIA (IIB) string theory can be regarded as classical solutions in the open string field theory living on a system of non-BPS D9-branes (or D9-anti-D9-brane pair).

In this section we will show how one can use SFT to check these conjectures. We will mainly concentrate on the first conjecture and will say only few words about others. Before proceed let us summarize what was obtained in the literature.

The first Sen's conjectures was numerically verified almost in all SFTs listed in the Introduction. In Table below we collect results of the computations of the ratio $(\mathcal{V}(0)-\mathcal{V}(t_c))/\tau$performed in different SFTs using the level truncation scheme.

SFT Type	level	number of the fields	$\frac{\mathcal{V}(0)-\mathcal{V}(t_c)}{\tau}$
Cubic SFT	10	102	0.9991
BI SFT	—	1	1 (exact)
Nonpolynomial SSFT	4	70	0.944
Modified Cubic SSFT	2	10	1.058
BI SSFT	—	1	1 (exact)

These results were presented in [19], [131], [7], [77] and [134] correspondingly. See also [81], [82], [7] about calculations of the tachyon potential in nonpolynomial SSFT [79], [80] and [83] about calculations in the Witten cubic SSFT. One sees that Background Independent SFTs give exact results for the minimum of the tachyon potential. The formulation of Background Independent SFT can be found in [118, 131], and for further developments related to tachyon see [121, 119, 120, 122, 123, 124, 132, 125, 126, 127, 128, 129].

The second Sen's conjecture was also verified almost in all SFTs [135, 136, 142, 143, 144, 145, 146, 147, 148, 149] and [150]. For superstrings the solution representing, for example, a decay non-BPS D9 $\to$ BPS D8 ($\bar{\text{D}}$8) is a kink (anti-kink) solution. There are also solutions which represent a decay D$p \to$ Dq. Such solutions can be obtained by combining several kink solutions representing the decay D$p \to$ D$(p-1)$. This chain relations between the solutions are called the brane descend relations [15, 16]. In the case of the bosonic strings the solution representing a decay D$p \to$ Dq is called lump solution of codimension $(p-q)$. Below we summarize some numeric results concerning lump solutions.

The third conjecture implies that there are different solutions of the equations of motion of the SFT which can be identified with different objects appearing in the string theory. For example, the vortex and kink solutions in the Super

SFT represent lower dimensional D-branes (of various types) in IIA(B) theory. Another example is lump solutions in the Bosonic SFT, which also represent lower dimensional D-branes.

5.2 CFT on Branes.

Dp-brane is incorporated into the string field theory by considering boundary CFT. This boundary CFT is constructed by $p+1$ bosons $X^\alpha(z,\overline{z})$ ($\alpha = 0,\ldots,p$) with Neumann boundary conditions and $25-p$ bosons $X'^i(z,\overline{z})$ ($i = p+1,\ldots,25$) with Dirichlet boundary conditions. But to be able to use vertex operators we need to consider T-dual CFT. This CFT consist of 26 bosons $X^\mu(z,\overline{z})$ with Neumann boundary conditions, but part of them is related to $X'^i(z,\overline{z})$ by T-duality transformation:

$$X^i(z,\overline{z}) = X^i_L(z) + X^i_R(\overline{z}) \quad \stackrel{T}{\Longrightarrow} \quad X'^i(z,\overline{z}) = X^i_L(z) - X^i_R(\overline{z}). \tag{5.1}$$

5.3 String Field Theory on Pair of Branes.

To incorporate more than one brane into string field theory one can use Chan-Paton factors. We will consider the string field theory for a pair of Dp-branes. To this end we need to introduce 2×2 Chan-Paton factors:

$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ denotes a string that begins and ends on the *first* brane;

$\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$ denotes a string that begins on the *first* and ends on the *second* brane;

$\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$ denotes a string that begins and ends on the *second* brane;

$\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$ denotes a string that begins on the *second* and ends on the *first* brane.

For these matrices the following compositions are true:

$$\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \tag{5.2a}$$

$$\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}. \tag{5.2b}$$

The string field is modified in the following way:

$$\hat{\Phi} = \Phi^{(1)} \otimes \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} + \phi \otimes \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} + \phi^* \otimes \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} + \Phi^{(2)} \otimes \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}. \tag{5.3}$$

The action is of the form

$$\begin{aligned} S[\hat{\Phi}] = &-\frac{1}{g_o^2} \operatorname{Tr} \left[\frac{1}{2} \langle\!\langle \hat{\Phi},\, \hat{Q}_B \hat{\Phi} \rangle\!\rangle + \frac{1}{3} \langle\!\langle \hat{\Phi},\, \hat{\Phi},\, \hat{\Phi} \rangle\!\rangle \right] = \\ &-\frac{1}{g_o^2} \left[\frac{1}{2} \langle\!\langle \Phi^{(1)}, Q_B \Phi^{(1)} \rangle\!\rangle + \frac{1}{2} \langle\!\langle \Phi^{(1)}, Q_B \Phi^{(3)} \rangle\!\rangle + \frac{1}{2} \langle\!\langle \phi^*, Q_B \phi \rangle\!\rangle + \frac{1}{2} \langle\!\langle \phi, Q_B \phi^* \rangle\!\rangle \right. \\ &\qquad + \frac{1}{3} \langle\!\langle \Phi^{(1)}, \Phi^{(1)}, \Phi^{(1)} \rangle\!\rangle + \frac{1}{3} \langle\!\langle \Phi^{(2)}, \Phi^{(2)}, \Phi^{(2)} \rangle\!\rangle \\ &\qquad \left. + \frac{1}{3} (\langle\!\langle \Phi^{(1)}, \phi^*, \phi \rangle\!\rangle + c.p.) + \frac{1}{3} (\langle\!\langle \Phi^{(2)}, \phi, \phi^* \rangle\!\rangle + c.p.) \right]. \end{aligned} \tag{5.4}$$

The dimensionless coupling constant g_o^2 is the same as in the action (3.36). The expression (5.4) can be simplified if we employ cyclic symmetry of the odd bracket:

$$\begin{aligned} S[\Phi^{(1)}, \Phi^{(2)}, \phi, \phi^*] = -\frac{1}{g_o^2} &\left[\frac{1}{2} \langle\!\langle \Phi^{(1)}, Q_B \Phi^{(1)} \rangle\!\rangle + \frac{1}{2} \langle\!\langle \Phi^{(2)}, Q_B \Phi^{(2)} \rangle\!\rangle + \langle\!\langle \phi^*, Q_B \phi \rangle\!\rangle \right. \\ + \frac{1}{3} (\langle\!\langle \Phi^{(1)}, \Phi^{(1)}, \Phi^{(1)} \rangle\!\rangle + \langle\!\langle \Phi^{(2)}, \Phi^{(2)}, \Phi^{(2)} \rangle\!\rangle) &\left. + \langle\!\langle \Phi^{(1)}, \phi^*, \phi \rangle\!\rangle + \langle\!\langle \Phi^{(2)}, \phi, \phi^* \rangle\!\rangle \right]. \end{aligned} \tag{5.4$'$}$$

Let us consider the following string fields

$$\Phi^{(1)}(w) = \int \frac{d^{p+1}k}{(2\pi)^{p+1}}\, i\delta\chi_j(k_\alpha) \mathrm{V}_V^j(w, k_\alpha, 0), \tag{5.5a}$$

$$\phi(w) = \int \frac{d^{p+1}k}{(2\pi)^{p+1}}\, t(k_\alpha) \mathrm{V}_t(w, k_\alpha, \tfrac{b^j}{2\pi\alpha'}) + i u_i(k_\alpha) \mathrm{V}_V^i(w, k_\alpha, \tfrac{b^j}{2\pi\alpha'}), \tag{5.5b}$$

$$\phi^*(w) = \int \frac{d^{p+1}k}{(2\pi)^{p+1}}\, t^*(k_\alpha) \mathrm{V}_t(w, k_\alpha, -\tfrac{b^j}{2\pi\alpha'}) + i u_i^*(k_\alpha) \mathrm{V}_V^i(w, k_\alpha, -\tfrac{b^j}{2\pi\alpha'},) \tag{5.5c}$$

where V_V^i is a *primary* operator representing the vector field (3.47) and V_t is the tachyon vertex operator. And we introduce additional restrictions $b^i u_i(k_\alpha) = 0$

and $b^i u_i^*(k_\alpha) = 0$. We are interested in the following action

$$\tilde{S}[\delta\chi_i, t, u_i] = -\frac{1}{g_o^2}\left[\frac{1}{2}\langle\langle\Phi^{(1)}, Q_B\Phi^{(1)}\rangle\rangle + \langle\langle\phi^*, Q_B\phi\rangle\rangle + \langle\langle\Phi^{(1)}, \phi^*, \phi\rangle\rangle\right]. \quad (5.6)$$

The quadratic action can be easily read from (3.51) and it is of the form

$$\tilde{S}_2[\delta\chi_i, t, u_i] = \frac{1}{g_o^2 {\alpha'}^{\frac{p+1}{2}}}\int \frac{d^{p+1}k}{(2\pi)^{p+1}}\ \left[\frac{\alpha' k_\alpha^2}{2}\delta\chi_i(-k)\delta\chi^i(k)\right.$$
$$\left. - \left[\alpha' k_\alpha^2 + \alpha' m_b^2 - 1\right] t^*(-k)t(k) + \left[\alpha' k_\alpha^2 + \alpha' m_b^2\right] u_i^*(-k)u^i(k)\right], \quad (5.7)$$

where $m_b^2 = \left(\frac{b_i}{2\pi\alpha'}\right)^2$.

To compute the interaction terms we need to know the following correlation functions:

$$\langle\langle i\delta\chi_j(k_1)\mathrm{V}_V^j(k_1,0),\, t^*(k_2)\mathrm{V}_t(k_2, -\tfrac{b_i}{2\pi\alpha'}),\, t(k_3)\mathrm{V}_t(k_3, \tfrac{b_i}{2\pi\alpha'})\rangle\rangle = -ib_j\delta\chi^j(k_1)\times$$
$$\times\, t^*(k_2)t(k_3)\left[\frac{2}{\alpha'}\right]^{\frac{1}{2}}(2\pi)^p {\alpha'}^{-\frac{p+1}{2}}\delta(k_1+k_2+k_3) {f_1'}^{\alpha' k_1^2} {f_2'}^{\alpha' k_2^2 - 1} {f_3'}^{\alpha' k_3^2 - 1}\times$$
$$\times\,(f_1 - f_2)^{2\alpha' k_1 k_2}(f_1 - f_3)^{2\alpha' k_1 k_3}(f_2 - f_3)^{2\alpha' k_2 k_3 + 2}. \quad (5.8)$$

Substitution of the maps (3.35) into (5.8) leads to the following expression

$$-\left[\frac{2}{\alpha'}\right]^{\frac{1}{2}} ib_j\delta\chi^j(k_1)t^*(k_2)t(k_3)(2\pi)^p {\alpha'}^{-\frac{p+1}{2}}\delta(k_1+k_2+k_3)\gamma^{\alpha'(k_1^2+k_2^2+k_3^2)-2}. \quad (5.9)$$

Now we consider the correlation functions

$$\langle\langle i\delta\chi_i(k_1)\mathrm{V}_V^i(k_1,0),\, iu_j^*(k_2)\mathrm{V}_V^j(k_2, -\tfrac{b_i}{2\pi\alpha'}),\, iu_k(k_3)\mathrm{V}_V^k(k_3, \tfrac{b_i}{2\pi\alpha'})\rangle\rangle$$
$$= -\delta\chi_i(k_1)u_j^*(k_2)u_k(k_3) {f_1'}^{\alpha' k_1^2} {f_2'}^{\alpha' k_2^2} {f_3'}^{\alpha' k_3^2}\left[\frac{2}{\alpha'}\right]^{\frac{3}{2}}$$
$$\times\left\langle c\partial X^i e^{2ik_1\cdot X_L(f_1)} c\partial X^j e^{2ik_2\cdot X_L(f_2)} c\partial X^k e^{2ik_3\cdot X_L(f_3)}\right\rangle. \quad (5.10)$$

Using the fact that $u_i(k_\alpha)b^i = u_i^*(k_\alpha)b^i = 0$ we get the following expression

$$-\left[\frac{2}{\alpha'}\right]^{\frac{1}{2}}\frac{ib^i\delta\chi_i(k_1)}{2\pi}u_j^*(k_2)u^j(k_3) {f_1'}^{\alpha' k_1^2} {f_2'}^{\alpha' k_2^2} {f_3'}^{\alpha' k_3^2}\times$$
$$\times(f_1 - f_2)^{2\alpha' k_1 k_2}(f_1 - f_3)^{2\alpha' k_1 k_3}(f_2 - f_3)^{2\alpha' k_2 k_3}. \quad (5.11)$$

Substitution of the maps (3.35) into (5.11) leads to the following expression

$$\left[\frac{2}{\alpha'}\right]^{\frac{1}{2}} ib^i\delta\chi_i(k_1)u_j^*(k_2)u^j(k_3)(2\pi)^p {\alpha'}^{-\frac{p+1}{2}}\delta(k_1+k_2+k_3)\gamma^{\alpha'(k_1^2+k_2^2+k_3^2)}. \quad (5.12)$$

Combining (5.9) and (5.12) we get the interaction term of the action (5.6)

$$\tilde{S}_3[\delta\chi_i, t, u_i] = \frac{1}{g_o^2 {\alpha'}^{\frac{p+1}{2}}} \left[\frac{2}{\alpha'}\right]^{\frac{1}{2}} \int \frac{d^{p+1}k_1 d^{p+1}k_2 d^{p+1}k_3}{(2\pi)^{p+1}(2\pi)^{p+1}} \, \delta(k_1{+}k_2{+}k_3) \frac{ib_i \delta\chi^i(k_1)}{2\pi}$$
$$\times \gamma^{\alpha' k_1^2 + \alpha' k_2^2 + \alpha' k_3^2} \left[\gamma^{-2} t^*(k_2) t(k_3) - u_j^*(k_2) u^j(k_3)\right], \quad (5.13)$$

where $\gamma = \frac{4}{3\sqrt{3}}$. Let us choose $\delta\chi^i(x)$ to be a constant

$$\delta\chi^i(k) = (2\pi)^{p+1} \delta\chi^i \delta(k)$$

and let t, t^* and u_i^*, u_i be on mass shell. This simplifies the action (5.13). The whole action $\tilde{S}$ (5.6) is of the form

$$\tilde{S}[t, u_i] = \frac{1}{g_o^2 {\alpha'}^{\frac{p+1}{2}}} \int \frac{d^{p+1}k}{(2\pi)^{p+1}} \left[-\alpha' k_\alpha^2 - \alpha' \left[\frac{b_i}{2\pi\alpha'}\right]^2 + 1 + \left[\frac{2}{\alpha'}\right]^{\frac{1}{2}} \frac{ib_i \delta\chi^i}{2\pi}\right] t^*(-k) t(k)$$
$$- \left[\alpha' k_\alpha^2 + \alpha' \left[\frac{b_i}{2\pi\alpha'}\right]^2 - \left[\frac{2}{\alpha'}\right]^{\frac{1}{2}} \frac{ib_i \delta\chi^i}{2\pi}\right] u_i^*(-k) u^i(k). \quad (5.14)$$

One sees that the term proportional to $\delta\chi_i$ has natural interpretation as a shift of mass of the fields t and u_i. But we want to interpret the term with $\delta\chi_i$ as a shift of b_i. Therefore, we have to compare two shifts of the mass: one produced by $\delta\chi_i$ and another produced by a shift of b_i:

$$\alpha' \frac{2b_i \delta b^i}{4\pi^2 {\alpha'}^2} = -\left[\frac{2}{\alpha'}\right]^{\frac{1}{2}} \frac{ib_i \delta\chi^i}{2\pi}. \quad (5.15)$$

So we get

$$\delta\chi^i = i \left[\frac{\alpha'}{2}\right]^{\frac{1}{2}} \frac{\delta b^i}{\pi\alpha'}. \quad (5.16)$$

This expression can be obtained in a more simple way. Let us consider the following operator product expansion:

$$e^{2i\frac{\delta b_i}{2\pi} X_L^i(z)} e^{2i\frac{b_j}{2\pi} X_L^j(w)} =$$
$$= (z-w)^{2\alpha' \frac{\delta b_i b^i}{(2\pi)^2}} e^{2i\frac{\delta b_j + b_j}{2\pi} X_L^i(w)} \left[1 + (z-w)\frac{i\delta b_i}{\pi} \partial X^i(w) + O(z-w)^2\right]. \quad (5.17)$$

The state $|0, b\rangle$ is generated by the vertex operator $V_t(0, \frac{b_i}{2\pi\alpha'})$, therefore, the change of this operator under a small shift of b_i must be equal to $\delta\chi_i V_V^i(0, \frac{b_i}{2\pi\alpha'})$. So we get

$$\delta\left[c(w) e^{\frac{ib_j}{\pi\alpha'} X_L^j(w)}\right] = \left[\frac{\alpha'}{2}\right]^{\frac{1}{2}} \frac{\delta b_j}{\pi\alpha'} \, V_v(0, \tfrac{b_i}{2\pi\alpha'}), \quad (5.18)$$

which is equal to the previously obtained expression up to the vertex operator of the auxiliary field. So we can now identify $\delta\chi^i$ by using

$$i\delta\chi^i = \left[\frac{\alpha'}{2}\right]^{\frac{1}{2}} \frac{\delta b_j}{\pi\alpha'}. \tag{5.19}$$

One sees that up to the sign (that is easily explained) we get the same result as in (5.16).

Now we substitute (5.16) or (5.19) into (5.7) and get for the field δb^i the following action

$$\tilde{S}_2[\delta b_i] = \frac{1}{2\pi^2 g_o^2 \alpha'^{\frac{p+1}{2}}} \int \frac{d^{p+1}k}{(2\pi)^{p+1}} \frac{k_\alpha^2}{2} \delta b_i(-k)\delta b^i(k) \tag{5.20}$$

As a consequence we get the brane's tension:

$$\tau_p = \frac{1}{2\pi^2 g_o^2 \alpha'^{\frac{p+1}{2}}}. \tag{5.21}$$

5.4 Superstring Field Theory on non-BPS D-brane.

To describe the open string states living on a single non-BPS D-brane one has to add GSO$-$ states [15]. GSO$-$ states are Grassmann even, while GSO+ states are Grassmann odd (see Table 4).

Name	Parity	GSO	Superghost number	Weight (h)	Comments
$\mathcal{A}_+$	odd	$+$	1	$h \in \mathbb{Z}$, $h \geqslant -1$	string
$\mathcal{A}_-$	even	$-$	1	$h \in \mathbb{Z}+\frac{1}{2}$, $h \geqslant -\frac{1}{2}$	fields
Λ_+	even	$+$	0	$h \in \mathbb{Z}$, $h \geqslant 0$	gauge
Λ_-	odd	$-$	0	$h \in \mathbb{Z}+\frac{1}{2}$, $h \geqslant \frac{1}{2}$	parameters

Table 4: Parity of string fields and gauge parameters in the 0 picture.

The unique (up to rescaling of the fields) gauge invariant action unifying GSO+ and GSO$-$ sectors is found to be

$$\begin{aligned} S[\mathcal{A}_+, \mathcal{A}_-] = \frac{1}{g_o^2} \Big[& \frac{1}{2} \langle\langle Y_{-2} | \mathcal{A}_+, Q_B \mathcal{A}_+ \rangle\rangle + \frac{1}{3} \langle\langle Y_{-2} | \mathcal{A}_+, \mathcal{A}_+, \mathcal{A}_+ \rangle\rangle \\ & + \frac{1}{2} \langle\langle Y_{-2} | \mathcal{A}_-, Q_B \mathcal{A}_- \rangle\rangle - \langle\langle Y_{-2} | \mathcal{A}_+, \mathcal{A}_-, \mathcal{A}_- \rangle\rangle \Big]. \end{aligned} \tag{5.22}$$

Here the factors before the odd brackets are fixed by the constraint of gauge invariance, that is specified below, and reality of the string fields $\mathcal{A}_\pm$. Variation of this action with respect to $\mathcal{A}_+$, $\mathcal{A}_-$ yields the following equations of motion[7]

$$\begin{aligned} &Q_B\mathcal{A}_+ + \mathcal{A}_+\star\mathcal{A}_+ - \mathcal{A}_-\star\mathcal{A}_- = 0,\\ &Q_B\mathcal{A}_- + \mathcal{A}_+\star\mathcal{A}_- - \mathcal{A}_-\star\mathcal{A}_+ = 0. \end{aligned} \tag{5.23}$$

To derive these equations we used the cyclicity property of the odd bracket (see Appexdix of [8]). The action (5.22) is invariant under the gauge transformations

$$\begin{aligned} &\delta\mathcal{A}_+ = Q_B\Lambda_+ + [\mathcal{A}_+,\Lambda_+] + \{\mathcal{A}_-,\Lambda_-\},\\ &\delta\mathcal{A}_- = Q_B\Lambda_- + [\mathcal{A}_-,\Lambda_+] + \{\mathcal{A}_+,\Lambda_-\}, \end{aligned} \tag{5.24}$$

where $[\,,]$ $(\{\,,\})$ denotes $\star$-commutator (-anticommutator). To prove the gauge invariance, it is sufficient to check the covariance of the equations of motion (5.23) under the gauge transformations (5.24). A simple calculation leads to

$$\begin{aligned} &\delta(Q_B\mathcal{A}_+ + \mathcal{A}_+\star\mathcal{A}_+ - \mathcal{A}_-\star\mathcal{A}_-)\\ &\qquad = [Q_B\mathcal{A}_+ + \mathcal{A}_+\star\mathcal{A}_+ - \mathcal{A}_-\star\mathcal{A}_-,\,\Lambda_+] - [Q_B\mathcal{A}_- + \mathcal{A}_+\star\mathcal{A}_- - \mathcal{A}_-\star\mathcal{A}_+,\,\Lambda_-],\\ &\delta(Q_B\mathcal{A}_- + \mathcal{A}_+\star\mathcal{A}_- - \mathcal{A}_-\star\mathcal{A}_+)\\ &\qquad = [Q_B\mathcal{A}_- + \mathcal{A}_+\star\mathcal{A}_- - \mathcal{A}_-\star\mathcal{A}_+,\,\Lambda_+] + [Q_B\mathcal{A}_+ + \mathcal{A}_+\star\mathcal{A}_+ - \mathcal{A}_-\star\mathcal{A}_-,\,\Lambda_-]. \end{aligned}$$

Note that to obtain this result the associativity of $\star$-product and Leibnitz rule for Q_B must be employed. These properties follow from the cyclicity property of the odd bracket. The formulae above show that the gauge transformations define a Lie algebra.

5.5 Computation of Tachyon Potential in Cubic SSFT.

Here we explore the tachyon condensation on the non-BPS D-brane. In the first subsection, we describe the expansion of the string field relevant to the tachyon condensation and the level expansion of the action. In the second subsection we calculate the tachyon potential up to levels 1 and 4, and find its minimum.

[7] We assume that r.h.s. is zero modulo $\ker Y_{-2}$.

5.5.1 Tachyon string field in cubic SSFT.

The useful devices for computation of the tachyon potential were elaborated in [18, 17, 80]. We employ these devices without additional references.

Denote by $\mathcal{H}_1$ the subset of vertex operators of ghost number 1 and picture 0, created by the matter stress tensor T_B, matter supercurrent T_F and the ghost fields b, c, $\partial\xi$, η and ϕ. We restrict the string fields $\mathcal{A}_+$ and $\mathcal{A}_-$ to be in this subspace $\mathcal{H}_1$. We also restrict ourselves by Lorentz scalars and put the momentum in vertex operators equal to zero.

Next we expand $\mathcal{A}_\pm$ in a basis of L_0 eigenstates, and write the action (2.4) in terms of space-time component fields. The string field is now a series with each term being a vertex operator from $\mathcal{H}_1$ multiplied by a space-time component field. We define the level K of string field's component A_i to be $h+1$, where h is the conformal dimension of the vertex operator multiplied by A_i, i.e. by convention the tachyon is taken to have level 1/2. To compose the action truncated at level (K, L) we select all the quadratic and cubic terms of total level not more than L for the space-time fields of levels not more than K. Since our action is cubic, $L \leq 3K$.

To calculate the action up to level $(2,6)$ we have a collection of vertex operators listed in Table 5. Note that there are extra fields in the 0 picture as compared with the picture -1 (see Section 4.4). Surprisingly the level $L_0 = -1$ is not empty, it contains the field u. One can check that this field is auxiliary. In the following analysis it plays a significant role. Only due to this field in the next subsection we get a nontrivial tachyon potential (as compared with one given in [83]) already at level (1/2, 1).

As it is shown in Appendix of [8] the string field theory action in the restricted subspace $\mathcal{H}_1$ has $\mathbb{Z}_2$ twist symmetry. Since the tachyon vertex operator has even twist we can consider a further truncation of the string field by restricting $\mathcal{A}_\pm$ to be twist even. Therefore, the fields r_i, s_i can be dropped out. Moreover, we impose one more restriction and require our fields to have the ϕ-charge (see 4.42)

Level L_0+1	Weight L_0	GSO $(-1)^F$	Twist Ω	Name	Picture -1	Picture 0
					Vertex operators	
0	-1	$+$	even	u	—	c
1/2	$-1/2$	$-$	even	t	$ce^{-\phi}$	$e^{\phi}\eta$
1	0	$+$	odd	r_i	$c\partial c\partial\xi e^{-2\phi}$	$\partial c,\ c\partial\phi$
3/2	1/2	$-$	odd	s_i	$c\partial\phi e^{-\phi}$	$cT_F,\quad \partial(\eta e^{\phi})$ $bc\eta e^{\phi},\quad \eta\partial e^{\phi}$
2	1	$+$	even	v_i	$\eta,\ T_F ce^{-\phi}$ $\partial\xi c\partial^2 ce^{-2\phi}$ $\partial^2\xi c\partial ce^{-2\phi}$ $\partial\xi c\partial c\partial e^{-2\phi}$	$\partial^2 c,\ cT_B,\ cT_{\xi\eta}$ $cT_\phi,\ c\partial^2\phi,\ T_F\eta e^{\phi}$ $bc\partial c,\ \partial c\partial\phi$

Table 5: Vertex operators in pictures -1 and 0.

equal to 0 and 1. String fields (in GSO$\pm$ sectors) up to level 2 take the form[8]

$$\begin{aligned}\mathcal{A}_+(z) &= u\,c(z) + v_1\,\partial^2 c(z) + v_2\,cT_B(z) + v_3\,cT_{\eta\xi}(z) + v_4\,cT_\phi(z)\\ &\qquad + v_5\,c\partial^2\phi(z) + v_6\,T_F\eta e^{\phi}(z) + v_7\,bc\partial c(z) + v_8\,\partial c\partial\phi(z),\\ \mathcal{A}_-(z) &= \frac{t}{4}\,e^{\phi(z)}\eta(z).\end{aligned} \tag{5.25}$$

5.5.2 Tachyon potential.

Here we give expressions for the action and the potential by truncating them up to level (2, 6). Since the field (5.25) expands over the levels 0, $\frac{1}{2}$, and 2 we can truncate the action at levels (1/2, 1) and (2, 6) only. All the calculations have been performed on a specially written program on Maple. All we need is to give

[8]The string fields are presented without any gauge fixing conditions.

to the program the string fields (5.25) and we get the following lagrangians

$$\mathcal{L}^{(\frac{1}{2},1)} = \frac{1}{g_o^2\alpha'^{\frac{p+1}{2}}}\left[u^2+\frac{1}{4}t^2+\frac{1}{3\gamma^2}ut^2\right], \tag{5.26}$$

$$\begin{aligned}\mathcal{L}^{(2,6)} = &\frac{1}{g_o^2\alpha'^{\frac{p+1}{2}}}\Bigg[u^2+\frac{1}{4}t^2+(4v_1-2v_3-8v_4+8v_5+2v_7)u\\ &+4v_1^2+\frac{15}{2}v_2^2+v_3^2+\frac{77}{2}v_4^2+22v_5^2+10v_6^2+8v_1v_3-32v_1v_4+24v_1v_5+4v_1v_7\\ &-16v_3v_4+4v_3v_5-2v_3v_7+12v_3v_8-52v_4v_5-8v_4v_7-20v_4v_8+8v_5v_7+8v_5v_8\\ &+(-30v_4+20v_5+30v_2)v_6+4v_7v_8\\ &+\left(\frac{1}{3\gamma^2}u+\frac{9}{8}v_1-\frac{25}{32}v_2-\frac{9}{16}v_3-\frac{59}{32}v_4+\frac{43}{24}v_5+\frac{2}{3}v_7\right)t^2\\ &-\left(\frac{40\gamma}{3}u+45\gamma^3v_1-\frac{45\gamma^3}{4}v_2-\frac{45\gamma^3}{2}v_3-\frac{295\gamma^3}{4}v_4+\frac{215\gamma^3}{3}v_5+\frac{80\gamma^3}{3}v_7\right)v_6^2\Bigg],\end{aligned} \tag{5.27}$$

where $\gamma = \frac{4}{3\sqrt{3}}$. To simplify the succeeding analysis we use a special gauge choice

$$3v_2-3v_4+2v_5=0. \tag{5.28}$$

This gauge eliminates the terms linear in v_6 and drastically simplifies the calculation of the effective potential for the tachyon field. We will discuss an issue of validity of this gauge in Section 8 of [8]. The effective tachyon potential is defined as $\mathcal{V}(t) = -\mathcal{L}(t, u(t), v_i(t))$, where $u(t)$ and $v_i(t)$ are solution to equations of motion $\partial_u\mathcal{L} = 0$ and $\partial_{v_i}\mathcal{L} = 0$. In our gauge the equation $\partial_{v_6}\mathcal{L} = 0$ admits a solution $v_6 = 0$ and, therefore, the tachyon potential computed at levels (2, 4) and (2, 6) is the same. The potential at levels (1/2, 1) and (2, 6) has the following form:

$$\begin{aligned}\mathcal{V}_{\text{eff}}^{(\frac{1}{2},1)}(t) &= \frac{1}{g_o^2\alpha'^{\frac{p+1}{2}}}\left[\frac{81}{1024}t^4-\frac{1}{4}t^2\right],\\ \mathcal{V}_{\text{eff}}^{(2,6)}(t) &= \frac{1}{g_o^2\alpha'^{\frac{p+1}{2}}}\left[\frac{5053}{69120}t^4-\frac{1}{4}t^2\right]\end{aligned} \tag{5.29}$$

One sees that the potential has two global minima, which are reached at points $t_c = \pm 1.257$ at level (1/2, 1) and at points $t_c = \pm 1.308$ at level (2, 6) (see also Figure 3 and Table 6).

5.6 Tension of Non-BPS Dp-brane in Cubic SSFT and Sen's Conjecture.

To find a tension of Dp-brane following [81] one considers the SFT describing a pair of Dp-branes and calculates the string field action on a special string field. This string field contains a field describing a displacement of one of the branes and a field describing arbitrary excitations of the strings stretched between the two branes. For simplicity one can use low-energy excitations of the strings stretched between the branes.

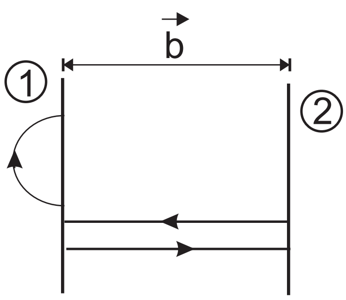

Figure 2: The system of two non BPS Dp-branes and strings attached to them.

The cubic SSFT describing a pair of non-BPS Dp-branes includes 2×2 Chan-Paton (CP) factors [81, 4] and has the following form

$$S[\hat{\mathcal{A}}_+,\hat{\mathcal{A}}_-]=\frac{1}{g_o^2}\left[\frac{1}{2}\langle\langle\hat{Y}_{-2}|\hat{\mathcal{A}}_+,\hat{Q}_B\hat{\mathcal{A}}_+\rangle\rangle+\frac{1}{3}\langle\langle\hat{Y}_{-2}|\hat{\mathcal{A}}_+,\hat{\mathcal{A}}_+,\hat{\mathcal{A}}_+\rangle\rangle\right.$$
$$\left.+\frac{1}{2}\langle\langle\hat{Y}_{-2}|\hat{\mathcal{A}}_-,\hat{Q}_B\hat{\mathcal{A}}_-\rangle\rangle-\langle\langle\hat{Y}_{-2}|\hat{\mathcal{A}}_+,\hat{\mathcal{A}}_-,\hat{\mathcal{A}}_-\rangle\rangle\right]. \tag{5.30}$$

Here g_o is a dimensionless coupling constant. The hatted BRST charge $\hat{Q}_B$ and double step inverse picture changing operator $\hat{Y}_{-2}$ are Q_B and Y_{-2} tensored by 2×2 unit matrix. The string fields are also 2×2 matrices

$$\hat{\mathcal{A}}_\pm=\mathcal{A}_\pm^{(1)}\otimes\begin{pmatrix}1&0\\0&0\end{pmatrix}+\mathcal{A}_\pm^{(2)}\otimes\begin{pmatrix}0&0\\0&1\end{pmatrix}+\mathcal{B}_\pm^*\otimes\begin{pmatrix}0&1\\0&0\end{pmatrix}+\mathcal{B}_\pm\otimes\begin{pmatrix}0&0\\1&0\end{pmatrix} \tag{5.31}$$

and the odd bracket includes the trace over matrices.

The action is invariant under the following gauge transformations:

$$\delta\hat{\mathcal{A}}_+=\hat{Q}_B\hat{\Lambda}_++[\hat{\mathcal{A}}_+,\hat{\Lambda}_+]+\{\hat{\mathcal{A}}_-,\hat{\Lambda}_-\},$$
$$\delta\hat{\mathcal{A}}_-=\hat{Q}_B\hat{\Lambda}_-+[\hat{\mathcal{A}}_-,\Lambda_+]+\{\hat{\mathcal{A}}_+,\hat{\Lambda}_-\}. \tag{5.32}$$

The fields $\mathcal{A}_\pm^{(1)}$ describe excitations of the string attached to the first brane, while $\mathcal{A}_\pm^{(2)}$ describe excitations of the string attached to the second one. Excitations of the stretched strings are represented by the fields $\mathcal{B}_\pm$ and $\mathcal{B}_\pm^*$ (see

Figure 2). The action for a single non-BPS D-brane (2.4) that we have used above is derived from the universal action (5.30) by setting $\mathcal{A}^{(2)}_{\pm}$, $\mathcal{B}_{\pm}$ and $\mathcal{B}^*_{\pm}$ to zero. Note also that we have not changed the value of the coupling constant g_o. The "$\pm$" subscript specify the GSO sector.

Let us take the following string fields $\hat{\mathcal{A}}_{\pm}$:

$$\hat{\mathcal{A}}_+ = \hat{A}^{(1)}_+ + \hat{B}^*_+ + \hat{B}_+, \qquad \hat{\mathcal{A}}_- = \hat{B}^*_- + \hat{B}_-, \tag{5.33}$$

where

$$\begin{aligned}
\hat{A}^{(1)}_+ &= \int \frac{d^{p+1}k}{(2\pi)^{p+1}}\, A_i(k_\alpha)\mathrm{V}^i_v(k_\alpha,0)\otimes\begin{pmatrix}1&0\\0&0\end{pmatrix},\\
\hat{B}_+ &= \int \frac{d^{p+1}k}{(2\pi)^{p+1}}\, iB_i(k_\alpha)\mathrm{V}^i_v(k_\alpha,\tfrac{\vec{b}}{2\pi\alpha'})\otimes\begin{pmatrix}0&0\\1&0\end{pmatrix},\\
\hat{B}^*_+ &= \int \frac{d^{p+1}k}{(2\pi)^{p+1}}\, iB^*_i(k_\alpha)\mathrm{V}^i_v(k_\alpha,-\tfrac{\vec{b}}{2\pi\alpha'})\otimes\begin{pmatrix}0&1\\0&0\end{pmatrix}\\
\hat{B}_- &= \int \frac{d^{p+1}k}{(2\pi)^{p+1}}\, t(k_\alpha)\mathrm{V}_t(k_\alpha,\tfrac{\vec{b}}{2\pi\alpha'})\otimes\begin{pmatrix}0&0\\1&0\end{pmatrix},\\
\hat{B}^*_- &= \int \frac{d^{p+1}k}{(2\pi)^{p+1}}\, t^*(k_\alpha)\mathrm{V}_t(k_\alpha,-\tfrac{\vec{b}}{2\pi\alpha'})\otimes\begin{pmatrix}0&1\\0&0\end{pmatrix}.
\end{aligned}$$

Here b_i is a distance between the branes, $\alpha = 0,\dots,p$ and $i = p+1,\dots,9$ and V^i_v and V_t are vertex operators of a massless vector and tachyon fields respectively defined by

$$\begin{aligned}
\mathrm{V}^\mu_v(k_\alpha,k_i) &= \frac{i}{2}\left[\frac{2}{\alpha'}\right]^{1/2}\left[c\partial X^\mu + c2ik\cdot\psi\psi^\mu - \frac{1}{2}\eta e^\phi\psi^\mu\right]e^{2ik\cdot X_L(0)},\\
\mathrm{V}_t(k_\alpha,k_i) &= \frac{1}{2}\left[c2ik\cdot\psi - \frac{1}{2}\eta e^\phi\right]e^{2ik\cdot X_L(0)}.
\end{aligned} \tag{5.34}$$

These vertex operators are written in the 0 picture and can be obtained by applying the picture changing operator (4.28) to the corresponding operators in picture -1. The Fourier transform of $A_i(k_\alpha)$ has an interpretation of the Dp-brane's coordinate up to an overall normalization factor [4]. Further we will assume that $b^iB_i(k_\alpha) = 0$.

The action for the field (5.33) depending on the local fields $B_i(k_\alpha)$, $B_i^*(k_\alpha)$, $t(k_\alpha)$, $t^*(k_\alpha)$ and $A_i(k_\alpha)$ is given by

$$S[A_i,B_i,t] = \frac{\alpha'}{g_o^2 \alpha'^{\frac{p+1}{2}}} \int \frac{d^{p+1}k}{(2\pi)^{p+1}} \left\{ \frac{k_\alpha^2}{2} A^i(-k)A^i(k) + B_i^*(-k)B^i(k)\left[k_\alpha^2 + \tfrac{b_i^2}{(2\pi\alpha')^2}\right] \right.$$
$$+ t^*(-k)t(k)\left[k_\alpha^2 + \tfrac{b_i^2}{(2\pi\alpha')^2} - \tfrac{1}{2\alpha'}\right] + \int \frac{d^{p+1}p}{(2\pi)^{p+1}}\, \gamma^{2\alpha'(k^2+p^2+p\cdot k+\frac{b_i^2}{(2\pi\alpha')^2})} \times$$
$$\left. \times \left[B_i^*(-p-k)B^i(k) + \gamma^{-1}t^*(-p-k)t(k)\right] \frac{b_j A^j(p)}{2\pi\alpha'\sqrt{2\alpha'}} \right\}, \quad (5.35)$$

where $\gamma = \frac{4}{3\sqrt{3}}$. Let us now consider the constant field $A_i(\xi) = \text{const}$, where ξ^α are coordinates on the brane. Its Fourier transform is of the form $A_i(p) = (2\pi)^{p+1}A_i\delta(p)$. Let also $B_i(k)$, $B_i^*(k)$ and $t(p)$, $t^*(p)$ be on-shell, i.e.

$$k_\alpha^2 + \frac{b_i^2}{(2\pi\alpha')^2} = 0$$

and

$$p_\alpha^2 + \frac{b_i^2}{(2\pi\alpha')^2} - \frac{1}{2\alpha'} = 0.$$

In this case the action (5.35) is simplified and takes the form:

$$\tilde{S}[t,B_i] = \frac{\alpha'}{g_o^2 \alpha'^{\frac{p+1}{2}}} \int_{\text{on-sh.}} \frac{d^{p+1}k}{(2\pi)^{p+1}} \left[k_\alpha^2 + \frac{b_i^2}{(2\pi\alpha')^2} - \frac{1}{2\alpha'} + \frac{b_i A^i}{2\pi\alpha'\sqrt{2\alpha'}}\right] t^*(-k)t(k)$$
$$+ \frac{\alpha'}{g_o^2 \alpha'^{\frac{p+1}{2}}} \int_{\text{on-sh.}} \frac{d^{p+1}k}{(2\pi)^{p+1}} \left[k_\alpha^2 + \frac{b_i^2}{(2\pi\alpha')^2} + \frac{b_i A^i}{2\pi\alpha'\sqrt{2\alpha'}}\right] B_j^*(-k)B^j(k). \quad (5.36)$$

The terms proportional to A_i have natural interpretation as shifts of masses of the fields t and B_i. We want to interpret the term with A_i as a shift of brane coordinate b_i. Therefore, we have to compare two shifts of the mass: one produced by A_i and another produced by a shift of b_i

$$\delta(m^2) = \frac{2b_i\delta b^i}{4\pi^2\alpha'^2} = \frac{b_i A^i}{2\pi\alpha'\sqrt{2\alpha'}}. \quad (5.37)$$

So one gets

$$A^i = \frac{1}{\pi}\left[\frac{2}{\alpha'}\right]^{\frac{1}{2}} \delta b^i. \quad (5.38)$$

This formula determines the normalization of the field A_i. Therefore, we can introduce the profile of the first non-BPS Dp-brane as $x_i(\xi) = \pi\left[\frac{\alpha'}{2}\right]^{1/2} A_i(\xi)$.

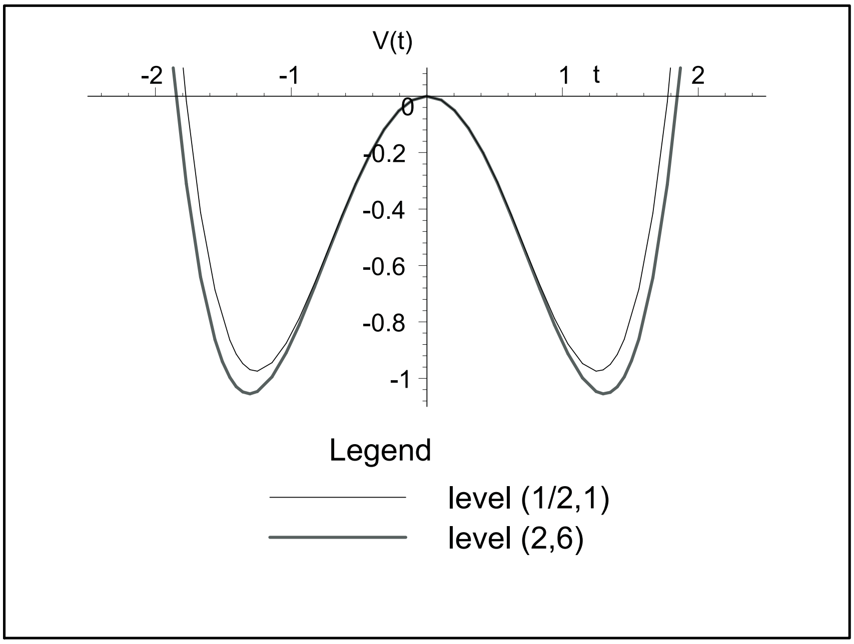

Figure 3: Graphics of the tachyon potential at the levels (1/2, 1) and (2, 6). "-1" is equal to the minus tension of non BPS Dp-brane $\tilde{\tau}_p = \frac{2}{\pi^2 g_o^2 \alpha'^{\frac{p+1}{2}}}$.

Substitution of the $A_i(\xi)$ into the first term of the action (5.35) yields

$$S_0 = -\frac{2}{g_o^2 \pi^2 \alpha'^{\frac{p+1}{2}}} \int d^{p+1}\xi \; \frac{1}{2} \partial_\alpha x^i(\xi) \partial^\alpha x_i(\xi). \tag{5.39}$$

The coefficient before the integral is the non-BPS Dp-brane tension $\tilde{\tau}_p$:

$$\tilde{\tau}_p = \frac{2}{g_o^2 \pi^2 \alpha'^{\frac{p+1}{2}}}. \tag{5.40}$$

As compared with the expression for the tension given in [81] we have the addition factor 4. The origin of this factor is the difference in the normalization of the superghosts β, γ.

Now we can express the coupling constant g_o^2 in terms of the tension $\tilde{\tau}_p$. Hence the potential (5.29) at levels (1/2 1) and (2 6) takes the form (see also Figure 3)

$$
\begin{aligned}
\mathcal{V}_{\rm eff}^{(\frac{1}{2},\,1)}(t) &= \frac{\pi^2\tilde{\tau}_p}{2}\left[\frac{81}{1024}t^4 - \frac{1}{4}t^2\right], \\
\mathcal{V}_{\rm eff}^{(2,\,6)}(t) &= \frac{\pi^2\tilde{\tau}_p}{2}\left[\frac{5053}{69120}t^4 - \frac{1}{4}t^2\right].
\end{aligned} \tag{5.41}
$$

The critical points of these functions are collected in Table 6. One sees that the

Potential	Critical points	Critical values
$\mathcal{V}_{\rm eff}^{(\frac{1}{2},\,1)}$	$t_c = 0$	$\mathcal{V}_c = 0$
	$t_c = \pm\frac{8\sqrt{2}}{9} \approx \pm 1.257$	$\mathcal{V}_c \approx -0.975\tilde{\tau}_p$
$\mathcal{V}_{\rm eff}^{(2,\,6)}$	$t_c = 0$	$\mathcal{V}_c = 0$
	$t_c = \pm\frac{24}{5053}\sqrt{75795} \approx \pm 1.308$	$\mathcal{V}_c \approx -1.058\tilde{\tau}_p$

Table 6: The critical points of the tachyon potential at levels (1/2, 1) and (2, 6).

potential has a global minimum and the value of this minimum is 97.5% at level (1/2, 1) and 105.8% at level (2, 6) of the tension $\tilde{\tau}_p$ of the non-BPS Dp-brane.

5.7 Test of Absence of Kinetic Terms Around Tachyon Vacuum.

In this section we perform calculations of the effective potential at level 0, i.e. we take only low levels fields:

$$
\begin{aligned}
\mathcal{A}_+ &= \int \frac{d^{p+1}k}{(2\pi)^{p+1}}\, u(k)\; c(w)e^{2ik\cdot X_L(w)}\Big|_{w=0}, \\
\mathcal{A}_- &= \int \frac{d^{p+1}k}{(2\pi)^{p+1}}\, t(k)\; \frac{1}{2}\left[c(w)2ik\cdot\psi(w) - \frac{1}{2}\eta(w)e^{\phi(w)}\right] e^{2ik\cdot X_L(w)}\Big|_{w=0}.
\end{aligned} \tag{5.42}
$$

Note that the expression for the tachyon field can be obtained by applying rasing picture changing operator X to the tachyon vertex operator in picture -1. The action in the momentum representation is of the form

$$
S[u,t] = \frac{1}{g_o^2\alpha'^{\frac{(p+1)}{2}}}\int \frac{d^{p+1}k}{(2\pi)^{p+1}}\Big[u(-k)u(k) - \frac{1}{2}t(-k)t(k)\;\left[\alpha' k_\alpha^2 - 1/2\right] \tag{5.43}
$$

$$
-\frac{1}{3\gamma^2}\int \frac{d^{p+1}p}{(2\pi)^{p+1}}\, t(-k-p)t(p)u(k)\,(\gamma)^{2\alpha'(p^2+k^2+kp)}\Big].
$$

This expression can be shortly rewritten in the x-representation

$$S[u,t] = \frac{1}{g_o^2 {\alpha'}^{\frac{(p+1)}{2}}} \int d^{p+1}x \left[u(x)u(x) - \frac{1}{2}t(x)\left(-\alpha'\partial_\alpha\partial^\alpha - 1/2\right)t(x) \right.$$
$$\left. -\frac{1}{3\gamma^2}\tilde{u}(x)\tilde{t}(x)\tilde{t}(x) \right], \quad (5.44)$$

where

$$\tilde{\phi}(x) = \exp(-\alpha' \log\gamma\, \partial_\alpha\partial^\alpha)\phi(x). \quad (5.45)$$

The operation $\tilde{}$ has the following properties

- $\tilde{t}_0 = t_0$ if $t_0 = \text{const}$,
- $\int dx\ \tilde{t}(x) = \int dx\ t(x)$,
- $\int dx\ \tilde{a}(x)b(x) = \int dx\ a(x)\tilde{b}(x)$.

Using the second property we can move $\tilde{}$ from $u(x)$ on the term $\tilde{t}(x)\tilde{t}(x)$. After this one can integrate over $u(x)$ and gets an effective action

$$S_{\text{eff}}[t] = \frac{1}{g_o^2 {\alpha'}^{\frac{(p+1)}{2}}} \int d^{p+1}x \left[\frac{1}{2}t(x)\left(\alpha'\partial_\alpha\partial^\alpha + \frac{1}{2}\right)t(x) - \frac{\left(\widetilde{\tilde{t}\tilde{t}}\right)^2(x)}{36\gamma^4} \right]. \quad (5.46)$$

Expanding $t(x) = t_0 + t_1(x)$ and keeping only terms that are quadratic in t_1 and linear in α' we get

$$S^{(2)}_{2,shifted}[t] = \frac{a}{2}\int d^{p+1}x \left[1 + \frac{10t_0^2}{9\gamma^4}\log\gamma\right] t_1(x)\,\alpha'\partial_\alpha\partial^\alpha t_1(x). \quad (5.47)$$

So to vanish the kinetic terms for t_1 we have to have $t_0 = 1.099$, which is not far from the critical value $t_0 = 1.257$ for the minimum of the tachyon potential at level $(\frac{1}{2}, 1)$.

5.8 Lump Solutions.

Here we give a brief review of the numeric computations of lump solutions (see review [142] for more details). There is an intresting [151, 152] relations with p-adic strings [137, 138, 139, 140, 141].

First, let us define what is the lump solution. Let us denote by Φ_0 the string field representing the solution of momentum independent equations of motion. In other words, Φ_0 is a tachyon condensate. We want to find a lump solution Φ of codimension one, which represents a decay D25 $\to$ D24. This means that string field Φ is a static configuration depending on one space coordinate, say $x^{25} \equiv x$. This configuration has nontrivial boundary conditions

$$\Phi(x) \to \Phi_0 \quad \text{as} \quad x \to \pm\infty.$$

Physically this condition means that we want to have true string vacuum at plus and minus infinity. We expect that the solution is concentrated near the origin and, therefore, it distinctly differs from Φ_0 only in small domain around the origin.

Second, to be able to find a solution numerically we have to have finite number of fields. To this purpose we use level truncation scheme for the computations of the tachyon potential. To find the lump solution authors of the paper [143] proposed to use a modified level truncation scheme. Their idea consists of two steps:

1. We compactify the direction x on a circle of radius $R\sqrt{\alpha'}$. This allows us to represent component fields $\phi_i(x)$ of the string field $\Phi(x)$ by Fourier series rather than Fourier integral:
$$\phi_i(x) \sim \sum_n \phi_{i,n} e^{\frac{inx}{R\sqrt{\alpha'}}}. \tag{5.48}$$
2. We modify the level grading:
$$\mathrm{lvl}_R(\phi_{i,n}) = \frac{n^2}{R^2} + N_i + 1, \tag{5.49}$$
where N_i is oscillator number operator evaluated on the vertex operator corresponding to the field ϕ_i and the normalization of the grading was chosen in such a way that it is equal to 1 on the zero momentum tachyon field. Using this level grading we can define (M, N)-level approximation of the action. This means that we get all fields with $\mathrm{lvl}_R \leqslant M$ from the string field Φ and keep only the terms in the action for which $\mathrm{lvl}_R \leqslant N$.

Third, we have to choose the Hilbert space $\mathcal{H}$ in which we are going to find our solution. The analysis performed in [143] shows that without loss of generality we can consider the following reduced space:

$$\mathcal{H} = \mathcal{H}^{\mathrm{univ}'}_{\mathrm{matter}} \oplus \mathcal{H}_{\mathrm{ghost}} \oplus \mathcal{H}_{\mathrm{matter}}(x)\Big|_{even}, \tag{5.50}$$

where $\mathcal{H}^{\text{univ}'}_{\text{matter}}$ is a universal (background independent) Hilbert space [143] generated by matter Virasoro generators restricted to the directions $x^1, \dots, x^{24}$, $\mathcal{H}_{\text{ghost}}$ is a Hilbert space spanned by all ghost operators and $\mathcal{H}_{\text{matter}}(x)$ is a Hilbert space generated by all oscillators α^{25}_n.

Now consider a simple example of the computations. Let us choose $R = \sqrt{3}$ and assume that we have only tachyon field $t(x)$. Since it has to be even we can write

$$t(x) = \sum_{n=0}^{\infty} t_n \cos\Bigl(\frac{nx}{R\sqrt{\alpha'}}\Bigr). \tag{5.51}$$

The terms t_n have the following grading:

$$\text{lvl}_{\sqrt{3}}\, t_n = 1 + \frac{n^2}{3}.$$

Now substitution into the cubic action (3.51), (3.56′) yields the following expressions at levels $(0,0)$ and $(\frac{1}{3}, \frac{2}{3})$:

$$\mathcal{V}^{(0,0)} = -\frac{1}{2} t_0^2 + \frac{1}{3\gamma^3} t_0^3, \tag{5.52a}$$

$$\mathcal{V}^{(\frac{1}{3},\frac{2}{3})} = -\frac{1}{6} t_1^2 + \frac{1}{2\gamma^{\frac{11}{3}}} t_0 t_1^2, \tag{5.52b}$$

where $\gamma = \frac{4}{3\sqrt{3}}$ and $\mathcal{V}$ is related to S by formula

$$S = \frac{2\pi R\sqrt{\alpha'}\,\text{Vol}_{24}}{g_o^2 \alpha'^{13}}\, \mathcal{V}. \tag{5.53}$$

The solution of the equations of motion obtained from (5.52) is presented on the left pane on Figure 4.

One can also compute the ratio of tension of the lump solution and the tension of original D25-brane. One expect that the tension of the lump solution of codimension 1 is equal to the tension of D24-brane. Therefore, we expect that

$$\frac{\tau(\text{lump})}{\tau_{\text{D24}}} = 1, \quad \text{where} \quad \tau(\text{lump}) = \frac{2\pi R\sqrt{\alpha'}}{g_o^2 \alpha'^{13}}\, (\mathcal{V}(\Phi) - \mathcal{V}(\Phi_0)). \tag{5.54}$$

There are several ways to check this equality numerically. We can express the tension of D25 brane through the coupling constant g_o as [18]

$$\tau_{\text{D25}} = \frac{1}{2\pi^2 g_o^2 \alpha'^{13}}.$$

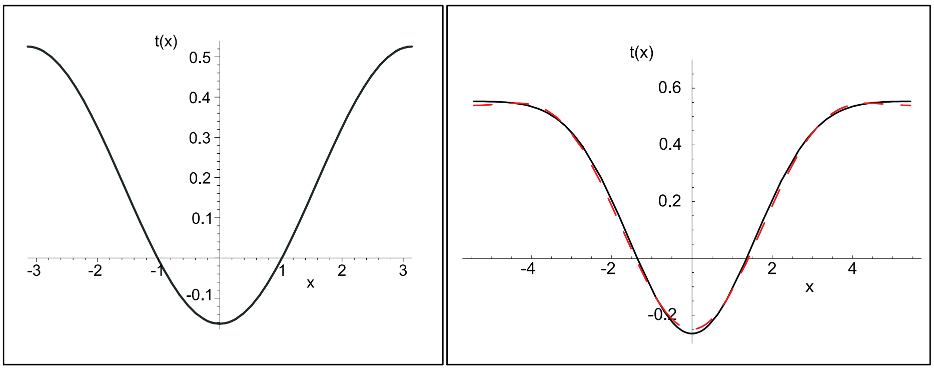

Figure 4: Plot of $t(x)$ for $R = \sqrt{3}$ on different levels. On the left pane the solid line represents $t(x)$ at level $(\frac{1}{3}, \frac{2}{3})$. On the right pane the dashed line shows a plot of $t(x)$ at level $(\frac{7}{3}, \frac{14}{3})$ and solid line shows a plot of $t(x)$ at level $(3, 6)$.

Substitution in (5.54) yields

$$r = \frac{\tau(\text{lump})}{\tau_{\text{D24}}} = R(2\pi^2\mathcal{V}(\Phi) - 2\pi^2\mathcal{V}(\Phi_0)). \tag{5.55}$$

So, now we can compute the value of r for our approximate solution and check how accurately it approaches unity. Since we know that $2\pi^2\mathcal{V}(\Phi_0) = -1$, we can write two expressions for r:

$$r^{(1)} = R\Big[2\pi^2\mathcal{V}^{(M,N)}(\Phi) + 1\Big], \tag{5.56a}$$

$$r^{(2)} = 2\pi^2 R\Big[\mathcal{V}^{(M,N)}(\Phi) - \mathcal{V}^{(M,N)}(\Phi_0)\Big]. \tag{5.56b}$$

By adding more fields one can increase the level and make computations of the solution more and more accurate. Now we present the results of such computations, which were obtained in [143] for radius $R = \sqrt{3}$. On the right pane of Figure 4 we present a plot of $t(x)$ at levels $(\frac{7}{3}, \frac{14}{3})$ and $(3, 6)$. In the Table below one can find the values of $r^{(1)}$ and $r^{(2)}$ depending on the level.

Level	$r^{(1)}$	$r^{(2)}$	Level	$r^{(1)}$	$r^{(2)}$
$(\frac{1}{3}, \frac{2}{3})$	1.32002	0.77377	$(\frac{7}{3}, \frac{14}{3})$	1.07358	0.984467
$(\frac{4}{3}, \frac{8}{3})$	1.25373	0.707471	$(3, 6)$	1.06421	0.993855
$(2, 4)$	1.11278	1.02368			

So, it seems that $r^{(1)}$ and $r^{(2)}$ converge to unity from top and bottom correspondingly.

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